Logic, Epistemology, and the Unity of Science 31

Maria van der Schaar *Editor*

Judgement and the Epistemic Foundation of Logic

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VOLUME 31

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Preface

In September 2009, the authors of this book came together in Leiden for a workshop called *Days of Judgement*. The majority of the chapters presented here are based upon a selection of the talks held at the Leiden workshop. Right from the start, the idea was to show the importance of the *history* of the notion of judgement for philosophy today. As one may learn from Wayne Martin's book *Theories of Judgment*, Cambridge University Press, 2006, the field of *judgement* is broad, and one needs to give a direction to the topic. The general idea of both the workshop and the book presented here is to take Per Martin-Löf's constructive type theory as a starting point, because the notion of judgement plays a central role there. Our logical system is not only in need of propositions; it also needs judgements in which propositions are asserted to be true and known. According to Martin-Löf, one is entitled to make a judgement if one has a ground for it. It is thus that the notion of judgement is related to the notions of truth, knowledge and ground. It is precisely the relation between these notions that has given a focus to the topic of the book presented here.

The book starts with two chapters that were not part of the workshop. In the first chapter, Martin-Löf gives a clear explanation of the way he understands the notion of judgement, and he relates his position to that of the logical positivists. The first part of the chapter is a reprint from the paper "Verificationism Then and Now", published in *The Foundational Debate* (W. DePauli-Schimanovich et al. (eds.). Dordrecht: Kluwer, 1995, 187–196). Martin-Löf has added to the paper a *postscript*, in which he makes an amendment to the paper. Göran Sundholm was asked to write an afterword to his paper "Constructions, Proofs and the Meaning of the Logical Constants", which appeared in 1983 in the *Journal of Philosophical Logic* (volume 11: 151–172). In this afterword, the second chapter here, Sundholm gives an overview of the history of constructive type theory of the last 30 years, focusing on the notions of construction, demonstration and judgement and the ambiguities in these notions. This afterword may help the reader to find the important literature that appeared on the notion of judgement within constructive type theory.

May, 2012

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Introduction

Maria van der Schaar

Assertion is a much discussed topic in philosophy today. Interest in the speech act of assertion seems to be new in philosophy. Assertion, though, is the linguistic counterpart to the notion of judgement, which has been a central notion in the history of philosophy and logic. As a first explanation of the notion of judgement, one may take Frege's understanding of it: acknowledging the truth of a thought or proposition. The aim of this book is (1) to give a historical introduction to the notion of judgement, in such a way that it becomes clear how the traditional theory of judgement relates to modern discussions on assertion, and (2) to understand how traditional theories of judgement can be used to give an epistemic foundation for logic. The aim of the introduction is to show how the notion of judgement could have disappeared from logic and how the notion can be brought back in order to give an epistemic foundation of logic.

Three moments in the history of logic have made it possible that the notion of judgement could disappear from modern logic. First, Bolzano in his *Wissenschaftslehre* (1837) accounts for the truth and falsity of judgements in terms of the truth and falsity of objective, Platonic propositions. The truth-bearer is no longer conceived as a product of an act of judgement, but as something that is independent of a judging and thinking mind. This is an important step away from term logic in the direction of propositional logic, but it comes with a price. For, how do we have epistemic access to these objective propositions that are supposed to be the contents of our judgements? Furthermore, Bolzano explains the validity of inferences in terms of relations of consequence between objective propositions. A non-epistemic foundation of logic is thus proposed, in which the act of judgement plays only a secondary role. Finally, Bolzano's explanation of validity and analyticity in terms of semantic variation has been understood as replacing the explanation of these notions in terms of the

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epistemic notion of containment, as we can still find it in Kant (see Sundholm's paper on analyticity). The knowing subject and the act of judgement no longer play a role in the explanation of fundamental logical notions. This is an important step in the direction of modern logic and the modern concept of analyticity as truth come what may, but again it has another side to it. For, we need both the purely formal and the epistemic notion in order to explain how an act of inference may bring us from known premises to new judgements known and to explain that an analytic truth may be known.

Second, Hilbert's idea that formal systems are objects of study for metamathematical research and his idea that meanings can arbitrarily be given to formal systems and their axioms have had a great influence on the model-theoretical tradition. On Hilbert's account, as soon as an axiom system is consistent, it specifies a class of models. For Hilbert, the question whether the axioms are judgements made or whether they are true does not arise. In this sense, his idea of axiom radically differs from Frege's. Whereas for Hilbert, the proof of the consistency of a set of axioms is essential, for Frege, the consistency of the axioms follows from the fact that these are true: a consistency proof is thus not required. In answer to Brouwer, Hilbert developed a finitary point of view of mathematics in the 1920s. There is an epistemic privileged part of mathematics, which relies only on a purely intuitive basis of signs. But, Gödel's incompleteness theorems showed that a finitary consistency proof of Hilbert's system cannot be given. Although the discussion whether mathematics can be given an epistemic ground on Hilbertian terms has not ended, one can safely conclude that the model-theoretical tradition that originated with Hilbert excludes epistemology from logic. The modern notion of axiom as a non-epistemic starting point of a system has replaced the traditional concept of axiom as judgement made whose truth can neither be proved, nor is in need of proof, because its truth is understood upon apprehension of the concepts involved. And the idea of a formal structure that can be given different interpretations, enables one to speak about different models in which a certain sentence or proposition is true or false. The notion of truth in a model is completely unrelated to the notions of judgement and knowledge and is therefore not the kind of truth Frege was speaking of. Hilbert's metamathematical approach also gave rise to the idea that language is not a universal medium, as Frege held, but is rather a calculus, to which an endless variety of interpretations can be given.

Third, the logical positivists were strongly influenced by Hilbert's early conception of mathematics. In the *Logical Syntax of Language* (1934), Carnap conceives of the language of logic as a calculus (§ 2, § 46): a symbol or expression in logic does not have any meaning, and the notion of judgement that still played a central role in Frege's *Begriffsschrift* is now replaced by that of a sentence without meaning.[1]

[1] "But the development of logic during the past 10 years has shown clearly that it can only be studied with any degree of accuracy when it is based, not on judgments (thoughts, or the content of thoughts) but rather on linguistic expressions, of which sentences are the most important, because only for them is it possible to lay down sharply defined rules" (Carnap 1937, 1).

The idea of the priority of syntax, the Hilbertian concept of axioms and Carnap's aim to solve the conflict with respect to the foundations of mathematics gave rise to Carnap's famous principle of tolerance, relating to mathematics and logic: "let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols." (Carnap 1937, xv). The criterion what mathematics or logic one will use in a certain situation is for Carnap a pragmatic one; for, justification can be given only within a system. "The first attempts to cast the ship of logic off from the *terra firma* of the classical forms were certainly bold ones, considered from the historical point of view. But they were hampered by the striving after 'correctness'. Now, however, that impediment has been overcome, and before us lies the boundless ocean of unlimited possibilities." (Carnap 1937, xv).

Furthermore, early logical empiricists, such as Moritz Schlick, were critical of the Kantian use of pure intuition in mathematics and of the notion of immediate evidence that is traditionally conceived as the characterizing mark of foundational truths. The general idea of the logical positivists that no epistemic foundation of logic can be given, made it possible that logic and epistemology became two separated fields. The basic notion in logic became either the sentence or the objective proposition, rather than the notion of judgement, because the traditional notion of judgement was thought to be infected with psychological elements. In the logical empiricist's conception of formal system, there is no place for the notion of judgement, that is, for the acknowledgement of the truth of a proposition or sentence.

As a result of these three moments in the history of logic, logic is now considered as either having no foundation at all, or as founded on a Platonic realm of propositions. In both cases, the notion of judgement and the epistemic agent have disappeared from logic. On such an account, a logical system cannot be used to provide proofs for a judging agent, either because the system concerns nothing but relations between abstract notions in a Platonic realm, whose accessibility by a judging agent is not understood, or because there is only an arbitrary relation between the formulas and the interpretation given. An important question that arises for logic today is therefore how to relate logic to epistemology. One way in which logic and epistemology can be related is by giving the notion of assertion or judgement a proper place in logic: axioms and theorems are judgements and assertions made.

Recent discussions of the notion of assertion show two different ways in which the notion of assertion can be explained. The condition under which one is entitled to make an assertion may either be understood in epistemic or in non-epistemic terms. With respect to judgement, one may defend either an epistemic account of judgement, or a non-epistemic account. On a non-epistemic account, one is entitled to judge, precisely if the thought contained is true, where truth is explained in a non-epistemic way. On an epistemic account of judgement, one is, for example, entitled to judge precisely if one has a ground or reason for one's judgement. It is precisely such an epistemic understanding of the notion of judgement that is needed when we rethink the relation between logic and epistemology. What may count as a ground for a judgement is determined by the judgemental content, or, if one prefers, by the meaning of the sentence by means of which the judgement is made manifest.

There is thus an internal relation between a judgement and its possible ground. In order to understand this relation between judgement and ground, this volume will investigate the rationalist tradition up to Kant, as we will see below.

Such an epistemic notion of judgement can already be found in Brouwer and his pupil Arend Heyting. In his paper on the reliability of logical principles from 1908, Brouwer explains that the law of excluded middle demands that a thesis is either correct or incorrect. For mathematics, this amounts to the thesis that we can either give a construction such that we are entitled to affirm the thesis or we can show that the thesis leads to absurdity, which means that we are entitled to the denial of the thesis. Such a strict epistemic demand on affirmation and denial shows that the law of excluded middle does not hold for mathematics insofar as it is concerned with infinite totalities. For Brouwer, the question of the validity of the law of excluded middle is directly related to Hilbert's thesis that there exist no unsolvable problems in mathematics. "It follows that the question of the validity of the *principium tertii exclusi* is equivalent to the question whether unsolvable mathematical problems can exist. There is not a shred of proof for the conviction, which has sometimes been put forward, that there exist no unsolvable mathematical problems" (Brouwer 1908, 156; 1975, 109). Brouwer should have been sensitive here, as he was elsewhere, to the distinction between the thesis that every mathematical problem is solvable, which is equivalent to the law of excluded middle, and the weaker claim that there are no unsolvable mathematical problems, which is correct on an intuitionistic account.[2] Per Martin-Löf will take up this topic in the first paper of this volume, where it is discussed in relation to Schlick's thesis that there are no unanswerable questions.

In a paper by Heyting from 1930, we find an explanation of the intuitionistic notion of the meaning of a sentence, and of the notion of assertion: "A proposition [declarative sentence] *p* like, for example, 'Euler's constant is rational', expresses a problem, or better yet, a certain expectation (that of finding two integers *a* and *b* such that $C = a/b$), which can be fulfilled [réalisée] or disappointed [déçue]" (Heyting 1930, 307). Heyting's explanation of the intuitionistic notion of assertion or judgement is clearly epistemic: "To satisfy the intuitionistic demands, the assertion must be the observation of an empirical fact, that is, of the realization of the expectation expressed by the proposition *p*. Here, then, is the Brouwerian assertion of *p*: *It is known how to prove p* [by construction]" (idem). Martin-Löf's explanation of judgement, presented in the paper below, essentially captures this notion of assertion and judgement.

We are in need of a logic that reconsiders its relation to epistemology by means of an epistemic notion of judgement. Per Martin-Löf has developed a new conception of logic, in which the concepts of knowledge and judgement are brought in right from the start. For him, logic is a demonstrative science, in which the epistemic act

[2] In the Brouwer archive, one may find a note from the writings for the 1907 dissertation: "Can one ever demonstrate of a question, that it can never be decided? No, because one would have to do so by *reductio ad absurdum*. So one would have to say: assume that the proposition has been decided in the sentence *a*, and from that deduce a contradiction. But then it would have been proved that not *a* is true, and the question *is* decided" (Van Dalen 2001, 174 note a, my translation).

of judgement plays a fundamental role. Acts of inference, which are acts of judgement based upon already known judgements, result in knowledge of theorems. And the axioms result from acts of immediate insight, which are also acts of judgement. Thus, going back to the conception of axioms before Hilbert, and Carnap's *Logical Syntax of Language*.

The papers can be understood as presenting a historical context for the notions that play a central role in Martin-Löf's logic, such as the notion of judgement, judgemental content, analyticity, the *a priori*, sufficient ground and the assertion sign. The epistemic conception of logic in constructive type theory goes back to Husserl's phenomenology, on the one hand, and to Frege's idea of logic, on the other hand. Husserl's elucidation of a cognitive act in terms of intention and fulfilment, in the sixth of the *Logical Investigations*, has been of importance for Heyting's explanation of a proposition as an expectation that can be fulfilled or disappointed and for the epistemic notion of judgement that he gives. In Frege's *Begriffsschrift*, the judgement stroke is not only a sign that the judgemental content is judged to be true, it is also a sign that the content is true and known. For Frege, axioms are pieces of knowledge and the logical truths that can be derived from them give the most general kind of knowledge of the world. All axioms and theorems in the *Begriffsschrift* are thus preceded by the judgement stroke. Logic, for Frege, is not a mere calculus; it is a language with meaning, and the most universal science there is. In Frege's writings, the notion of judgement has a fundamental role in logic without making logic a psychological enterprise, although a subjective or personal element does play a role in the *Begriffsschrift*. Because the judgemental stroke precedes only what Frege has shown to be true and known, the axioms and theorems in the *Begriffsschrift* are the judgements made by Frege. One can also make the point in a less subjective way. The judgements made in the ideal *Begriffsschrift* are those made by an ideal judger. The judgement stroke plays an important role in the early *Begriffsschrift*, but seems to lose its importance on the more Platonic conception of logic that we find in Frege's later writings. Nevertheless, judgement plays a role in Frege's later conception of logic insofar as he asserts that the assertive force contains the clearest indication of the essence of logic.[3]

No doubt we cannot simply go back to Frege's idea of logic because his logicist project is inconsistent and because his later theory of judgement presupposes a Platonism with respect to judgemental contents. We are thus in need of a new conception of judgement that is to play a role in a theory of inference. Such a notion can also be used in mathematics and in science in general. For, science does not consist of sentences or abstract propositions but of assertions and judgements made.

Knowledge of Kant's theory of judgement and of the Kantian tradition makes it possible to understand how the notions of judgement and judging agent can be used in logic and philosophy in general without reducing logic and philosophy to psychology.

[3] "Now the thing that indicates most clearly the essence of logic is the assertoric force with which a sentence [the German has 'ein Gedanke'] is uttered [the German has 'ausgesprochen', which can also be translated as 'expressed']" (Frege 1915, 252).

Ground or reason is a central notion in the Kantian account of judgement, and it is thus that the rationalist tradition and its notion of sufficient ground become relevant for the notion of judgement. The notion of judgement that needs to be brought back to logic stands in a long tradition, in which the notion of judgement is essentially related to the notion of ground or reason, and the rationalist tradition may therefore play an inspiring role in bringing back the notion of judgement to logic.

In Part I, the stage is set by Per Martin-Löf and Göran Sundholm. Martin-Löf's paper on the verification principle explains what a judgement is from a constructivist point of view. The meaning of a judgement is fixed by laying down what it is that you must know in order to have the right to make the judgement in question. Starting with one of the basic judgemental forms *A is true*, where *A* is a proposition, we can say that *A* is true if there exists a verification of *A*, that is, if a proof of *A* has been constructed. We thus have obtained a verification principle of truth. We can see now in what sense both the idea of a judging agent and that of an objective reason or ground play a central role in Martin-Löf's theory. What one has to know in order to be entitled to make the judgement is a ground for the judgement. On the one hand, what counts as a ground is given by the explanation of the judgement in question, or, if one prefers, it is given by the meaning of the sentence that one uses to make the judgement. The ground for the judgement, and thereby the judgement itself, is in this sense objective. On the other hand, the ground has to be known to the person who makes the judgement. To put the point in terms of proofs and propositions, the judging agent needs to construct a proof for the relevant proposition, or to understand that something counts as a proof for the proposition, in order to be entitled to judge that the proposition *A* is true. In this sense, the notion of judging agent cannot be neglected. Because there is thus a strict conception of judgement and assertion, the law of excluded middle in its positive formulation does not hold. Instead, a negative formulation of the law can be defended. The constructivist's thesis that there are no propositions of which neither the truth nor the falsity can be known leaves open the possibility that there are many propositions which we do not know how to decide whether they are true or false. To put it in terms of the solvability of problems: although we can say that there is no proposition that we know to be undecidable, we are not allowed to assert that every proposition is decidable. We are thus not entitled to assert that every question can in principle be answered, and it is in this sense that the constructivist's position differs from that of a logical positivist such as Schlick, who says: "Whenever there is a meaningful problem, one can, in theory, always show the way that leads to its solution."[4]

Martin-Löf also gives a constructivist interpretation for the logical positivist's thesis that the meaning of a proposition is the method of its verification, that is, of the verification principle of meaning. For a constructivist, the meaning of a proposition is its method*s* of verification. And a method of verification can be understood as a proof for a proposition. The constructivist explanation of a proposition in terms of

[4] "Wo immer ein sinnvolles Problem vorliegt, kann man theoretisch stets auch den Weg angeben, der zu seiner Auflösung führt" (Schlick 1930, 7).

proofs thus expresses a verification principle of meaning. Martin-Löf's paper is a reprint from 1995, and an amendment is added for this occasion as postscript.

Sundholm's afterword to the paper on the explanation of the logical constants in terms of constructions and proofs gives a history of constructive type theory since the paper appeared in 1983.[5] The afterword may serve as a guideline to the central notions in constructive type theory, and indicates where the reader can find the relevant literature. The paper from 1983 gives an analysis of Arend Heyting's distinction between assertion (*Behauptung*) and proposition (*Aussage*). Whereas a proposition expresses a certain expectation or intention, the assertion of the proposition signifies the fulfilment of the intention by a certain construction. Heyting is thus already making the distinction between judgement and proposition, which is so essential to an epistemic conception of logic. Not unimportant for a wider understanding of the constructivist project, Heyting uses the distinction between intention and fulfilment that Husserl introduced in his elucidation of the cognitive act in the sixth *Logical Investigation*.

Sundholm's paper on analyticity shows how the concept of analyticity is fundamental to logic and that the two ways in which analyticity may be explained – as epistemic containment and as variation – have determined two developments in logic from Aristotle on. This means that right from the start, one can find both an epistemic and a purely formal conception of logic and that the former account needs a place besides the latter.

Part II deals with Descartes and Spinoza. In the seventeenth century, the epistemic account of logic and judgement is developed within a rationalist conception of knowledge and truth. In Descartes, judgement in accordance with certain epistemic rules is the key to science (as we see in the paper by Elodie Cassan). And the principle of sufficient reason plays an important role in the development of a rationalist conception of judgement in Spinoza (as Michael Della Rocca shows). Spinoza's idea that there is an essential connection between reason and judgement has had a great influence on Leibniz, and thereby on philosophers and logicians that were influenced by Leibniz.

In Part III, Wolff is the mediating figure between the rationalist and the Kantian approach to judgement. For Wolff, a logical analysis of judgements into condition and statement is central to his account of judgement. The condition of the judgement is to be understood as the (sufficient) ground for its truth. Such a ground of the judgement provides a demonstration for the judgement in question and thereby a possibility that it can be known. The younger Kant broadens this notion of condition in such a way that it does not consist in a sufficient reason for the judgement but in the epistemic source of sensibility and understanding of the concepts united in the judgement. Kant is thereby able to use the logical analysis of judgement for his criticism of rationalist metaphysics, while changing the rationalist and metaphysical notion of sufficient ground into an epistemic and logical notion (see the paper by Johan Blok).

[5] The reader is kindly invited to read the paper 'Constructions, Proofs and the Meaning of the Logical Constants', *Journal of Philosophical Logic* (volume 11, 1983: 151–172).

In the nineteenth century, we see two developments – on the one hand, a line developing Kant's conception of judgement. The neo-Kantian Windelband transforms the Kantian thesis that judgement is at the same time a logical and an epistemic notion by claiming that judgement is an epistemic assessment ('Beurteilung'; see the paper by Arnaud Dewalque) and thereby not a value-free process. Judgement is understood as assessing the truth-value of a propositional content. On the other hand, the rationalist metaphysics of Spinoza, Leibniz and Wolff is transformed by Bolzano in a rather anti-Kantian way. Whereas Kant explains the epistemic distinction between *a priori* and *a posteriori* judgements in epistemic terms, Bolzano explains this distinction in terms of non-epistemic, semantic properties of propositions (see the paper by Stefan Roski). In a similar way, Bolzano explains logical validity in terms of the possibility of semantic variation in objective propositions, thereby making the judging subject irrelevant to the objectivity of logic. It is the tension between these two lines of thinking about the notion of judgement, the Kantian and the Bolzanian one, which determines the account of judgement and logic given by Husserl, Frege and Russell.

In Part IV, we see the influence of Bolzano, Lotze and the neo-Kantians on Husserl, Frege and Russell. They give an account of the relation between the act of judgement, the propositional content and the object of judgement, in such a way that an objective foundation of logic can be given without neglecting the judging and knowing subject (for Husserl, this is shown in the paper by Robin Rollinger). Jeremy Kelly's account of judgemental force in Frege and early Russell shows that this notion can be interpreted in non-psychological, logical terms. The syntactical difference between the finite form of the verb and the participial form, so essential to the distinction between asserted and unasserted propositions in Russell's *Principles of Mathematics*, is thereby given a logical interpretation, thus allowing for the objectivity of logic without losing sight of the idea of judgemental force.

We thus see that one does not have to go back far in the history of logic to understand how the notion of judgement can play a role in logic. By going farther back into the rationalist and Kantian tradition, it is also possible to understand from which broader backgrounds these theories have emerged, while both traditions on their own are still of value for the development of a notion of judgement and assertion in which the notion of ground or reason plays a central role.

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Part I
Constructivism, Judgement and Reason

Chapter 1
Verificationism Then and Now

Per Martin-Löf

The term verificationism is used in two different ways: the first is in relation to the verification principle of meaning, which we usually and rightly associate with the logical empiricists, although, as we now know, it derives in reality from Wittgenstein, and the second is in relation to the theory of meaning for intuitionistic logic that has been developed, beginning of course with Brouwer, Heyting and Kolmogorov in the twenties and early thirties but in much more detail lately, particularly in connection with intuitionistic type theory. It is therefore very natural to ask how these two forms of verificationism are related to one another: was the verificationism that we had in the thirties a kind of forerunner of what we have now, or was it something entirely different? I would like to discuss this question by considering a very particular problem, which was at the heart of Schlick's interests, namely, the problem whether there might exist undecidable propositions or, if you prefer, unsolvable problems or unanswerable questions: it is merely a matter of wording which of these terms you choose. As I said, it is a problem which was at the heart of Schlick's interests: it is explicitly discussed already in his early, programmatic paper *Die Wende der Philosophie* in the first volume of *Erkenntnis* from 1930, and there is a short later paper, which has precisely *Unanswerable Questions?* as its title, from 1935, and he discussed it on several occasions in between also.

So what is the problem? Well, simply this: is it conceivable that some propositions, or some problems, may be such that they just cannot be decided, or cannot be settled, that is, is it conceivable that a proposition may be such that it can neither be proved nor be disproved, or, what amounts to the same, that it can neither be known to be true nor be known to be false? To be very specific, is it, for instance, conceivable that $x^n + y^n$ is in reality different from z^n for arbitrary natural numbers x, y and z when

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n is greater than 2, but that we are somehow blocked from knowing it, so that all our attempts at trying to prove this will be in vain, or is that conceptually excluded, which is to say that, if this cannot be known to be so, then it is actually false? Or, to vary the example, is it conceivable that there are as a matter of fact infinitely many twin primes, although we cannot prove it by any means, or is that conceptually excluded? Now it is clear from the outset that this is a question about the proper conceptual connections between the notions in terms of which it is formulated, and these are roughly the notions of

proposition,
truth,
falsity,
knowledge,
possibility.

The last of these, the notion of possibility, enters in the guise of the verb can in the formulation of the question: might there exist propositions which *can* neither be known to be true nor be known to be false? So it is a question about the conceptual connections between these few notions, about half a dozen notions, and hence, a definite answer to this question cannot he given unless we decide upon a sufficiently precise interpretation of these notions. The interpretation that I shall develop in the following is the intuitionistic interpretation, and I want to show that, on this interpretation, the question can be definitely answered in the negative.

Now, before the notions of proposition, truth and falsity, basic as they are, there comes in the order of conceptual priority an even more basic notion, namely, the notion of judgement. Indeed, the three first notions on our list, proposition, truth and falsity, are associated with the three forms of judgement

A is a proposition,
A is true,
A is false,

of which the first is used to say that something is a proposition, the second to hold a proposition true and the third to hold a proposition false. Of course, there are many other forms of judgement, for instance, forms of hypothetical and general judgement, and even more elaborate forms of judgement in type theory, but in this talk I will only need to consider these three. So, to our list of notions that need to be clarified, we have to add the notion of judgement.

What is a judgement? Well, the notion of judgement is an essentially epistemic notion, which means that it is connected with the notion of knowledge, the fourth on our list of notions to be clarified, and I think that the most natural explanation is to say that the meaning of a judgement is fixed by laying down what it is that you must know in order to have the right to make the judgement in question. Or, in another formulation, which is the same in substance, though, a judgement is defined simply by what knowledge it embodies: a judgement is a piece of knowledge, and you have to clarify what knowledge.

Now connected with the notion of judgement is the notion of evidence: just as the notion of proposition is coupled with the notion of truth, the notion of judgement is coupled with the notion of being evident, and they are related in the following way: a judgement is evident if it has been known or demonstrated or justified or warranted. There are many terminological possibilities here, but, although there may be different shades of meaning between these in natural language, it does not matter which of these terms I choose in the logical analysis, because it is solely the structure into which they fit which is important, and the structure is one and the same irrespectively of whether I choose to express myself using one or the other of these terms. After all, with the possible exception of known, they are all metaphorical in nature: just as demonstrated is connected with shown, evident is connected with seen, whereas justified and warranted, which was Dewey's preferred term, both seem to be of legal origin.

So much for the notion of evidence of a judgement, but we also have the notion of truth of a judgement. However, since we also have the more well-known notion of truth of a proposition, it is sometimes wise, and quite common, to try to use a different word together with judgements, and the natural choice then is to use correctness, or objective correctness, in connection with judgements. Now what is the connection between the notions of evidence and truth for judgements? Well, simply this: a judgement is true or correct, by definition, if it *can* be made evident. So true or correct for judgements means evidenceable or knowable or demonstrable or justifiable or warrantable: you may choose whichever formulation you prefer here. This analysis of the notion of truth of a judgement in terms of the notions of evidence and possibility validates the Cartesian criterion of truth, which says that, if a judgement is evident, then it is true, in the classical formulation, *quod clare et distincte percipio verum est*, what I clearly and distinctly perceive is true, true in the sense of correct. Indeed, that principle becomes a consequence of my explanation of the notion of truth of a judgement and an even more basic principle, namely, the principle that the scholastics formulated as *ab esse ad posse valet consequentia (illatio).* And why? Because evident means actually known and true means knowable, that is, possibly known, and hence, by the principle that, if something is actual, then it is possible, the Cartesian criterion follows: it becomes simply an instance of the *ab esse ad posse* principle. This will have to be enough about the notions of judgement, evidence and truth of a judgement, so that I can pass on to the notions of proposition and truth.

What is a proposition? Once this question is posed, you see immediately the connection with the general explanation of the notion of judgement that I have just given, and why that explanation had to come first. As I said, a judgement is defined by laying down what it is that you must know in order to have the right to make it, and to ask: what is a proposition? is precisely to ask what you must know in order to have the right to make a judgement of the form A is a proposition, or, equivalently, what knowledge is embodied in a judgement of this form. And here I am going directly to the intuitionistic explanation of the notion of proposition, although we know that it is a relatively late one. So recall the explanations of the meanings of the logical constants, the connectives and the quantifiers, given by Brouwer, Heyting and Kolmogorov: they all follow the common pattern that, whatever the logical constant

may be, an explanation is given of what a proof of a proposition formed by means of that logical constant looks like, that is, what is the form, and, more precisely, canonical or direct form, of a proof of a proposition which has that specific logical constant as its outermost sign. It is clear from this what ought to be the general explanation of what a proposition is, namely, that a proposition is defined by stipulating how its proofs, more precisely, canonical or direct proofs, are formed. And, if we take the rules by means of which the canonical proofs are formed to be the introduction rules, I mean, if we call those rules introduction rules as Gentzen did, then his suggestion that the logical constants are defined by their introduction rules is entirely correct, so we may rightly say that a proposition is defined by its introduction rules.

Now what I would like to point out is that this is an explanation which could just as well be identified with the verification principle, provided that it is suitably interpreted. Remember first of all what the verification principle says, namely, that the meaning of a proposition is the method of its verification. The trouble with that principle, considered as a formula, or as a slogan, is that it admits of several different interpretations, so that there arises the question: how is it to be interpreted? Actually, there are at least three natural interpretations of it. On the first of these, the means of verifying a proposition are simply identified with the introduction rules for it, and there is then nothing objectionable about Wittgenstein's formula, provided that we either, as I just did, replace method by means, which is already plural in form, or else make a change in it from the singular to the plural number: the meaning of a proposition is the method*s* of its verification. Interpreted in this way, it simply coincides with the intuitionistic explanation of what a proposition is, or, if you prefer, the Gentzen version of it in terms of introduction rules. For instance, using this manner of speaking, there are two methods of verifying a disjunctive proposition, namely, the two rules of disjunction introduction, and absurdity is defined by stipulating that it admits of no method of verification.

A second interpretation of the term method of verification, perhaps the most natural one, is to use it as a synonym for proof of a proposition, because what is a proof of a proposition on the intuitionistic conception? Well, in general, it need not be in canonical form, that is, it need not have one of the forms displayed in the meaning explanation of the proposition in question, but a proof in general is at least a method which, when it is executed, yields a canonical proof of the proposition as result, so it is very natural to call a proof a method of verification, more precisely, a method of direct, or canonical, verification. But, of course, we are then using the term "method of verification" in a sense which is entirely different from the first one, and which is in conflict with the verification principle.

Now, as a matter of fact, it is in neither of these two senses that the term method of verification was used by Schlick and the Vienna Circle: rather, for them, method of verification meant method of empirical veri- or falsification, that is, method of testing by observation whether the proposition is true or false. So a method of verification was for them simply a decision method, where in addition it is required that the decision, or testing, is to be on empirical grounds. However, in the case of pure mathematics, it is excluded that it could be an empirical testing, so, if we remove that empiricist element, which was absent, by the way, from Wittgenstein's

own discussions of the verification principle, what remains is the idea that a method of verification is a method of veri- or falsifying the proposition, that is, a method of deciding whether it is true or false, and such a method is for the intuitionist the same as a proof of $A \vee \neg A$, where A is the proposition in question. Indeed, a proof of $A \vee \neg A$ is a method which, when executed, yields a canonical proof of $A \vee \neg A$ as result, and, by the definition of disjunction, a canonical proof of $A \vee \neg A$ consists either of a proof of A, together with the information that it is a proof of the left disjunct, or of a proof of $\neg A$, together with the information that it is a proof of the right disjunct, so that we can read off which of the two alternatives is the case. To sum up, the outcome of this discussion of the verification principle is that, on the first of the three interpretations that we have considered, the verification principle of meaning is fine as a formulation of what a proposition is on the intuitionistic conception, but that is *not* the interpretation that was actually given to it by the logical empiricists.

Correlated with the verification principle of meaning is the verification principle of truth, which explains what it means for a proposition in the sense that has just been made precise to be true, and the explanation is now very simple, namely, that A is true is taken to mean that there exists a proof of A, a proof which need not necessarily be direct or canonical. The term proof is of course synonymous with verification here. This definition of the notion of truth of a proposition reduces it to two notions, namely, the notion of proof or verification and the notion of existence, and it is because of this that it is very natural to use the term verificationism in connection with the theory of meaning for intuitionistic logic: the term verification is used to stress the fact that the notion of truth is not taken as a primitive notion, like in a truth conditional theory of meaning, but is rather defined in terms of an underlying notion of verification by the principle that A is true if there exists a verification of A. Now, if A is a proposition, then we know of course what a proof of A is, because a proposition is defined precisely by stipulating how its proofs are formed, so we cannot know a proposition without knowing what a proof of the proposition is, but there remains the question how the notion of existence here is to be understood. Normally, we take the notion of existence to be expressed by means of the existential quantifier, and we have a careful explanation of what the existential quantifier means, but it is very clear that the notion of existence as it enters here cannot possibly be expressed by means of the existential quantifier, so we have to give a direct explanation of what we mean by existence here. According to the general explanation of what a judgement is, this means that we have to lay down what it is that you must know in order to have the right to judge that A is true, that is, that there exists a proof of A, and the intuitionist explanation is that to know that there exists a proof of A is to have constructed, or found, a proof of A, that is, to have a proof of A in your possession.

Let me now pass on to the notion of falsity. It has an explanation which is entirely analogous to that of the notion of truth: a proposition A is false, by definition, if there exists a disproof, or refutation, of A. Now I need not say anything more about the notion of existence here, because I have already done that in my discussion of the notion of truth, but, instead, it remains to explain the notion of disproof, or refutation, which is a new notion. And here the explanation is the following: a disproof of a proposition A is a hypothetical proof of absurdity from A. This definition

of the notion of disproof presupposes that, among our propositions, we have the special proposition, called absurdity, which is by definition false. Like any other proposition, its meaning is fixed by giving the introduction rules for it, and in this case there are no introduction rules: in the case of disjunction, we have two introduction rules, but, in the case of absurdity, we have zero introduction rules, so the meaning of absurdity is fixed by stipulating that it has no canonical proof and therefore no proof at all. Now, once we have introduced absurdity, symbolized by $\bot$, we can explain the notion of disproof by saying that a disproof of a proposition A is a hypothetical proof of $\bot$ from A, or, what amounts to the same, a function which takes a proof of A into a proof of $\bot$. So, in type theoretical notation, a disproof f of a proposition A is an object of the function type $(A)\,\bot$.

$$f : (A)\,\bot\,.$$

Of course, this constructive notion of falsity, defined in terms of the notion of disproof, or refutation, goes back to Brouwer: to know that a proposition A is false is to have constructed, or found, a refutation of A, that is, to have a refutation of A in your possession.

Once the notion of falsity has been constructed, there arises the question as to what the formal laws are that govern its use. Actually, there are three such laws, and, formulated in natural deduction style, they read as follows. First of all, in addition to the usual assumption rule, which allows us to assume a given proposition to be true, there is a new assumption rule which allows us to assume a given proposition to be false instead. Second, if we have proved, from the assumption that a proposition A is true, that $\bot$ is true, we may conclude that A is false,

$$\frac{\begin{array}{c}(A\ \text{true})\\ \bot\ \text{true}\end{array}}{A\ \text{false}},$$

and, third, if one and the same proposition A has been demonstrated to be both true and false, we may conclude that $\bot$ is true,

$$\frac{A\ \text{true} \qquad A\ \text{false}}{\bot\ \text{true}}.$$

So these are the three formal laws of falsity, provided now that you introduce the notion of falsity into your object language, which is not common, of course: normally, we express the falsity of A by the truth of $\neg A$. Then the rule of assuming a proposition to be false becomes a special case of the usual rule of assuming a proposition to be true, and the two remaining laws of falsity reduce to the negation laws. Now, from these new rules of falsity, it follows immediately that a proposition A is false if and only if $\neg A$ is true, which is to say that the two rules

$$\frac{A \text{ false}}{\neg A \text{ true}}, \qquad \frac{\neg A \text{ true}}{A \text{ false}}$$

are valid as derived rules, and this is of course why it works to define the falsity of A as the truth of $\neg A$, but, nevertheless, falsity is a notion in its own right, and deserves to be treated as such, even if you can do without it from a purely formal point of view.

Now the notion of knowledge, the fourth on our list of notions to be elucidated, I have already dealt with in connection with the notion of judgement: my discussion of the notion of judgement and the notions of evidence and truth, or correctness, of a judgement was a treatment in brief of the epistemic notions that are needed, whereas the notions of proposition, truth and falsity are non-epistemic in nature. And now there remains on our list only the notion of possibility, which I have already used in defining the notion of truth of a judgement as knowability. Concerning this notion of possibility, I have nothing more to say, except that it is the notion of logical possibility, or possibility in principle, as opposed to real, or practical, possibility, which takes resources and so on into account. It is something that was repeated over and over again by Schlick that, in the verification principle, it is absolutely necessary to understand the –able in verifiable as logically possible, or possible in principle, to verify, and, although I am not adhering to the verification principle as interpreted by Schlick, I am as dependent as he was on the notion of logical possibility, or possibility in principle, so I will allow myself to use it without further ado in this discussion.

Now the ordinary logical laws, the laws of propositional and predicate logic, are properly characterized as laws of truth, laws that allow us to derive consequences, which say that one proposition, the consequent, is true provided certain other propositions, the antecedents, are true. It is therefore very natural to ask, once we have seen the correspondence between the non-epistemic notions and the epistemic ones, in particular, between the notion of truth of a proposition and the notion of truth of a judgement, whether there are some general laws that we can formulate for judgements and their truth, which means knowability as we have seen, and indeed there are three such laws. If the ordinary, object linguistic logical laws are characterized as laws of truth, it is natural to refer to these as metalinguistic laws, or laws of knowability. Now the first of these laws is so trivial that maybe it should not be spelled out as a separate law, but I will do it anyway.

First Law (reflection). *If the premises of a valid inference are knowable, then so is the conclusion.*

The justification is simple: if the premises of a valid inference are knowable, or demonstrable, then it is clearly possible to demonstrate, that is, to get to know, the conclusion by first demonstrating the premises and then applying the very inference that is under consideration, the one that is valid by assumption.

The first law, if we choose to call it a law, allows us to lift every object linguistic rule of inference into a metalinguistic rule of inference. So, instead of saying, in an object linguistic mode: $J_1, \ldots, J_n$, therefore J, we say, in a metalinguistic mode: if $J_1, \ldots,$

J_n are knowable, then J is knowable. For example, the usual rule of conjunction introduction

$$\frac{A \text{ true} \qquad B \text{ true}}{A \,\&\, B \text{ true}},$$

is lifted into the metalinguistic law which says: if two propositions A and B can both be known to be true, then $A\&B$ can be known to be true.

Second Law (absolute consistency). *Absurdity cannot be known to be true*.

In other words, the judgement

$$\bot \text{ true}$$

is unknowable. And how do you see this? Well, as always in the case of an axiom, by reflection on the meanings of the terms involved. Remember how absurdity was defined: like any other proposition, it was defined by its introduction rules, and, in the particular case of absurdity, there are none. This means that there is no canonical proof of absurdity, and, since an arbitrary, possibly noncanonical proof is a method, or program, which yields a canonical proof as result, there is no noncanonical proof either. Hence, it is impossible to know a proof of absurdity, and, by the definition of truth, this amounts to the same as saying that it is impossible to know that absurdity is true.

It is noteworthy that the absolute consistency is more basic even than the law of contradiction, in the sense that the law of contradiction follows as a corollary from it.

Corollary (law of contradiction). *One and the same proposition cannot both be known to be true and be known to be false.*

Put differently, the two judgements A true and A false, which presuppose A to be a proposition, are not both knowable, or correct. To see why, remember that

$$\frac{A \text{ true} \qquad A \text{ false}}{\bot \text{ true}}$$

is a valid rule of inference: it is the second of the two rules of inference associated with the notion of falsity. Hence, by the first law of knowability, if the judgements A true and A false are both knowable, then so is the judgement $\bot$ true. But that is excluded by the second law, so A true and A false cannot both be knowable, or correct, which is precisely what the law of contradiction states.

Now, just as the law of contradiction follows as a corollary from the second law, the answer to the question with which I began this talk will follow as a corollary from the third law of knowability.

Third Law (unknowability of truth entails knowability of falsity). *If a proposition cannot be known to be true, then it can be known to be false*.

Since, as we have seen, the judgement A false, where A is a proposition, is interderivable with the judgement $\neg A$ true, the third law may just as well be rendered: if a

proposition cannot be known to be true, then its negation can be known to be true. What is more, this is the only formulation available if you choose to define falsity in terms of negation and truth rather than to take it as a primitive notion. And how do you convince yourself of the third law? Well, let a proposition, say A, be given, and suppose that the judgement A true is unknowable. By the definition of truth, knowing that A is true amounts to the same as knowing a proof of A. Hence, using type theoretic notation, the assumption that A cannot be known to be true means that the epistemic situation

$$\mathrm{a} : A$$

cannot arise: it is impossible that we arrive at a judgement of this form. Now, from this negative piece of information, I have to get something positive, namely, I have to show that we actually *can* know a refutation of A, and a refutation of A is a hypothetical proof of $\bot$ from A, or, equivalently, a function which takes a proof of A into a proof of $\bot$. The argument is this: we simply introduce a hypothetical proof of $\bot$ from A, call it R. In type theoretical terms, this means that we introduce an object R of the function type $(A)\,\bot$, in symbols,

$$R : (A)\,\bot,$$

and it only remains for us to make this judgement (in fact, axiom) evident. So what does it mean? Well, by the semantical explanation of the function type, it means that $R(\mathrm{a}) : \bot$ provided that $\mathrm{a} : A$, and, moreover, that $R(\mathrm{a}) = R(\mathrm{b}) : \bot$ provided that $\mathrm{a} = \mathrm{b} : A$. Thus, the crucial judgement $R : (A)\,\bot$ may be considered as a licence to infer by the two rules

$$\frac{\mathrm{a} : A}{R(\mathrm{a}) : \bot}, \qquad \frac{\mathrm{a} = \mathrm{b} : A}{R(\mathrm{a}) = R(\mathrm{b}) : \bot},$$

which are both vacuously valid. This is obvious in the case of the first rule, since, by assumption, its premise can never be demonstrated, and it is equally obvious in the case of the second rule, since its premise carries with it the two presuppositions $\mathrm{a} : A$ and $\mathrm{b} : A$, which can never be demonstrated either. So we may safely judge R to be an object of type $(A)\,\bot$, that is, to be a refutation of the proposition A. This finishes the explanation why a proposition which cannot be known to be true, in recompense, can be known to be false. Observe how similar it is to the justification of the rule of absurdity elimination, the rule that was referred to as *ex falso sequitur quodlibet* by the scholastic logicians.

Corollary (law of excluded middle). *There are no propositions which can neither be known to be true nor be known to be false.*

In short, there are no absolutely undecidable propositions. And why does this follow from the third law? Well, suppose that we had a proposition which could neither be known to be true nor be known to be false. Then, in particular, it cannot be known to be true, so, by the third law, it can instead be known to be false. But that contradicts the assumption that the proposition could not be known to be false either. So the answer to the question with which I began this talk – might there exist absolutely

undecidable propositions? – is in the negative, and this is precisely the conclusion, in both senses of the word, that I wanted to reach.

Let me just finish by comparing the preceding treatment with the way in which absolutely undecidable propositions were excluded by Schlick. For him, it was much easier, because a proposition was for him defined by its method of verification, where method of verification was interpreted as method of veri- or falsifying the proposition, that is, as method of deciding whether the proposition is true or false: if it has no clear method of verification, the alleged proposition simply is not a proposition, that is, it is not meaningful. So Schlick's interpretation of the verification principle actually validates the law of excluded middle in its positive formulation, which says that every proposition can either be known to be true or be known to be false, and clearly so: simply execute the method of verification, or decision method, that defines the proposition in question. As concerns the foundations of mathematics, Schlick was most strongly influenced by Hilbert, and at least one source of his interest in the question of unsolvable problems must have been Hilbert's mathematical problems paper from 1900, in which he just states as an axiom, or a conviction, which every mathematician certainly shares, that every mathematical problem can be solved, that is, that every mathematical proposition can either be proved or be disproved. Schlick's way of justifying that axiom was to say that a proposition is defined by its method of verification, that is, by its decision method, and hence, by being a proposition, it is necessarily decidable. Here we see that we have had to go a considerably more roundabout way to reach the weaker conclusion that there are no absolutely undecidable propositions. It is the price that we have had to pay for being able to make sense of quantification over infinite domains, like the domain of the natural numbers. There are many propositions whose meanings we understand perfectly well although we do not known how to decide whether they are true or false.

Postscript, January 2012

As a result of having reread the preceding article after seventeen years, I have become dissatisfied with the treatment of what I called the third law and its corollary, and therefore propose the following amended treatment.

Third Law (unknowability of truth entails falsity). *From the unknowability of the truth of a proposition, its falsity may be inferred.*

Since, as we have seen, the judgement A false is interderivable with the judgement $\neg A$ true, the third law may just as well be rendered: from the unknowability of the truth of a proposition, the truth of its negation may be inferred. What is more, this is the only formulation available if you choose to define falsity in terms of negation and truth rather than to take it as a primitive notion. And how do you convince yourself of the third law? Well, let a proposition, say A, be given, and suppose that the judgement A true is unknowable. By the definition of truth, knowing that A is true amounts to the same as knowing a proof of A. Hence, using type theoretic notation, the assumption that A cannot be known to be true means that the epistemic situation

$$a : A$$

cannot arise: it is impossible that we arrive at a judgement of this form. Now, from this negative piece of information, I have to get something positive, namely, I have to construct a refutation of A, and a refutation of A is a hypothetical proof of $\bot$ from A, or, equivalently, a function which takes a proof of A into a proof of $\bot$. The argument is this: we simply introduce a hypothetical proof of $\bot$ from A, call it R. In type theoretical terms, this means that we introduce an object R of the function type $(A)\,\bot$, in symbols,

$$R : (A)\,\bot,$$

and it only remains for us to make this judgement (in fact, axiom) evident. So what does it mean? Well, by the semantical explanation of the function type, it means that $R(\mathrm{a}) : \bot$ provided that $\mathrm{a} : \mathrm{A}$, and, moreover, that $R(\mathrm{a}) = R(\mathrm{b}) : \bot$ provided that $\mathrm{a} = \mathrm{b} : \mathrm{A}$. Thus, the crucial judgement $R : (A)\,\bot$ may be considered as a licence to infer by the two rules

$$\frac{\mathrm{a} : \mathrm{A}}{R(\mathrm{a}) : \bot}, \qquad \frac{\mathrm{a} = \mathrm{b} : \mathrm{A}}{R(\mathrm{a}) = R(\mathrm{b}) : \bot},$$

which are both vacuously valid. This is obvious in the case of the first rule, since, by assumption, its premise can never be demonstrated, and it is equally obvious in the case of the second rule, since its premise carries with it the two presuppositions $\mathrm{a} : A$ and $\mathrm{b} : A$, which can never be demonstrated either. So we may safely judge R to be an object of type $(A)\,\bot$, that is, to be a refutation of the proposition A. It now only remains to make the inference

$$\frac{R : (A)\,\bot}{A \text{ false}}$$

in order to reach the desired conclusion that A is false. This finishes the explanation why, from the unknowability of the truth of a proposition, we may infer its falsity. Observe how similar it is to the justification of the rule of absurdity elimination, the rule that was referred to as *ex falso (sequitur) quodlibet* by the scholastic logicians.

Corollary (law of excluded middle). *There are no propositions which can neither be known to be true nor be known to be false.*

In short, there are no absolutely undecidable propositions. Whichever way it is formulated, however, the negative existential: there are no …, as it occurs in either of the two formulations, needs careful explanation, since it cannot be expressed by means of an ordinary negated existential quantifier. What the corollary says, in detail, is that it is impossible to give a counterexample to the law of excluded middle in its positive formulation: every proposition can either be known to be true or be known to be false, which Brouwer correctly identified with Hilbert's solvability axiom. Such a counterexample would have to be a proposition for which it had been established that it can neither be known to be true nor be known to be false. What the corollary says is therefore that the epistemic situation

$$A \text{ prop},$$
$$(A \text{ true}) \text{ unknowable},$$
$$(A \text{ false}) \text{ unknowable},$$

cannot arise. To see why, first apply the third law to the first two judgements in order to reach the conclusion A false, from which $\neg\, A$ true follows by the first of the two rules

$$\frac{A \text{ false}}{\neg\, A \text{ true}}, \qquad \frac{\neg A \text{ true}}{A \text{ false}}.$$

Then use the second of these rules to conclude, from the unknowability of A false, that $\neg A$ true is likewise unknowable. A second application of the third law now yields $\neg A$ false. We have thus arrived at both $\neg\, A$ true and $\neg\, A$ false, which is impossible by the law of contradiction. The epistemic situation determined by the three judgements above is hence impossible, which is to say that it is impossible to find a counterexample to the law of excluded middle in its positive formulation, and this is precisely what the law of excluded middle in its negative formulation says: tertium non datur.

Chapter 2
Demonstrations Versus Proofs, Being an Afterword to *Constructions, Proofs, and the Meaning of the Logical Constants*

Göran Sundholm

The spring of 1980 I spent as visiting lecturer at Utrecht. The volume of Heyting's *Collected Papers* had not yet been put together, and his philosophical papers could not be found at Oxford. Accordingly, I availed myself of the opportunities offered by Dutch libraries and read the relevant papers. A couple of years earlier, I had learned about Constructive Type Theory from Per Martin-Löf, and Michael Beeson, who had just written a paper on a theory of constructions, was an eager sparring partner in almost daily discussions at Utrecht. The outcome of these ponderings was the paper on which you are reading now as an afterword. It was ready toward the end of the summer 1981, and my Oxford Professor Dana Scott suggested to me that I should submit it to Richmond Thomason, the editor of the *Journal of Philosophical Logic*, at a meeting of authors for the *Handbook of Philosophical Logic* at Bad Homburg. I did so and the paper was readily accepted; however, a special issue on intuitionism was being prepared, and Thomason suggested that I might want to wait in order to have it appear in that issue. Thus, the paper appeared only in 1983 but had circulated rather widely in the intervening time.

In his seminal account from the Stanford LMPS congress, Kreisel had used a predicate $\Pi(A, c)$ with two argument places, for, respectively, propositions and constructions, with the meaning "construction c is a proof of proposition A" (Kreisel 1962). I summarized the perspective of his "theory of constructions" in four theses:

1. When A is a proposition, $\Pi(A, x)$ is a *decidable* predicate over the "universe of all constructions": $\forall x(\Pi(A, x) \vee \neg \Pi(A, x))$.
2. Proofs of $\supset$ -propositions are constructions that transform constructions proving the implicational antecedent into proofs of the consequent proposition and similarly

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for ∀- propositions. In the Kreisel interpretation, there are "second clauses" for ⊃ and ∀ demanding a second component that serves as proof that the construction from the first clause does what it is supposed to do.
3. When A is a mathematical proposition, then so are Π(A, c), Π (Π(A, c), d)…
4. There is a universe of all constructions.

The use of the Π predicate that ranges over the "vast generality" of all possible intuitionistic proofs goes back to Gödel, as is clear from Volume III of his *Collected Works*. W. W. Tait has pointed out that because of this alleged impredicativity (of quantification over "all constructions"), Gödel considered his *Dialectica* interpretation superior to Heyting's proof-explanation.[1] With the benefit of—what we might call the Curry-Howard—hindsight, we know that this was a mistake on Gödel's part. Kreisel, in conversation at Burres-sur-Yvettes 1978, indicated to me that the decidability of his proof-predicate Π on constructions was motivated entirely by the corresponding situation regarding Gödelized proof-predicates for formal systems: as is well known, they are primitive recursive or even Kalmar elementary. His intention was to use the formal theory of constructions for formal derivability work in connection with his investigations into finitist autonomous progressions. The Gödelized proof-predicates for arithmetical and other theories can then serve as models of the Π-predicate. Thus, in Kreisel's framework, the second clauses for ⊃ and ∀ had the sole task to bring about the required decidability of the Π-predicate also for these connectives and all constructions. In my Utrecht debate with Beeson, this was a recurring topic: one of his main concerns was to *deny* the decidability of the Π-predicate while retaining the second clauses. Since the latter had been introduced by Kreisel solely to guarantee that decidability, I found Beeson's theory lacking proper motivation as well as wanting in simplicity (Beeson 1979).

The main contribution of the *Constructions…* paper, apart from the careful examination of the writings of Heyting and Kreisel, was to note a multiple ambiguity in the term *construction* that I formulated as follows:

(a) Process of construction
(b) Object obtained as the result of a process of construction
(c) Construction-process as object (rather than as something "dynamic")

The "second clauses" then pertain to objectified construction processes rather than to construction objects in the usual sense, and I prefaced the paper with a quotation from Hao Wang that in my opinion well expressed what had happened in Kreisel's theory: the construction objects (b) were conflated with their processes of constructions (a).

[1] Tait (2006, §12). The pitfalls involved in construing the BHK interpretation after the fashion of Kreisel and his (partial) follower Goodman are presented by Scott Weinstein (1983, 264–266), where a threatening paradox and the ensuing need for stratification of the single universe of constructions are clearly spelled out. Of course, today we know that the best way to achieve such stratification is by *typing*.

My purpose in the present postscript is to outline later developments regarding Heyting's "proof-semantics". I wish to stress, as I did already in the *Constructions*… paper, that I find the by now customary terms proof (or BHK, for Brouwer, Heyting, and Kolmogoroff) *interpretation* seriously misleading: instead, I much prefer the term proof-*explanation*. The explanations offered by Heyting do not constitute a mathematical (re-) interpretation like the realizability and Dialectica interpretations: their role is properly *meaning* theoretical.

In the following year, 1984 Diller and Troelstra, when commenting on my *Constructions*… account, coined the felicitous terminology "proof-object" for what I there called "object… of construction". In the same year, Per Martin-Löf's 1980 lectures from Padova appeared as a book on Constructive ("*Intuitionistic*") *Type Theory*. Here, as in the lecture (1982) at the Hannover LMPS VI in 1979, he introduced a fundamental distinction between judgements and propositions, and the Heyting explanations of constructive propositions were given a streamlined formulation that draws upon a distinction between *canonical* and non-canonical proof(−object)s: a non-canonical, indirect proof is a method, or program, that evaluates, or executes, to a canonical one. This "canonical" terminology goes back to Brouwer's talk of *kanonische Beweisführungen* in his demonstration of the bar theorem. It was used by Dummett in his seminal 1973 lecture on "The Philosophical Basis of Intuitionistic Logic" at the Bristol Logic Colloquium. For Dummett, canonical proofs were the idealized formal proofs used in Heyting's explanations. These were placed in a dichotomy against what Dummett called *demonstrations,* that is ordinary, non-formalized proofs in mathematical texts, say.

At this point, I follow Martin-Löf in making both a terminological and conceptual distinction. Canonical/non-canonical is a distinction at the level of *propositions*. Demonstrations, on the other hand, are not of propositions, but of judgements. Judgements are not explained in terms of proof- (or, derivatively, in terms of truth-) conditions but in terms of *assertion* conditions. Traditionally, all proving took place at the level of judgements; theorems are judgements made by an act of demonstration. The idea that propositions have proofs is a novum in the history of logic and mathematics that was introduced by intuitionism.[2]

Secondly, also in 1984, Mulligan, Simons, and Smith published their paper on *Truth-Makers*. However, its significance to the present context was not immediately apparent but became clear only after a decade, when I realized that the constructivist account of truth

proposition A is true = there exists a proof of A

[2] Per Martin-Löf's lectures (1985) that constitute a shorter summary of a full course of lectures given at Siena in 1983 attempted to avoid the use of proof-objects jettisoning type theory in favour of predicate logic instead. However, the experiment did not work, and soon, Martin-Löf returned to his type-theoretical conception.

constitutes a version of the true-maker analysis of truth, and published a paper on this in 1994.[3] Here, it also became clear that the true-maker relation is not a propositional function. In Wittgenstein's Tractarian terminology, it is an *internal* relation. Kreisel's Π-predicate Π(A, c), which is an ordinary, that is, "external", propositional function over Gödel's "vast generality" of all intuitionistic proofs, gets matters wrong here. The type-theoretical c: Proof (A), on the other hand, does exhibit the desired internality.

A little bit earlier, Per Martin-Löf gave a talk at Paris on the epistemology of type theory and suggested the apt term *proof-trace* for what is left of the act when it has been completed, or carried out, for instance, a written record, that also someone else could use for gaining the same item of knowledge, namely, the demonstrated theorem. Other examples of similar traces of acts would be the written recipe of a beautiful dish, the score of a chess game, or the music of a symphony. At this time, Martin-Löf also showed how to apply the notion of analyticity to the judgements of his type theory. My paper *Questions of Proof* from 1993 was an attempt to integrate these conceptual advances that had been made since the first formulation a decade earlier.

With the aid of these conceptual clarifications, the tripartite distinction I drew in *Constructions*… can be rephrased as

(a′) Proof-act
(b′) Proof-object
(c′) Proof-trace

Also, the term *demonstration*, then, is ambiguous between

(a′′′) Demonstration act
(c′′′) Demonstration trace

It is important to distinguish the act and its trace from the product ("object") of the act. In the case of a demonstration, this object is the judgement made, that is, the demonstrated theorem.

Michael Dummett did not avail himself of the above distinction between judgement and proposition (nor, accordingly, of the concomitant one between proof(−objects) and demonstrations). In the absence of the distinction, his identification of non-canonical proof with demonstration lies close at hand. From my perspective, it is misjudged since proofs pertain to propositions, whereas demonstrations demonstrate theorems rather than the propositions to which a theorem ascribes truth according to the constructivist version of the true-maker scheme

A is true = there exists a proof of A.

[3] The *truth-maker* terminology was coined in analogy with truth-*bearer*. However, a proposition is made *true,* and so, *true*-maker might be more felicitous. The participants of the workshop *Truth-Makers and Proof-Objects*, ENS, Paris, 23–25 November 2011, including two of the authors of the original *Truth-Makers* publication, agreed that it was.

Furthermore, this is a general point from the theory of true-making that does not pertain solely to the constructivist true-maker account; the existence involved is not that of the existential quantifier on pain of an ever-descending explanatory regress. The existential quantifier is explained in terms of the truth-condition for the ∃ quantifier and that truth-condition in turn is explained in terms of the existence of a proof for the existentially quantified proposition. If that existence were to be taken in the sense of ∃, a meaning-explanation would never take off. Instead, it is the Brouwer-Weyl constructive judgemental notion of existence that is involved here. When α is a type, then *α exists* is a judgement with assertion conditions

$$\frac{\text{a is an } \alpha}{\alpha \text{ exists.}}$$

Hence, the judgement "A is true" is elliptical and grounded in one of the form "a is a proof of A". Dag Prawitz has been a steadfast opponent of this way of looking at things; in particular, he has resisted the distinction between propositions and judgements and especially the distinction between proof(−object)s and demonstrations. I tried to address our differences in a *Festschrift* contribution for his 60th birthday, but Prawitz remained unconvinced and our debate continues to this day.

During sabbaticals spent at Stockholm in 1994–1995, I read all of Frege as well as the first two volumes of Bolzano's *Wissenschaftslehre*. This preparation formed the background to a detailed investigation of the notion of *inference* during the second half of the 1990s when I wrote a series of seven papers devoted to this topic. I argued that the customary reduction, which I named after Bolzano, of the validity of an inference to the holding of a relation of logical consequence among propositions is unsatisfactory: it confers validity on "*blind*" *(*ungrounded*)* inferences. The notions of proposition and truth are ontological ("alethic") and so are those of consequence and their holding, be it logical or not. Inference and validity, on the other hand, essentially involve judgements, and just like judgement, they are *epistemic* notions.

Crucial in my treatment was the realization that whereas consequence, be it logical or not, involves preservation of truth from antecedent propositions to consequent proposition, regarding inference matters were different. Here, in the validation of inference, one uses not the ordinary natural-deduction notion of assumption (*Voraussetzung*) (that a proposition is true) that is familiar from the work of Gentzen but *epistemic assumptions* (epistemic hypotheses) that the premise(−judgement)s are *known*, and from this, one has to show how to know the conclusion judgement. Complex inferences are validated by linking premises and conclusion by means of chains of immediate inference starting from immediately evident axioms. Inference transmits hypothetical knowledge by means of chains of immediate inferences.

Accordingly, we have to distinguish four closely related notions and their associated kinds of *Geltung*, namely,

The *implication*-proposition A ⊃ B is true.
The *conditional* (judgement) B is true if A is true.
B is true on condition (hypothesis, assumption) that A is true.

The *consequence* $(A \Rightarrow B)$ holds.

The *inference* $\dfrac{A \text{ is true}}{B \text{ is true}}$.

The first demands a proof-object a: Proof($A \supset B$) (that is evaluable to $\supset$-introductory form), whereas the second demands a hypothetical proof:

$$b{:}\mathrm{Proof}(B)(x{:}\mathrm{Proof}(A).$$

The third demands a function:

$$f{:}\mathrm{Proof}(A) \rightarrow \mathrm{Proof}(B).$$

It should be noted that the three notions are different; the assertion conditions of the judgements are not the same. However, all three judgements are equi-assertible, that is, when one is entitled to assert one, it requires little or no effort justifying the other two assertions.

$$\text{The inference } \frac{A \text{ is true}}{B \text{ is true}} \text{ finally}$$

demands for its validity a chain of immediately evident axioms and inferences that link the premise judgement A is true to the conclusion judgement B is true, and that is another matter. However, apart from the above equi-assertibility, another reason why these four notions sometimes have been difficult to keep apart might be the fact that all four are refuted in the same epistemic situation, namely, one in which the judgements *A is true* and *B is false* are both known.

Handbooks have a tendency to be long in the making, or even very long, and that was true for both the *Handbook of Epistemology* and for *The Development of Modern Logic* that appeared in 2004 and 2009, respectively. My contribution to the first was written in 1999 and related constructivist ideas to traditional notions from epistemology, in particular the traditional theories of truth, for which various roles were sought. The correspondence theory, under the guise of the proof version of the true-maker analysis, allows propositions to be defined, whence it plays a semantic role. The evidence theory according to which the true judgement is an evidenceable (justifiable, knowable, warrantable, assertible…) one fitted my conception perfectly, whereas the pragmatic, coherence, and consensus accounts I see as methodological instruments for overall theory-revision in the face of inconsistency or incoherence. I also worked out the links to Kantian analyticity that I had first adumbrated at Kirchberg for the Wittgenstein centenary.[4] The typing relation of Constructive Type Theory is not a propositional function.[5] In Tractarian terminology, it is an internal relation and being of a certain type is an internal property of an object. Both the Tractarian "object a has internal property P" and the type-theoretical "a is an object

[4] I could also draw on Martin-Löf (1994) where the Kantian connection was worked out in some detail.

[5] The computer scientist's legitimate wish for keyboard convenience has replaced Martin-Löf's set-theoretical epsilon with the colon.

of type α" are counterparts to the Kantian analytic judgements where the predicate P famously is "contained" in the subject S.

A Century of Judgment and Inference: 1837–1936 was written in 2001 and dedicated to Per Martin-Löf on the occasion of his 60th birthday. Here, I traced the development of logic from Bolzano to Gentzen, drawing upon the briefer lecture from the Cracow LMPS that was published as (2002). I stressed that Bolzano had a(n almost) perfect account of (logical) consequence among propositions and also that Frege's views on inference are generally right. The uniformly harsh opposition to Frege on inference is unwarranted and might be due to the fact that he is erroneously taken to have talked about (logical) consequence, which notion, however, plays no significant role within Frege's logical system. It is only with the system of Gentzen's 1936 "first consistency" proof that we encounter a framework that is able to cope both with consequence *and* inference. Consequence here occurs in the guise of a natural-deduction sequent $(A_1, A_2, \ldots, A_k) \Rightarrow C$ that lists all assumptions as antecedent propositions to the left of the arrow and with one succedent proposition only. Such systems are sequent calculi in name only: logical inference takes place to the right of the arrow using both introduction and elimination rules. My paper (2006) was written already in 1999 and dealt with the interpretation, from a contentful perspective, of the two different styles of natural-deduction derivations: derivations in the standard format, with (possibly open) assumption-formulae as top leaves in the proof trees, should be seen as notations for proof-*objects,* whereas the derivations in the sequential (1936) format give demonstration(−trace)s of acts of demonstrations that consequence relations hold between propositions.

Finally, a recent more general insight pertaining to the ontology of constructions was noted in my joint paper with Mark van Atten (2008). For a long time, I have been attracted by Fichte's characterization of the two perspectives of epistemology and spoke about it already at the 1991 Uppsala LMPS under the title *Ontologic versus Epistemologic*, in which paper further details can be found. Today, I even wish to characterize my own position as a (1) metaphysical objectivism (I do have a norm of rightness for epistemic acts), (2) ontological Platonism (abstract objects, such as propositions and numbers, are real), and (3) epistemological idealism (ontology does not ground the epistemology).

Thus, I reject *Ontological Descriptivism* with respect to meaning and rightness: the ontology does not provide the norm of rightness for the conceptions of meaning and knowledge. I do not define, or reduce, the epistemological norm of correctness for our judgement(−act)s to the obtaining of states of affairs in our ontology. In this way, I manage to combine a Platonist ontology of abstract entities with epistemological idealism. Such a position is not common, for sure. Berkeley and Brouwer were both ontological *idealists*. Their views of epistemology, on the other hand, can both be seen as *realist*. Questions of meaning and epistemic correctness are reduced to matters of fact in their respective ontologies, albeit that in both cases this ontology is an *idealist* one. Thus, the Brouwerian intuitionist and the Platonist adopt similar stances with respect to the relation epistemology/ontology: in both cases, ontology has the upper hand. For Brouwer, the obtaining of states of affairs among his mental constructions ultimately serves to ground issues of meaning and correctness within intuitionism, just as the obtaining of states of affairs among abstract entities

provides similar grounds in the Platonist case. My constructivism rejects this: the rightness of acts—of meaning and of knowledge—is *sui generis* and is approached, "filled in", via the *error* phenomenon and in particular via the theory-revision required by diagnosed or suspected error.[6]

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[6] The paper "Error", Sundholm (2012), offers further details here.

Chapter 3
Containment and Variation; Two Strands in the Development of Analyticity from Aristotle to Martin-Löf

Göran Sundholm

My original training as a philosopher, at Uppsala and at Oxford, was ruggedly analytical. Also the notion of an analytic judgement, or 'proposition', or 'sentence', or 'statement', (one did not overly distinguish these notions) was repeatedly treated of by excellent teachers and colleagues. There were *aficionados* of Quine and experts on Kant among them, but no names, no pack-drill! If there was one central topic in traditional epistemology on which I felt philosophically at ease, it was that of analyticity. In the early 1980s, I entered for the first time a pluralist philosophical environment in the Philosophy Department of the Catholic University at Nijmegen, with ample representation in phenomenology, Hegelian idealism, and (neo) Thomism. To my considerable surprise, I discovered that it could be enjoyable as well as instructive talking to such rare birds in the philosophical aviary. A colleague drew my attention to Thomas Aquinas' *Five Ways*, which I had never read, having adopted, from the exposition in Anders Wedberg's *History of Philosophy,* the opinion that, like Kant's transcendental deduction, Aquinas' demonstrations were 'worthless'. However, the *Summa Theologica* was readily available on open shelves in the library at Nijmegen, and my curiosity got the better of me. Upon consultation of its second question, my shock was great. In a discussion of whether the judgement *Deus est* admits of demonstration, Aquinas introduces the notion of a *propositio per se nota*, that is, an S is P *judgement known in, or*—perhaps better—*from itself*: The explanation offered is that the predicate P is included, or contained, in the notion (= concept) of the subject S. Needless to say, in view of my previous deep and thorough (as I misguidedly thought) exposure to analyticity, I had a powerful *déjà lu* experience, pertaining to Kant, four centuries later. Clearly, I had been choused.

Dedicated to Wolfgang Künne on the occasion of his retirement

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What was the hidden tale behind this, and why had my eminent teachers not told me that the notion of an analytic judgement was known long before Kant?

Investigation speedily revealed that both crucial Kantian notions of *analytic* and *a priori* went back to Aristotle, in particular to the second of his 'sweet Analytics'.[1] At the outset of the *Analytica Posteriora*, Aristotle asks about the ultimate justification for a known judgement ('proposition'). Clearly, some are justified in terms of other known propositions, and they in terms yet of others; however, sooner or later, on pain of an infinite regress *in probando*, we shall ultimately have to reach judgements that are in a crucial sense *self-evident*, that is, their evidence does in no way rest upon anything outside the very formulation of the judgement in question and thus knowable from or in terms of itself. It should be stressed though that 'self-evident' does not mean *obvious* or *patent*. When such an axiomatic principle makes use of highly involved concepts, it need not be at all easy or quick to recognize its self-evidence. On the contrary, it might take a very long time to acquire sufficient familiarity with the concepts in question. The self-evidence of the axiomatic judgement in question is guaranteed by its formulation that contains all that is needed for knowing it. Bringing that to the fore, making it explicit, might, however, be a challenging task indeed.

> Warning: *Evidence* and *proposition* are dangerously ambiguous terms in contemporary analytical philosophy. Here *evidence* is used with its proper, original meaning—the first given in the OED—as the quality pertaining to what is evident. We are concerned with the evidence *of* what is evident, but not with the Anglo-American '*legal*' sense of supporting evidence *for*. As far as I know, English is the only language that uses the *evidence for* construction (even though one may find German *philosophers* of science that, under the influence of their Anglo-American professional reading, consider also *Evidenz* ***für***). *Proposition* similarly underwent a sense-disturbing meaning shift when Russell mistranslated Frege's *Gedanke* in his Frege Appendix to *The Principles of Mathematics*: Thereby he moved the proposition, something that can be propounded, from the level of judgement to the level of judgemental *contents*. Yet a further change was effected with the introduction of the propositional and predicate calculi, where a proposition is whatever a propositional letter stands for. My own uses will as a rule follow that of the philosopher discussed.

With respect to the kind of judgements that may serve as self-evident points of departure ('first principles') within demonstrative science, Aristotle notes that they should be (1) general (the particular cannot be the subject of demonstrative knowledge), (2) the (general) predications should be *per se* (*kath' auto*), and (3) universal in the sense of topic neutral, that is, *metabasis eis allo genos* must be avoided: Arithmetical knowledge, say, must not be gained using biological or geometrical principles. The principles must be truly universal in that they be applicable within all areas of discourse, be it biology, geometry, the human soul, or what have you ….

Concerning the notion of *perseity*, Aristotle notes four classes, of which the first two are especially relevant for my topic. Perseity of the first kind is exemplified by the proposition 'Man is rational'; here the predicate *rational* is included in the 'formula' (*logos*, definition) of the predicate *Man* since the sense of *Man* is *rational animal*. Such judgements, after definitional resolution, amount to what are known as *identical* judgements, that is, those of the forms *A is A* or *AB is A*.

[1] The lemmas by Heinrich Schepers on *Analytisch* and *Apriori* in the *Historisches Wörterbuch der Philosophie* were most helpful to me, as was the study of Wolfgang Künne's *Abstrakte Gegenstände* (1983, Ch. 5), 'Verstehen und Evidenz'.

The second Aristotelian form of *perseity* is slightly more complex. Here Aristotle considers examples such as 'a is odd' or 'a is straight'. For the first judgement to make good sense, the object a to which oddness is ascribed has to be a natural number: *Number* has to be a part of the logos for a. In brief, *a is odd* is **meaningful** only given that *a is a number* is **true**. In contemporary terms, this is a question of (Frege-Strawson) *presuppositions*:

!x King of France(x) is an individual,

and

Bald(!x King of France(x)) is a proposition
under the presupposition that ∃!x King of France(x) is a true proposition.

Thus, both notions of *perseity* pertain to questions of *meaning*: The first is a question of (Kantian) analyticity and the second one of presupposition. This is seen also from the medieval tag *ex vi terminorum* ('from the power of the terms'). As far as I have been able to discover, it was first used in this sense by Duns Scotus in his discussion with Henry of Ghent about the question whether Divine Illumination is required for insight into the first principles; Duns, the Subtle Doctor, denies this and instead opts for grounding them by or in 'the power of terms'.[2] However, that a *propositio per se nota* is grounded via meaning was also seen already by Aquinas, who characterized them as being 'known as soon as you know their terms' (*quae statim notis terminis cognoscuntur*) in the *Summa contra Gentiles*.[3] Furthermore, for later reference, I note that according to Saint Thomas a *propositio per se nota* has the property that its 'opposite cannot be thought' (*quod eius contrarium cogitari non possit*).

Perseity applies at the level of propositions ('judgements'). A similar kind of epistemic immediacy can be found also regarding the validity of inference in the fourteenth-century treatments of *consequentiae*. Here there are two main accounts. The first holds that an inference is valid when the conclusion is 'naturally understood' in the premises: *Understanding* (i.e. 'meaning knowledge') of the inferential form, premises, and, conclusion, plus *knowledge* of the premises, allows one to know the conclusion. Not all inferences that hold (*tenere* is the Latin term commonly used in connection with *consequentiae*) hold in virtue of such immediate epistemic containment. It could well be that one needs a whole chain of immediate containments for a complex containment

[2] Duns Scotus, 'Concerning Human Knowledge', Wolter edition, 126. *Ex vi terminorum* is another notion that has undergone a sense-disturbing meaning shift; already in Garland the Computist do we find this expression. There, however, it is placed against *ex vi syllogisticae*: In modern terms, the two notions correspond to material, respectively, formal validity. For instance, what terms we use in a syllogism in modus Barbara is immaterial for the correctness of the inference, whereas in the inference

Socrates is a man.
Therefore, Socrates is mortal.

its validity is highly dependent on the specific terms *man* and *mortal.* See Desmond Paul Henry (1984, 82).

[3] See *Summa contra Gentiles*, Chapter 10. The edition and German translation offered in Seidl (1986) I have found particularly helpful. His German translation of *per se nota* as 'an sich Erkanntes' seems to me more felicitous than any of the English translations I have seen.

to be seen to hold. The situation is similar to Aristotle's use of the *perfect* syllogisms of the first figure to validate other syllogisms. A perfect syllogism needs no further support for its validity to be seen than what is contained in its formulation, and other syllogisms are justified by means of chains of (conversions and) perfect syllogisms. Grasping a perfect syllogism is enough for seeing that it is valid.[4]

The squarely epistemic nature of these notions is patent. An analytic judgement is knowable *a priori* as such by resolution, or analysis, of the terms it contains, whence they have a priori demonstrations that may be obtained by systematically replacing what is defined by its definition. The reduction considered above, in connection with perseity of the first kind, of the proposition *Man is rational* to an identical proposition is a case in point. Furthermore, analytic judgements are (conceptually) necessary. The containment account for judgements as well as consequences was given prominence by Leibniz, even to the extent of making *all* truth analytic: Resolution of the terms in an *S is P* truth yields an a priori proof.[5] However, since Leibniz allows for terms of infinite complexity, the patently unwanted inference that *all* truths, including contingent, empirical ones, are necessary and knowable a priori via this a priori proof is blocked. The resolution of an infinitely complex term, in general, does not terminate. God, on the other hand, does not use discursive reasoning but knows these judgements intuitively. Accordingly, here the epistemic aspect of analytic judgements is mixed up with ontological issues concerning the complexity of terms.

Kant gave three characterizations of analytic judgements in the *Critique of Pure Reason*. The containment one is well known, as is the account in terms of the law of contradiction: It is (self-)contradictory to deny an analytic judgement, and as we saw above, both are found already in Thomas Aquinas.[6] A third Kantian characterization holds that analytic judgements constitute mere elucidations but yield no amplification of our knowledge.[7] This fits squarely with the resolution of concepts. The containment required for analyticity of an ***S is P*** judgement need not be explicit from the terms ***S*** and ***P*** as given in the formulation of the judgement in question but may be buried deep down in the definitions of definitions of definitions … of the constituent terms.[8] The analysis may take many steps of resolution in order eventually to bring out either of the required 'identical' forms *A is A*, or *AB is A*.

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[4] An Pr, 24b24.

[5] See the fragment called 'primary propositions', in Parkinson (1973, 87).

[6] Containment, KrdrV (A6, B10), Contradiction (A151, B190).

[7] (A7, B11).

[8] Kant—if he be the author of these remarks. There is scholarly controversy on the issue—in the *Jäsche Logik*, §37, notes this regarding '*tautologies*'. Warning: *Definition* is another term that has changed its meaning. From Frege onwards, a definition consists of a definiendum and definiens joined together by definitional equality:

$$Definiendum =_{df} definiens$$

is the form of a definition, whereas earlier in the tradition,

definitum = definitio

used to be the form. Thus, in current terminology, the definition is an *equation* whose right-hand-side definiens used to be called definition.

This containment account of analyticity and validity first arose for Aristotelian predications, that is, judgements.[9] Subsequently, and perhaps inspired by the Aristotelian use of perfect syllogisms, it was exported also to the validity of inferences by medieval logicians. An alternative medieval account of validity, the *incompatibility* account, generalizes the account of analyticity in terms of self-contradictory denial. Here one says that an inference, that is, *consequentia*, holds (is valid) if the truth of the premises is *incompatible* with the falsity of the conclusion.

Valid (A true. Therefore: B true) iff
Incompatible (A true and B false) iff
Necessarily (if A true, then B true)

When this necessity is read as 'truth in all variations' or 'truth under all *interpretations*', a variational account is accordingly obtained from the incompatibility account. This variational account operates at the level of inferences (consequences) and offers a kind of *consequentia formalis* ('formal consequence') that knows many definitions. The most common one is perhaps that it should hold 'in all terms':

Consequentia formalis est illia quae tenet in omnibus terminis.[10]

Similar locutions are found also in, for instance, Buridan and in Ockham. I have searched for, but have not found, medieval examples of variational accounts also at the level of propositions (judgements). When applied to propositional truth rather than to inferential holding, the 'in all instances' account seems a relatively recent notion.

In his exposition of the theory of the syllogism, Aristotle uses schematic letters that systematically allow for legitimate substitutions. This, of course, is similar to the variational account, but it is not explicitly presented as such. It must be stressed that schematic holding is not confined to the *incompatibility* account: Also, *per se nota* and other analytic *containment* claims may hold schematically in all terms. Thus, for instance, I already made such a claim when noting that the judgement *AB is A* is 'identical'.

The variational account applied to propositions, or to consequences among propositions, holds sway in contemporary logic, either in the form of logically true propositions, or '*tautologies'* as they are called in Wittgenstein's Tractarian terminology, or as consequences holding logically, in all 'variations' (Bolzano) or *interpretations* (Tarski). It was given prominence in Bolzano's magisterial *Wissenschaftslehre* from 1837, where the notions of a logically analytic proposition, that is, a logically true proposition, and *Ableitbarkeit* are deployed with great effect.[11] We should take careful note of the fact that Bolzano moved analyticity from

[9] Aristotle actually formulated his account in terms of '*belongings*' rather than predications.

[10] Pseudo-Duns Scotus cited after Kneale and Kneale (1962, 278).

[11] *Wissenschaftslehre* § 148:3 is the *locus classicus* for Bolzano's notion of (logical) analyticity, whereas his notion of (logical) consequence, called deducibility (*Ableitbarkeit*), is treated of in § 155. His reduction of epistemic notions to alethic, ontological ones can be found in §§34 and 36, whereas the fourth chapter of the *Wissenschaftslehre*, §§ 223–268, bears the title 'Of Inferences' but deals exclusively with the holding of consequences. In Vol. III, Bolzano also gives a treatment of *Vermittlungen*, that is (what I would call) *inferences*, namely, mediately grounded (not grounded in Bolzano's sense though!) acts of judgement, but then rather as a part of individual psychology and not in his usual objectivist terms. For him, objective grounding is a relation not among judgements but among propositions.

Kant's epistemic level of judgement to the alethic, ontological level of propositions, or in his own terminology, *Sätze an sich*.[12] This manner of proceeding is characteristic for Bolzano's way of thinking, in which ontology holds pride of place, and where epistemological notions are reduced to matching ontological ones. A philosophical weakness in his account is that an inference is held to be valid when a certain ontological relation (of logical consequence) merely *happens* to hold 'under all variations' between antecedent propositions and a consequent proposition.[13]

In the next crucial contribution to logic, a notion of analyticity also plays a major role: In his *Grundlagen der Arithmetik,* Frege felt compelled to introduce a fairly sophisticated novel kind of analyticity, even though he considers it to be merely a more explicit version of the Kantian notion.[14] Thus, for instance, according to Paul Boghossian, there is even a *semantic* notion of 'Frege analyticity' that holds of a proposition that 'is *transformable into a logical truth by the substitution of synonyms for synonyms*'. Boghossian also notes that '[s]ome may regard the attribution of precisely this notion to Frege controversial'.[15] I would not call it controversial but rather plain wrong. First, Frege's notion from the *Grundlagen* does not belong to semantics but to epistemology. Secondly, *logical truth* is a notion neither known to nor deployed by Frege.[16] Thirdly, substitution of synonyms for synonyms is not a part of Frege's account. In fact, Boghossian's notion is Quinean rather than Fregean.[17]

So what does *Frege* do? Both in the Preface to the *Begriffsschrift* and in *Grundlagen* §3 he runs the familiar Aristotelian regress of questioning his way back from a

[12] Künne (2007) treats of the vicissitudes of the variational notion from Bolzano to Quine in lucid detail.

[13] §36; Bolzano was aware of a certain tension at this point §314. Adherents of today's anti-anti-realism tend to make a virtue out of necessity, whence this tension is held to be a defining mark of realism: 'True but unknowable propositions' are its hallmark. Here truth and knowability are both applied to *propositions*. However, propositions are not really what is known. An object of knowledge surely must take the form that a proposition is true. (*Know a proposition* properly speaking means being familiar with the words, knowing what it says, but not knowledge that it is true.) The surprising impoverishment of current English for epistemological purposes makes itself felt here: Thus, for instance, the fine verb *to wit* has been jettisoned in contemporary parlance. Fortunately, it(s etymological equivalent) is retained in Germanic languages (e.g. *kennen* vs. *wissen*) or in Latin ones (e.g. *connaitre* vs. *savoir*). A further consequence of the abolition of *to wit* is that German *Gewissheit* has no good equivalent in English. '*Certainty*', the translation commonly used, for instance, in discussions of Wittgenstein's final work, properly speaking is the translation of *Sicherheit*. English philosophy as a consequence tends to ignore *Gewissheit* that is the objective side of knowing and considers only the psychological dimension of certainty.

[14] *Grundlagen*, §§ 3 and 4, especially footnote 1 on p. 3.

[15] Boghossian (1997, 337) and especially footnote 13.

[16] As far as I know, the only place where logical truth is even remotely considered by Frege is his final article *Gedankengefüge* from 1923. It is best seen as a Critical Notice of Wittgenstein's *Tractatus*, and it is through reflection on that work that Frege considers assertions of trivial truths such as 'if A, then A' or 'A or not A'.

[17] For Quine, see 'Two dogmas of empiricism', Quine (1951), in particular §1, where Boghossian's crucial phrase 'can be turned into a logical truth by putting synonyms for synonyms' is to be found.

known truth. A judgement made is for Frege a grounded holding true of a proposition. He then considers the demonstration (*Beweis*), that is, the tree of successive groundings offered for such a grounded holding true and traces that back to primitive claims that are no longer grounded in other claims. If all these topmost self-grounded judgements in the tree of grounding (perhaps we may call them 'leaves') are *general logical laws*, applicable within all areas of knowledge, or definitions (including any 'presuppositions' upon which the admissibility of the definitions in question depend), then the original judgement is analytic. Thus, what Frege considers is not propositions but theorems, judgements made, and they owe their analyticity to the kind of demonstration offered. In particular, one should be clear that Frege gives an epistemological account of analyticity: A *logical law* is not (what is today called a) logical truth. A logical truth is a proposition that is true, come what may, independently of what is the case, or perhaps, one that holds in all variations with respect to non-logical constants. A logical law, on the other hand, is a judgement, and Frege is emphatic on this: Analyticity does not pertain to judgemental *contents* but is a matter for the entitlement to judge (*die Berechtigung zur Urtheilsfällung)*.[18] I have dealt with Frege's notion of analyticity at some length elsewhere and here only wish to register my disagreement with the ascription of the *Quinean* notion of 'Frege analyticity' to Frege.[19] Thus viewed, Frege falls outside the pattern of the two strands in the development. However, in view of the firmly epistemic mould in which his theory is cast, comprising as it does strongly Aristotelian elements, and the (modern) concept of logical truth being absent from his *oeuvre*, Frege seems closer to the epistemic (containment) tradition than to the variational account.

If is difficult to place Frege in either tradition; Wittgenstein's *Tractatus,* on the other hand, does belong to both: In that work, both lines of development come together. The *Tractatus* may well be seen as a grand, albeit flawed, attempt at fusing the containment and variational accounts of analyticity. The elaborate Tractarian edifice revolves around the pivot of the state of affairs (*Sachverhalt*) and its 'obtaining' (*bestehen*). This *bestehen* is bivalent: A *Sachverhalt* either obtains or does not obtain, and everything else can remain the same. The propositions of logic, the logical truths, are *tautologies*, that is, propositions that are true come what may, independently of what is the case. A tautology remains true independently of what the state of affairs obtains (or not, as the case may be). Theses 6.1 and 6.11 are crucial for our topic:

6.1 Die Sätze der Logik sind Tautologien.
6.11 Die Sätze der Logik sagen also nichts. (Sie sind die analytischen Sätze.)

Thus, the analytical propositions are propositions of logic, 'tautologies', that is, *logical truths*. We note that Wittgenstein, just as Bolzano, applies analyticity to *propositions* rather than to judgements. Furthermore, we note that the analytical

[18] *Grundlagen*, §3.

[19] Cf. my 'A Garden of Grounding Trees' (2011, 53–64).

propositions, being tautologies, *say* nothing. This is because tautologies are not *bipolar*. A tautology rules nothing out but leaves the whole of logical space open. Thus, Wittgenstein agrees with Kant at least to this extent: Analytical propositions offer no amplification of our knowledge, and at best, they can provide elucidation.[20] The tautologies, being logical truths, are a paradigm instance of the variational account. However, also the containment account has a matching parallel in Wittgenstein's work. Ascriptions of internal properties to their bearers really are *per se nota*. An internal property is represented in language not by means of a symbol but by means of a *feature* (*Zug*) of the sign in which the symbol is given material form (4.126). It can be read off *am Symbol allein* and does not need to be established by means of confrontation or comparison with the world (6.113).[21] In the *Tractatus*, the containment account holds for ascriptions of internal properties. When the entity a has an internal property α, this property α can be read off from—is shown in or by—the symbol used to present the entity a, be it an object, a sentence, a state of affairs, a fact, or what have you. The Tractarian explanation of internality deploys contradiction (4.1223): A property is internal when it is *unthinkable* that its object does not have it. (My emphasis GS) This must not be taken in a propositional sense, having an internal property P is *not* expressed by means of a proposition, the negation of which would be a contradiction. Rather, it is unthinkable in the sense that there is no thought there to be had. Furthermore, the tautologies, that is, the logical truths or the sentences ('propositions') of logic, are variational with a vengeance. They hold come what may, independently of what is the case: No matter how the world is varied, no matter what states of affairs obtain or do not obtain, the tautology always comes out true.

We may thus regard Wittgenstein's *Tractatus* as an attempt to merge the containment and variational accounts. The analytical propositions are logical truths, 'tautologies', but, for Wittgenstein, the property of being a tautology is an *internal* property of sentences: Accordingly, it must be possible to read it off *am Symbol allein* by means of mechanical calculation. Unfortunately, owing to the undecidability of first- and higher-order predicate logic, no such calculation method can be had, whence the intended Tractarian merger is doomed to fail. This undecidability result in logic was obtained only in 1936 by Alonzo Church and Alan Turing and was of course unknown at the time when the *Tractatus* was written.

Kant's containment and non-contradiction characterization of analyticity have been traced in the *Tractatus*. Also, the third Kantian feature is well covered there: A proposition of logic, that is, a tautology, *says nothing*. Indeed, the choice of the rhetorical term *tautology*, which goes back at least to Quintilian, for the propositions

[20] I am expressing myself with considerable care here; Wittgenstein's notion of an elucidation (3.262) is a difficult and much discussed one. One should note though that *Satz* is used differently by Wittgenstein and Bolzano; for Wittgenstein, a *Satz* is anchored in language, whereas Bolzano's *Sätze* (*an sich*) is *sui generis* and serves as the content of linguistic *Sätze*.

[21] I carried out the comparison between Kantian analytic judgements and Tractarian ascriptions of internal properties in some detail in my (1989).

of logic might well be due to their saying nothing. In the terminology of John Locke's *Essay concerning Human Understanding* (Bk IV, Ch. viii), they are *trifling*, but not instructive, and as we already saw in Kant, they only offer *elucidation* but give no amplification of our knowledge. The trifling propositions are of course also found at the corresponding location in Leibniz' *Nouveaux Essais*, where they are charmingly called '*frivoles*'. However, the emptiness of such sayings is noted already in Thomas Aquinas (for instance, at ST Iª q. 11 a. 1 arg. 3) and is there described as *nugatory*. (Leibniz also linked his frivolities to Scholastic *nugatoriae* in his discussion in the *Nouveaux Essais*.) Finally, similar passages can be found already in Aristotle, for instance, at *Met.* A 9 (at 991a 20ff) where '*kenologein*' is used for empty or hollow speech.[22]

The most influential entry in the 'ampliatory' tradition from Aristotle onwards was made by Frege in the opening section of *Über Sinn und Bedeutung*, where he famously noted that:

> a = a und a = b sind offenbar Sätze von verschiedenem Erkenntniswert: a = a gilt *a priori* und ist nach Kant *analytisch* zu nennen, während Sätze von der Form a = b oft sehr wertvolle Erweiterungen unser Erkenntnis enthalten und *a priori* nicht immer zu begründen sind.[23]

Frege's use of (Kantian) analyticity (query: Why does Frege not deploy *his own* notion of analyticity from *Grundlagen*?) and amplification (*Erweiterung*) places his

[22] I am indebted to my Leiden colleague Jeroen van Rijen for help with the Aristotelian Greek.

[23] SuB, 25. Black (1948) translated it; thus,

> a = a and a = b are obviously statements of differing cognitive value; a = a holds a priori and, according to Kant, is to be labelled, analytic, while statements of the form a = b often contain valuable extensions of our knowledge and cannot always be established a priori.

The translation of *Satz* is crucial here. Should it be a *Gedanke* ('proposition') or a sentence? Black uses statement, which is multiply ambiguous, but is the usual equivalent of German *Aussage,* rather than of *Satz*. Michael Beaney (1997, 151) retains *statement*. Feigl (1949, 85) on the other hand opts for *sentence* as the translation of Frege's *Satz* and uses *cognitive* ***significance*** for *Erkenntniswert*.

Secondly we note that Frege does *not* place his identity sentences within quotation marks. Frege, in the final lines of his preface to *Grundgesetze der Arithmetik*, Vol. I, 1893, is, after all, the source of our current neurotic use of quotation marks and the foremost precursor of Quine's distinction between use and mention in *Mathematical Logic* (Quine 1940, § 4). Against this background, his omission is quite remarkable, the more so since quotation marks do get used around the identity sentence in Frege's footnote on the same page. (Recalling crucial changes with respect to quotation marks in various printings of Russell's *On Denoting*, I deemed it wise to check the original *Fette Fraktur* printing of Frege (1892, 25), and it agrees with the way the quote is given above.) Black and Beaney follow suit in their translations but do not comment upon the matter. Feigl, on the other hand, clearly smelled a rat since he *inserted* quotation marks where none are found in Frege's text. Later in the text, for instance, on page 32, Frege uses *Satz* for *Behauptungssatz*, that is, *declarative sentence*. However, he does not use *sentence* as it is commonly used in current philosophy of language, where a sentence is an 'expression', that is, a certain thing (entity, object …) void of meaning. Frege's sentences, on the other hand, do have thoughts as contents. So it remains a mystery why those quotation marks were left out on his first page.

discussion squarely within the Kantian epistemological tradition.[24] In the light of this, the Fregean *Erkenntniswert* should be rendered as an epistemological notion. Hence, the popular options of *information value* or *information content* from present-day semantics, irrespective of their putative merits within philosophy of language, will not serve as proper translations here. The bearer of *Erkenntniswert* is the declarative sentence. For Frege, such a sentence expresses a thought that serves as content of the assertion made by means of an utterance of the sentence in question. Accordingly, from an epistemic point of view, the Fregean *Erkenntniswert* of a declarative sentence is captured by means of its *assertion condition*. This condition lays down what one has to know in order to have the right to make the assertion in question. However, a careful analysis of Frege's examples from the present *epistemic* point of view would take us too far away from the topic of my talk and will have to wait for another occasion.

Wittgenstein's attempted fusion of the containment and variational approaches to analyticity failed: Tautologicity is *not* a decidable property. The *Tractatus* is the foremost example of a realist true-maker analysis of propositional truth.[25] However, Wittgenstein's treatment lacks explicit notations for true-makers of complex propositions. An elementary proposition is true when the matching state of affairs obtains ('exists'). For complex propositions, the truth conditions are given recursively, but specific truth-makers are not provided by Wittgenstein's account. Matters are different in the constructivist case. To each (constructive) proposition A, there is associated a *type* Proof(A) of 'proof-objects'. These are defined via a by now familiar recursion on (canonical) proof-objects. A proof(−object) is a programme or method for obtaining a canonical proof(−object), much in the same way that complex numerical expressions may be evaluated in virtue of the canonical notations in standard systems of numerals such as the Arabic, the binary, or the basic *unary* system that is based on 0 and successor s. Propositional truth is then explained as existence of a proof of A. Existence here has to be taken in a constructive sense (Sundholm 1994). (If it is not, it is trivial to validate the law of excluded middle (Sundholm 2006).) These ideas are made precise in the Constructive Type Theory of Per Martin-Löf.[26] One is entitled to assert the elliptical, and synthetic, judgement

[24] 'Placed within the Kantian tradition' might be too weak here. The passage may well have a direct Kantian source. The quote from 'Über Sinn und Bedeutung' reads as a symbolization of § 37, 'Tautologische Sätze', of Kant's *Jäsche Logik*. From the *Grundlagen der Arithmetik*, §12, p. 19, we know that Frege was familiar with this book, whence a direct influence cannot be ruled out.

[25] The *locus classicus* account of truemaking is Mulligan, Simons, and Smith (1984). On the term 'truemaking', see above Chap. 2, 'Demonstrations and Proofs', footnote 3.

[26] The analogy between Kantian analytic judgements, Tractarian predications of internal properties, and type ascriptions in Martin-Löf's Constructive Type Theory, was the main theme of my (1989) and especially at fn. 21. Martin-Löf treats of parallels between his Type Theory and Kant in his (1994). My (2004) gives a reasonably full description of Constructive Type Theory and spells out relations to a number of traditional epistemological issues, including analyticity. In particular, the significance of Gödel's theorem is treated of.

$$\text{A is true}$$

only if one has already made some judgement

$$\text{c is a Proof}(\text{A}).$$ [27]

As is well known, algorithms for 'type-checking' allow one to decide mechanically whether a judgement of the form

$$a : \alpha$$

is correct, that is, *demonstrable*. These algorithms, by applying the instructions coded in the proof-objects 'backwards', produce search trees that, when the judgement in question is correct, constitute the required demonstration. Here we have a clear case of Kantian analyticity, which is established (à la Wittgenstein) by mechanical calculation from the signs in question. The analyticity of the judgements that result from the proof relation is analogous to the Tractarian internality of the true-maker relation. This does not mean that propositional *truth* is decidable. Given a proposition A and an alleged proof-object c for A, it can be decided whether c: Proof(A). Truth is not decidable, whereas proof-hood is, and this is how Constructive Type Theory accommodates Kantian analyticity.

In order to understand how the variational account turns out in CTT, we need to understand the difference between truth and *logical* truth for propositions (and similarly consequence vs. *logical* consequence). This is best considered via two examples. I wish to demonstrate the judgement

The proposition A&B⊃A is true.

1.	A: prop	(known) premise;
2.	B:prop	(known) premise;
3.	A&B:prop	(1), (2), &-formation;
4.	z:A&B	(3), (assumption);
5.	$\&E_1$ (A,B, z): A	(4), & elimination to the left;
6.	$\supset$I(A&B,A, (z)$\&E_1$ (A,B, z)) : A&B$\supset$A	(4), (5), $\supset$ introduction.

The (variable-binding) notation '(x)t' serves as a kind of lambda abstraction; it obeys the computation rule

$$(\text{x})\text{t}(\text{a}) = \text{t}[\text{a}/\text{x}].)$$

[27] Martin-Löf notes that all judgments of the form [A true] are *synthetic* since their explicit form is Proof(A) exists. Kant stressed that every existential judgement is synthetic. Here we go even further: A correct existential judgement is always grounded in an analytic one that instantiates the existence claim in question.

On line (6) of this demonstration, we have a fully explicit proof-object that makes true the proposition A&B ⊃ A. On the other hand, given this proof-object and the proposition, we can apply the rules backwards from (6) upwards in order to obtain a demonstration.

The judgement

The proposition A&B ⊃ A is *logically true*

takes a little more effort to demonstrate since it demands generality with respect to propositions. This generality ('true in all propositional instances') is not quantificational. Generality with respect to propositions is not quantificational from a constructive point of view since the domain in question, namely, the type of propositions, is not inductively generated in terms of canonical propositions. This is the price tag attached to the unresolved impredicativity of second-order quantification. Second-order quantification over propositions (that are construed as sets of proof-objects) does not make sense constructively. Accordingly, the generality will not be obtained using universal quantifiers and their canonical proof-objects (i.e. function *sets*) but will instead draw upon function *types* formed from the types of propositions. Accordingly, I now consider not the truth but the logical truth of the proposition A&B⊃ A. The demonstration of its logical truth proceeds large in parallel to the demonstration above that established the truth of the proposition A&B ⊃ A:

1.	X: prop	(assumption)
2.	Y:prop	(assumption)
3.	X&Y:prop	(1), (2), &-formation
4.	z:X&Y	(3), (assumption)
5.	$\&E_1$ (X,Y, z): X	(4), & elimination to the left
6.	⊃I(X&Y,X, (z)$\&E_1$ (X,Y, z)) : X&Y ⊃ X	(4), (5), ⊃ introduction
7.	(Y) ⊃ I(X&Y,X, (z)$\&E_1$ (X,Y, z)):(Y:prop)X&Y ⊃ X	(6), abstraction of Y:prop
8.	(X)(Y) ⊃ I(X&Y,X, (z)$\&E_1$ (X,Y, z)):(X:prop) (Y:prop)X&Y ⊃ X	(7), abstraction of X:prop

The verification-object given in the left-hand side of (8) does establish that whatever propositions C and D are chosen, the proposition C&D ⊃ C is true, that is, the proposition A&B ⊃ A is a *logical* truth. My use of the term *verification*-object (rather than of *proof-object*) is deliberate since, owing to the function-abstractions with respect to the type prop of propositions, it does not belong to a set of proof-objects but to a certain function type. Similar considerations, but considerably more complex from the point of view of notation, apply also to the treatment of the distinction between consequence and *logical* consequence.

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Wolfgang Künne's writings on analyticity have long been a source of inspiration, already from a Nijmegen reading group in 1983 on his *Abstrakte Gegenstände*. In 2004, I furthermore had the privilege to serve as 'Président de la séance' at an early Paris presentation of his 'Analyticity and logical truth: from Bolzano to Quine' (2007).

Prior commitments prevented me from attending the symposium that honoured him at his retirement from the Hamburg chair. Since Wolfgang has long-standing links to Leiden and was a key speaker, both at the prior ***Leiden Days of Truth*** dedicated to his *Conceptions of Truth* in 2004 and at the present event, and did not protest too much after my lecture, it is a great pleasure to dedicate this chapter to him.

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Part II
Judgement and Reason in the Seventeenth Century

Chapter 4
Descartes' Theory of Judgement: Warranted Assertions, the Key to Science*

Elodie Cassan

The presence of the concept of judgement in many parts of Descartes' philosophy, methodology, metaphysics and morals has been widely remarked upon.[1] But the reason why judgement is all-pervasive in Descartes' work has not been elucidated. This is the issue at stake in the present study of the nature and the function of the concept of judgement in the *Regulae ad directionem ingenii*, a text Descartes is likely to have been working on from 1618 to 1628 and which was not published during his lifetime. The *Regulae*, which belong to the earliest strata of Descartes' work, constitute the starting point not only of Descartes' scientific thought but also of his philosophy in general.

From the *Regulae* on, Descartes assumes that in so far as judgements are true, that is, in so far as they are based on cogent and maturely considered reasons, they are the key to science. Descartes' concept of judgement has three aspects: not only does

* In this chapter, the references to Descartes will be given to the following two editions : (a) *The Philosophical Writings of Descartes*, edited and translated by J. Cottingham, R. Stoothoof and D. Murdoch [= CSM],Cambridge, Cambridge University Press, 3 volumes (vol. 3 including A. Kenny's translation of selected philosophical letters, first published in 1970), 1985–1991; (b) *Oeuvres de Descartes*, edited by C. Adam and P. Tannery [=AT], Paris, CNRS/Vrin, 11 volumes, 1964–1974. In both cases, the references are by volume number and page.

[1] Jean-Claude Pariente has considered the impact of Descartes' approach to method as a key to judging well on the Logique de Port-Royal. See Pariente (1985). In *La liberté chez Descartes et la théologie*, Paris, Alcan, 1913, Etienne Gilson has contributed to the identification of the theological sources of the *Meditationes de Prima Philosophia* theory of judgement and in *La philosophie première de Descartes*, Paris, Flammarion, 1979; Jean-Marie Beyssade has shed light on the role of this theory of judgement in the framework of the metaphysical foundation of science. Descartes' use of the concept of judgement in the field of morals has been studied by Geneviève Rodis-Lewis, *La morale de Descartes*, Paris, PUF, 1956, and by Denis Kambouchner, *L' homme des passions*, Paris, Albin Michel, 1995.

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it refer to reason, which is treated as a capacity to judge well, but also to the act of forming judgements, and to judgements, in so far as they are formed and asserted and in so far as their truth is recognised or denied. These three aspects are articulated: the exercise of the capacity to judge well, which defines reason, consists in the making of actually true judgements. Once these judgements are produced, they can be expressed in any language and the truth of their propositional content is persuasive. This approach to judgement is connected to Descartes' project of reformulating science in the framework of the scientific revolution of the seventeenth century, without resorting to Aristotle's theory of scientific syllogism.

Accordingly, in this chapter, I will address the connection Descartes draws between judgement and science. How can Descartes' approach to judgement in terms of a warranted assertion be productive scientifically? To what extent does Descartes' conception of true judgements contribute to a reworking of scientific reasoning? Considering these two issues together will make it possible to understand Descartes' philosophical project, in so far as Descartes intends to deal with all knowledge.

My claim will be twofold. I will claim, first, that in the *Regulae,* Descartes, who conceives of judgement as the assent given to a propositional content, insists on the importance of forming true judgements because he thinks that the making of science is made possible by the determination of the conditions under which we legitimately, that is, objectively, give assent and assert things accordingly. Secondly, I will defend the view that the prominent part the concept of judgement plays in the production of science, according to the *Regulae,* illustrates a mathematical rather than a syllogistical approach to the link between the constituents of a proposition and that proposition itself.

I shall begin by examining the conceptual framework within which the concept of judgement is developed in the *Regulae.* Then, once I have made the case that Descartes' rejection of scholastic logic involves a redefinition of the concept of judgement, I will argue that the exposition, from *Rule I* to *Rule XII*, of the procedure for constructing science that is to say method is based on an understanding of judgement in terms of a capacity to assert truth. Finally, I shall consider the extent to which the second part of the *Regulae* makes use of this approach to judgement, in a section where a mathematical programme for the resolution of "perfectly understood"[2] problems is being set.

1 Descartes' Debate with Scholastic Logic over the Foundations of Science

In his treatment of judgement in the *Regulae,* Descartes does not content himself with discussing propositions in their role as bearers of truth and falsity, which provides Aristotle and the Stoics with a starting point for their theory of judgement.

[2] CSM I, 51; AT X, 429.

Since he is concerned with finding out how to produce propositions that are actually true, he replaces a semantical approach to judgement with an epistemological one. In the first part of this chapter, I intend to account for the different approach adopted by Descartes.

First, I will make some remarks concerning Descartes dispute with scholastic logic concerning the foundations of science. Along with simple apprehension and reasoning, judgement is one of the three operations of the intellect[3] that Aquinas identifies in Aristotle's *Organon,* the text which provides most of the materials for a study of logic in the scholastic tradition. A common view is that the *Isagoge* and the *Categories* correspond to the apprehension of simples; the *Peri Hermeneias* to composition and division; and the *Prior Analytics*, the *Posterior Analytics*, the *Topics* and the *Sophistical Refutations* to ratiocination. It is well known that in the *Regulae,* Descartes undermines the products of the first and of the third of these three operations, the categories and the syllogism.[4] In *Rule VI*, he rejects categorial predication. In addition to this, in *Rule II* and in *Rule X*, he underlines the formalism of syllogistic reasoning. Although a syllogism may exhibit valid reasons for drawing a conclusion rather than another, it does not conform with the logic required for making scientific discoveries. As a consequence, Descartes views it as useless, in so far as, in the *Regulae***,** he has the founding of science as an objective. But why does he not criticise the product of the act of judging, when he questions the correlates of the other two acts of the mind, the categories and syllogism?

According to a common view,[5] Descartes did not do so, because then he would remain a Thomist. His attributing of judgement to the intellect, in the second part of *Rule XII*, would conform with Aquinas' theory of judgement. However, this standard view does not account for the *Regulae*'s theory of judgement. Although Descartes, like Aquinas, connects judgement to the intellect, he reshapes Aquinas' approach to judgement. First, his criticism of the logical correlates of simple apprehension and reasoning prevents him from endorsing Aquinas' semantical atomism: neither does he present judgement as an operation which makes use of the matter provided by the apprehension of simples nor does he insist on the fact that propositions are the constituent parts of syllogisms. Second, he indicates in the title of *Rule I* that he pursues the project of directing the mind "toward having true and sound judgements about everything which comes before it".[6] His contention that judgements, in so far as they are true, play a prominent part in the forming of science marks a shift from an understanding of scientific reasoning in terms of rules of inference to a non-formalist approach to scientific discourse. This view differs from Aquinas'.

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[3] Sicut dicit Philosophus in III De anima, duplex est operatio intellectus: una quidem, quae dicitur indivisibilium intelligentia, per quam scilicet intellectus apprehendit essentiam uniuscuisque rei in se ipsa; alia est operatio intellectus scilicet componentis et dividentis. Additur autem est tertia operatio, scilicet ratiocinandi, secundum quod ratio procedit a notis ad inquisitionem ignotorum. (Aquinas, *In Libros Peri Hermeneias Expositio*, 7).

[4] See, for instance: Marion (1981) and Charrak (2005), 469–484.

[5] See, for instance: Laporte (1945), 37, Beck (1952), 361, Kenny (1972), 1–31, and 4.

[6] CSM I, 9; AT X, 359.

Accordingly, in the *Regulae,* neither does Descartes focus on the syntax of judgements nor does he consider the logical structure of the arguments judgements enter into. He defines judgements in terms of vehicles of knowledge set up by a mind, understood as a capacity to judge solidly. According to Descartes, one obtains reliable knowledge, which possesses the certainty of mathematics, through the resolution of scientific questions. His approach to knowledge entails that judgements are the setting in order of the elements of these questions. This conception of judgement is based on the presupposition that all things can be arranged serially in various groups not because they can be referred to some ontological genus, such as the categories, as Aristotle claimed, but because some things can be known on the basis of others. Judgement, built according to deductive order, no longer describes the nature of things but exhibits the mutually dependent relations a question contains. But what are the conditions of the making of such judgements?

2 The Rules for the Forming of True Judgements

It is common knowledge that the *Regulae,* which were supposed to comprise 3 parts containing 12 rules each, are actually only composed of 21 rules divided into 2 parts. While from *Rule XIII* to *Rule XXI* Descartes focuses on "perfectly understood"[7] questions, that is, on problems whose solutions functionally depend upon them, from *Rule I* to *Rule XII*, he is concerned with presenting the many ingredients of his method of science and deals with "simple propositions",[8] that is, with propositions whose truth could be directly intuited by a well-prepared mind. Here, I will analyse the claims he advances in the first section of this book.

In the first three *Rules*, Descartes presents his theory of how science is grounded in reason. In *Rule II*, in the context of a criticism of disputation, he claims that the science a knowing subject obtains through forming true judgements does not belong to her exclusively, because truth is immediately persuasive. According to him, to build science requires learning how to use one's mind in order to recognise something as true and to produce true judgements. As a consequence, Descartes invalidates the capacity of memory to provide one with knowledge since through memory, one only accesses a collection of opinions on a question. In other words, the fact that science relies on reason entails that science progresses if and only if one builds propositions based on an unquestionable content.

As is made clear from *Rule III* to *Rule XI*, the formation of this content results, first of all, from the two intellectual acts that are productive from the viewpoint of science: intuition and deduction. It also relies on the definition of a method for using intuition and deduction and, lastly, on the performance of exercises in order to reinforce them. Descartes, in *Rule III*, not only shows that the forming of true judgements is

[7] CSM I, 57; AT X, 429.

[8] CSM I, 50; AT X, 428.

made possible by the capacity to grasp clearly and distinctly any object under consideration, that is, by intuition, but he also adds that true judgements are related to deduction since they are elements one needs to put in the right order so as to conclude correctly on a given question. Thus, the making of true judgements depends on the method that governs the use of intuition and deduction, the truth-bearing operations of the intellect. This method, which is accounted for in *Rule IV* in mathematical terms and summarised in *Rule V*, has two steps. First, "complicated and obscure propositions"[9] are reduced to simpler ones. Second, we proceed from an intuition of the simplest back to the more complex. This workable procedure for finding an intuition and a deductive chain is all the more efficient once one has become used to resorting to intuition and deduction. Accordingly, *Rule IX* treats the art of entertaining intuition, that is, of gaining perspicacity, and *Rule X* explains the art of entertaining deduction, that is, of gaining sagacity. In other words, Descartes' approach to science as a capacity to draw true judgements leads him to base scientific discourse on an elucidation of the mind's cognitive abilities and of the conditions under which certainty obtains.

But how is it possible to combine mental elements, which are certain, into true judgements? This issue is dealt with in the first seven paragraphs of second part of *Rule XII*, where Descartes assesses ways of adequately composing judgements. First, unlike Aristotle, he does not account for judgement analytically. He does not set out to identify the terms that are parts of a judgement. Thus, although he writes that a judgement consists of an affirmation or a denial, which is a contention one could find in the *Peri Hermeneias,* he does not replace the semantic atomism of scholastic logic with a gnoseological atomism. In the first three paragraphs of the passage, he affirms that a judgement is made of simple natures, which are either purely intellectual, purely material or common to both. But neither are the simple natures of which a judgement is composed the mental terms a judgement is made of, nor do they constitute a reworking of Aristotle's categories. Descartes, having undermined the idea of substance in *Rule VI*, is not entitled to use simple natures as categories. As a consequence, simple natures, rather than being signs standing for things, or categories through which we access the essential properties of things, are elements allowing for the intelligibility of things. They are the epistemological starting point of the process of making sense of an object.

This is not to say, however, that simple natures have nothing to do with the building of a propositional complex. This is the reason why, afterwards, in the fifth paragraph of the second part of *Rule XII*, Descartes classifies the different kinds of relations between simple natures by recourse to the criterion of modality. According to him, a combination of simple natures is necessary if those natures cannot be distinctly apprehended when considered by themselves, as in the statement "shape is conjoined with extension".[10] Otherwise, it is contingent, like when we say "a man is dressed".[11]

[9] CSM I, 20; AT X, 379.

[10] CSM I, 45–46; AT X, 421.

[11] CSM I, 46; AT X, 421.

This conceptual distinction is treated in epistemological terms rather than in ontological terms; neither is it based on the Porphyrian tree nor is it built in connection with rules for the formation of either a necessary or a contingent set of simple natures. This phenomenon has a logical and an epistemological impact. It entails that, although judgements make use of simple natures, the combining of these natures, which is determined by the issue at stake, cannot be formalised. Accordingly, logic is replaced in the sixth and seventh paragraphs of the second part of *Rule XII* with the determination of the conditions under which one is entitled to give assent: first, we should not judge that we do not completely know a thing grasped through intuition; second, we should not give assent to a mental complex whose elements cannot be clearly inferred one from the other. Descartes' non-formalist approach to the matter of judgement also reveals that he wants science to describe what enters into the object we study from a functional point of view. Aristotle's approach to definition in terms of nearest genus and difference is thus rendered obsolete by Descartes' new scientific paradigm.

3 The Many Uses of the Concept of Judgement in Descartes' *Mathesis*

Now that we understand how to form true judgements, we can turn to the question of how helpful this procedure is in the resolution of scientific questions. In order to do so, it is necessary to focus on the second part of the *Regulae*. The concentration of the occurrences of the term "judgement" in the first part of the *Regulae* does not imply that the issue of the building of true and solid judgement is only relevant in this framework. Whereas the term judgement appears 26 times in the first 12 *Rules*, it comprises only 3 occurrences in *Rule XIII* and 4 in *Rule XIV*, where Descartes tackles "perfectly understood"[12] problems. As he says, "We must note that a problem is to be counted as perfectly understood only if we have a distinct perception of the following three points : first, what the criteria are which enable us to recognize what we are looking for when we come upon it; second, what exactly is the basis from which we ought to deduce it; third, how it is to be proved that the two are so mutually dependent that the one cannot alter in any respect without there being a corresponding alteration in the other".[13] This amounts to saying that the objective is no longer to deal with "simple propositions",[14] that is to say, to relate one simple mental content grasped by intuition to another. Rather, the project is to draw one conclusion out of a series of elements which are intermingled. This new programme is based on a theory of equations. This theory is commonly accounted for in terms

[12] CSM I, 51; AT X, 429.

[13] CSM I, 51; AT X, 429.

[14] CSM I, 50; AT X, 428.

of the progress Descartes makes in mathematics between 1619 and 1621, when he begins to write the *Regulae,* and 1628. Still, the introduction of numbers and algebra into geometry in the second part of the *Regulae* does not entail that this part breaks with the first part of the *Regulae.* Therefore, one can wonder to what extent the theory of judgement of *Rule XII* prepares the way for the study of "perfectly understood questions" and to what extent it is made use of when equations are set in order to solve these questions.

First of all, in *Rule XIII,* Descartes contends that question and judgement are related as a cause to its consequence. He claims that answering a question amounts to potentially making a true judgement on a topic. Judgements help to disentangle factors which are conjoined in a question, and to that extent, they constitute tools for solving that question. They are incorporated into an analytical procedure which consists in scrutinising in due order all the factors given in the problem at hand, once we have dismissed the factors which are irrelevant to the issue, holding onto those who are essential and inferring the solution from them. In this context, judgement plays a twofold role: not only is it used in order to connect the parameters of the question to each other, according to a deductive order, but it also provides the conclusion of the reasoning through which the question is solved.

In *Rule XIV*, Descartes shows that the resolution of perfectly determined questions, that is, the forming of true and solid judgements about the issues they concern, is determined by the model of the resolution of equations. Either the thing sought and the initial data participate equally in a certain nature or the common nature in question is not present equally in both but only by way of other relations or proportions which imply it. In both cases, the objective is to make visible the connection between what we are seeking and what we already know. In order to do so, one needs to make sure that the objects being compared to each other are commensurable. This consists in a twofold operation of reduction and transposition. One needs to reduce the problem at hand to a problem about extension, that is, about geometrical magnitude, to use figures for representing it distinctly to the mind and, if necessary, to draw the figures on paper as an aid to the imagination.

Rule XVI and the following rules describe in general terms the technique of translating a problem into an equation. First, one needs to use short symbols to denote those elements of a problem which must be kept in mind. Then, one shall disregard whether the terms are known or unknown and find their interrelations. Third, one shall use the operations of addition, subtraction, multiplication and division, noting down these interrelations as equations. Fourthly, one shall search for equations as many as there are unknown terms and apply a further procedure (*Rule XX*); lastly, one shall reduce the equations to a single one of lowest possible degree. Here, the text of the *Regulae* breaks off, prior to the sequel one would expect, which would consist in rules for deriving the solution of the problem from the equation arrived at in *Rule XXI*. Nevertheless, in the second part of the *Regulae,* it is clear that "the true and sound judgements"[15] the first 12 *Rules* sought to achieve consist in the establishment of certain proportions which contain both the thing sought and the things given.

[15] CSM I, 9; AT X, 359.

In defending this claim, I do not mean to say that the method of science which is put forward in the first *Rules* is identical to the *mathesis* which is treated in the second part of this book. Since the *Regulae* is a multilayered text, there would be no point in identifying the method of the first 12 *Rules,* which relies on Descartes' philosophy of mathematics, with the *mathesis* that is elaborated from *Rule XIV* on, which is based on his mathematical practice. My purpose is just to show that in both frameworks, judgement plays the same role. It always exhibits the relation of mutual dependence between the terms of a question. While in *Rule XII* a judgement is based on the intuitive grasp of the connection between given simple natures, from *Rule XIII* on Descartes works out a procedure for discovering these connections and for analysing them in terms of measure and order. To this extent, he makes explicit the conditions under which we make true judgements so as to solve scientific problems. His concern with judgement is thus motivated by his mathematical approach to scientific discourse.

In the history of logic, the Cartesian era is commonly depicted as one during which philosophers sought to reduce logic to psychology. The theory of judgement of the *Regulae* shows that this understanding of Cartesianism is overly simplistic. Although Descartes does not work out a set of formal rules for the making of scientific reasoning, and although he does not put forward a theory of linguistic signs, he is truly concerned with discursive thought. In this connection, he redefines judgement. Unlike Aristotle, he does not see judgement as the combination of constituents which are logically and temporally anterior to the act of grouping them. According to him, a judgement indicates the relation which exists between elements of thought and which is identified in the context of scientific problems. Influenced by a mathematical approach to scientific discourse, he thinks that the connection of judgements in the solution of scientific problems is determined by the requirements of mathematics in general and of algebraic thought in particular, rather than by the ontological claims relied on by the Aristotelian syllogistic. In so far as he makes a crucial appeal to algebra, Descartes' contribution to the history of logic is not merely negative.

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Chapter 5
Striving, Oomph, and Intelligibility in Spinoza

Della Rocca Michael

Spinoza's rationalism engenders a drive for unification. Because sharp breaks in reality are, for him, inexplicable and unintelligible, Spinoza's commitment to the principle of sufficient reason (hereafter: "the PSR") – the principle that for each thing that exists there is an explanation – dictates a rejection of such breaks. In Spinoza's philosophy of mind, the crucial unifying notion is that of representation. Thus, volitions, emotions, and also affirmations and negations, i.e., judgements, are all reducible to representations of one kind or another, representations of certain objects or states of affairs. For Spinoza ideas, representations, as such and by their very nature, are judgements or affirmations and, by their very nature, are volitions and affective states. There is no sharp break in Spinoza between representations and mental states such as judgements and volitions.

As I have argued elsewhere, the feature of representations in terms of which we can understand how all these mental states are generated is simply the *activity* or *power* or *striving* (*conatus*) of representations. All representations, for Spinoza, have some degree of power; they are not, as he famously puts it, "mute pictures on a tablet" (*Ethics* 2p43s, 2p49s).[1] Thus, the notion of activity or power or striving is a unifying notion even more fundamental than the notion of representation. The nature of the activity in the case of representations is the same as the nature of the activity in the case of bodies and modes of extension. For Spinoza, things, insofar as they are mental, are active and strive, and things, insofar as they are physical,

[1] See Della Rocca (2003a). Unless otherwise noted, all references to Spinoza's works are to the *Ethics*. I employ a standard method for referring to passages from this work. Thus, "2p43s" = *Ethics* Part 2, proposition 43, scholium.

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are active and strive. The notion of striving is thus not an inherently mental or inherently physical notion. And so, in order to understand Spinoza's notion of representation and affirmation and judgement, which are by their nature active mental states, we need to understand the fundamental and nonpsychological notion of striving.

And I believe that – despite the excellent efforts of wonderful people – we have not yet fully grasped Spinoza's notion of striving or, thus, his notion of judgement. Specifically, two deep problems for his account of striving need to be, but have not yet been, adequately addressed. The first is a fabulous problem Leibniz presses against Descartes' notion of striving, a notion in many ways similar to Spinoza's. The charge here is that Spinozistic striving is not a genuine exercise of force or is not genuinely a manifestion of – to use a technical term – *oomph*. By articulating a new understanding of Descartes' account of striving, I will try to show that the Leibnizian criticism of Descartes and the same criticism as applied to Spinoza is a misguided criticism of what are, in effect, the Humean accounts of striving that Descartes and Spinoza – in different ways – offer. But averting this first problem on Spinoza's behalf only makes more acute a second problem that threatens to render Spinoza's account of striving completely unworkable. For this second problem is the threat that, even if Spinoza has a notion of force, he has no legitimate way to say that one thing has more force, more *oomph*, than another. In this chapter, I will argue that, in order to show how these problems do not threaten Spinoza, we need to take up a new perspective on Spinoza's deepest metaphysical motivations and on the ways in which his system is – and, surprisingly, is not – committed to the intelligibility of all things.

1 Descartes and the Great Intelligibility Trade-Off

As is often the case in interpreting Spinoza, a natural place to begin is with Descartes. In this section, I will characterize briefly Descartes' notion of striving and dig down to its motivations. I will focus on what Descartes sees as the striving of bodies. (Descartes is careful to specify in *Principles* III 56 that his notion of striving is not inherently psychological.) Whether and to what extent Descartes offers the same account of striving for minds is a question I will leave aside here, except to note that Descartes does not seem to account for mentalistic notions such as representation or judgement in terms of the general notion of striving. This is one of the many respects in which Descartes does not treat minds as subject to the same principles that bodies are subject to. Thus – and not surprisingly – Descartes is less of a naturalist about the mind than is Spinoza.[2]

In presenting Descartes' account of striving, I will be especially concerned to articulate in a new way some of the differences between the Cartesian account and more traditional Aristotelian accounts of striving and causation in general. This depiction of Descartes' account of striving will help us to appreciate that, although Descartes and Spinoza share an appealingly austere account of striving

[2] For some discussion, see Della Rocca (2008a).

that is stripped of Aristotelian substantial (and other) forms, there are nonetheless important differences between their views of striving. These differences will reveal the different resources that Spinoza and Descartes have for responding to the problems that their views of striving face.

Broadly speaking, we can say that Descartes' aim in his physics is to give an austere account of the nature of matter and its workings. In particular, as is well known, Descartes seeks to reduce matter merely to extension in three dimensions and to motion. Banished from the realm of matter are Aristotelian substantial and accidental forms. These forms were, as Descartes sees it, invoked to account for the causal power of bodies and to make causal relations intelligible. Descartes says that substantial forms

> were introduced by philosophers solely to account for the proper actions of natural things, of which they were supposed to be the principles and bases. (AT III 506/CSMK 208)

But, Descartes also thinks, however admirable that goal is, Aristotelian forms are themselves unintelligible:

> No natural action at all can be explained by these substantial forms, since their defenders admit that they are occult and that they do not understand them themselves. (AT III 506/CSMK 208)

Because Descartes finds Aristotelian forms unintelligible, he sees the causes of bodily change (at least insofar as those causes are to be found in bodies themselves) as instead nothing more than the motion of bodies together with their extended features. These causes, unlike the mysterious forms, are themselves perfectly intelligible. And Descartes accounts for causal relations not by appealing to the operation of such (unintelligible) forms but simply by appealing to the tendency or striving that bodies exhibit in virtue of these features of extension and motion. Thus, Descartes offers the following characterization of striving: a body strives to be F, just in case its current state is such that it will be F unless it is prevented by external causes. This is the account of striving at work in *Principles* III 56 and in Descartes' account of the laws of nature in Part II of the *Principles* (though there the term he tends to use is not "strives" but "tends"). Descartes says in *Principles* III 56:

> When I say that the globules of the second element 'strive' [*conari*] to move away from the centers around which they revolve, it should not be thought that I am implying that they have some thought from which this striving proceeds. I mean merely that they are positioned and pushed into motion in such a way that they will in fact travel in that direction, unless they are prevented by some other cause.

In *Principles* II 43, Descartes says that a body's power or force consists in this austere striving: "this power consists simply in the fact that everything tends, so far as it can, to persist in the same state, as laid down in our first law."[3] For Descartes, bodies do have force, but this force is nothing but the tendency of bodies to be in certain states.

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[3] *Hic vero diligenter advertendum est, in quo consistat vis cujusque corporis ad agendum in aliud, vel ad actioni alterius resistendum: nempe in hoc uno, quod unaquaeque res tendat, quantum in se est, ad permanendum in eodem statu in quo est, juxta legem primo loco positam.*

But what determines what a body will do or be given its current state? As *Principles* II 43 indicates, Descartes answers this question by appealing to the laws of nature (the details of which need not concern us here). And what determines what the laws of nature are (and thus what determines what bodies tend to do)? Descartes' answer: God's nature and, in particular, God's immutability. When God first created matter, he also introduced a certain quantity of motion, and thus, bodies came to be individuated and began to be moved. Because God is immutable, at each moment God endeavors to maintain the same overall quantity of motion in the extended realm. Thus, at each moment, as bodies bump into one another, God moves these bodies around so as to preserve the same overall quantity of motion. Although God moves bodies around, that God does so does not mean, for Descartes, that bodies do not have power to move other bodies. Bodies do have force, as Descartes says in *Principles* II 43, and, as he also says there, this force, this power, consists in God's immutable action of introducing and maintaining a certain overall quantity of motion. Descartes does not – on this reading – deny that bodies have force, and so Descartes is not an occasionalist. Rather, Descartes reduces bodily force to the tendency or striving of bodies, and this striving on the part of a body consists in the fact that, because of God's immutability, the body will be in a certain state unless prevented by external causes.[4]

Of course, if Descartes' original goal was to avoid mysterious Aristotelian forms, then his move of appealing to God's immutable action in order to account for bodily force may not seem to be a big advantage in the intelligibility department. After all, God's activity – as Descartes admits and stresses – is itself incomprehensible and is far from exhibiting the austerity that Descartes' account of matter and its workings aspires to. But at the level of bodies themselves, the account is quite bare. The cause of bodily motion is simply a body with a certain size, shape, and motion. The power or force of this cause simply consists in this body's having a certain tendency to be in a certain state. And this tendency consists in the fact that the body will be in that state unless prevented by external causes. This is what may be called a merely conditional account of striving: *x*'s striving to be F consists in the fact that *if x* is not prevented by external causes, *then x* will be F. No appeal here is made to mysterious substantial forms as causes. And no appeal is made to the operations of such forms to account for the causal relations between bodies. All that is needed to account for the cause is the size, shape, and motion of bodies. All that is needed to account for the causal relation or the exercise of force or of *oomph* of bodies is the tendency that such bodies enjoy in virtue of their size, shape, and motion. And, again, this tendency is understood as a merely conditional fact about what a body will, in fact, do under certain circumstances.

However, although Descartes' account of bodily power makes for an increase in intelligibility as far as the cause is concerned, this increase is purchased at the price of a significant decrease in intelligibility along another dimension. For Descartes,

[4] For more on Descartes' account of power, see Della Rocca (2008a).

nothing about the cause, i.e., nothing about the current state of the relevant body, taken on its own, dictates that the effect will occur, dictates that the body will come to be in a certain state. From the bodily cause, taken on its own, i.e., from the body's current state of size, shape, and motion, one cannot see the effect coming. The cause is, as it were, intrinsically indifferent to the effect. What is needed to see the effect coming is the activity of God. It is this activity that makes the conditional true. Nothing about the cause – the body and its current state – taken on its own suffices for the truth of the conditional or points to what will be the succeeding state of the body. The cause, in other words, does not, taken on its own, make the effect intelligible. This is unlike the Aristotelian account on which there is something about the cause – viz., the relevant form – which is designed to enable us to see the effect coming. In this way, on the Aristotelian account, the cause makes the effect intelligible. Of course, the cause – i.e., the form – that the Aristotelians posited was itself unintelligible (at least in Descartes' eyes). But *given* this cause, the effect was made intelligible. In other words, the Aristotelian cause – unlike the Cartesian cause – was not intrinsically indifferent to the effect. That, in effect, was the whole point of the Aristotelian account (as Descartes recognizes): to point to something in the cause that enables us to see the effect coming. In this way, the Aristotelian aspiration is to make the causal relation intelligible.

In giving up the intelligibility of the causal relation in exchange for the intelligibility of the cause, Descartes makes more or less the same move Hume was to make much later. Like Descartes, Hume makes the cause fully intelligible: what could be more intelligible than the current state of an object, a state which is, in fact, constantly conjoined with other states. No mysterious forms for Hume. But this austere cause makes it impossible, Hume thinks, to render the causal relation itself intelligible. For, as Hume acknowledges, the cause taken on its own does not enable us to see the effect coming.[5] Unlike Descartes, though, Hume does not appeal to the incomprehensible activity of God in order to account for the causal relation itself. Nonetheless, this difference between Descartes and Hume should not blind us to the important similarity between their accounts: considered at the level of bodies themselves, the cause does not enable us to see the effect coming and thus does not render the effect intelligible. Thus, both Descartes and Hume espouse the intelligibility of the cause and also the unintelligibility of the causal relation. In this respect, Descartes' account is fundamentally Humean in spirit.

There is here, in effect, a trade-off. Each side wants intelligibility at one place and sacrifices it at another. This is not a criticism (yet) of either account, just a way of pointing out that neither Descartes nor the Aristotelians (nor Hume) give a fully rationalist account of causation, an account according to which both the cause and the causal relation are intelligible.

However, despite this similarity between Descartes and Hume, there is a different respect in which Descartes' account of causation is not Humean. Hume allows that, in some sense, one billiard ball causes another to move even though there is no

[5] Hume, *Treatise*, 266–267, *Enquiry*, 65 ("secondly"), 68 ("secondly"), 76.

intelligible causal relation between them. But causation in this sense (a sense which, I have argued, Descartes more or less embraces too) is not the only sense of causation that Hume recognizes. Indeed, for Hume, this notion of causation is, in some way, defective. For Hume, strictly speaking, a cause – if such there be – must make its effect intelligible; a cause must enable us to see the effect coming.[6] That is why, for Hume, the fact that the motion of one billiard ball bears no conceptual connection to the motion of another shows that the first ball is not a genuine cause of the motion of the second. Nonetheless, in order to capture something of the ordinary view that causal relations obtain, Hume appeals to his stripped-down, unsatisfactory account of causation as mere constant conjunction, an account in terms of which causes do not make their effects intelligible. As we will see, Spinoza agrees with Hume (and thus disagrees with Descartes) that genuine causation requires conceptual connection.

With these underpinnings of the Cartesian account of striving in mind, we can now raise a criticism of Descartes. This objection comes from Leibniz who argues that Descartes' merely conditional notion of striving does not do justice to the causal power and genuine force to resist that is present in things that strive. Leibniz puts the point this way in a letter to de Volder, a follower of Descartes:

> I admit that each and every thing remains in its state until there is a reason for change; this is a principle of metaphysical necessity. But it is one thing to retain a state until something changes it, which even something intrinsically indifferent [*per se indifferens*] to both states does, and quite another thing, much more significant, for a thing not to be indifferent, but to have a force and, as it were, an inclination to retain its state, and so resist changing. (March 24/April 3, 1699, Gerhardt II, 170, Ariew and Garber, 172)

Here, Leibniz objects to the way that Descartes has stripped down the notion of striving. For Leibniz, striving and the causal power to resist cannot be explained in terms of a mere if-then claim but must involve something more, a full-blooded causal *oomph*. This objection does seem powerful. The mere fact that a thing is such that it will do F, unless prevented by external causes, does not show that this thing actually strives against or resists such external causes. To use Daniel Garber's example, the mere fact that a child will keep playing with her doll unless her father gets her to do something else does not mean that when the time comes, she will resist doing something else (Garber 1994). On the contrary, she might go on to the new activity willingly. Similarly, the fact that a moving body will, unless external bodies intervene, keep moving does not by itself entail that when it comes into contact with those external bodies, it will resist them and it will exert some causal power against them. Leibniz thinks that bodies actively resist change in this way, and he seems to be right. Just feel the pressure against your hand as you try to stop the motion of a billiard ball. The point, for Leibniz, is that Descartes' merely conditional notion of striving cannot account for this seemingly obvious fact.

However, it is not at all clear that Leibniz's objection is fair to Descartes. Leibniz seeks – and claims that Descartes does not provide – a cause that, taken on its own,

[6] Hume, *Treatise*, 161–162, *Enquiry*, 63.

enables us to see the effect coming that somehow, by its nature, points to the effect. This is indicated by Leibniz's claim that the Cartesian causes are "intrinsically indifferent" to their effects. (Notice that I blatantly lifted this Leibnizian locution in my earlier statement of Descartes' position.) But Leibniz's worry seems to miss Descartes' point. Leibniz charges that Cartesian causes do not – unlike, say, Aristotelian causes – make their effect intelligible. Quite right, and this is precisely the heart of Descartes' trade-off with Aristotelianism. Descartes is willing to deny the intelligibility of the causal relation and embrace instead the intelligibility of the cause. And thus Descartes offers a surprising account of resistance: it is no more than the fact that a thing will do F unless impeded. The fact that the rock strives to go through the window toward which it is hurtling is nothing more than the fact that the rock, given its current state, will go through the window unless something prevents it from doing so. This is Descartes' story and he's sticking to it. In this light, Leibniz's objection seems actually not to be an argument so much as a mere observation that Descartes rejects the view that causes make their effects intelligible. But to this observation, Descartes can say, "So what? Rejecting such intelligibility was one of my aims all along."

Certainly, the view that Descartes espouses here may seem implausible, but Descartes would see the cost of rejecting his view that the causal relation is unintelligible as a return to unintelligible, mysterious causes such as Aristotelian forms. And very many philosophers in our Humean and post-Humean world would make precisely the same choice that Descartes makes in his typically ahead-of-his-time way, viz., the choice to embrace the unintelligibility of the causal relation.

But perhaps we can have our cake and eat it too. That is, perhaps we can have both the intelligibility of the causal relation *and* the intelligibility of the cause. This is where Spinoza comes in (and, as we'll see, Leibniz makes another appearance in this part of the story).

2 Strengthening Intelligibility

Spinoza's theory of striving owes a lot to Descartes'. Sharing Descartes' disdain for Aristotelian forms, Spinoza insists on the intelligibility of the cause. For this reason, Spinoza follows Descartes in offering an account of bodily causation simply in terms of extension and motion and in offering a merely conditional account of striving. However, unlike Descartes, Spinoza wants to make the causal relation – as well as the cause itself – intelligible: Spinoza thus (as is his wont) strives to have it all as far as intelligibility is concerned. He seeks to avoid the great intelligibility trade-off that Descartes and the Aristotelians were caught up in. But how can Spinoza, with his austere account based simply on extension and motion, avoid Descartes' conclusion that causes (in the extended realm) do not make their effects intelligible? Let us see.

We should note first that Spinoza does indeed have a stripped-down account of striving. Spinoza, like Descartes, holds that bodies as well as minds strive and that

striving is to be defined merely conditionally as Descartes defines it. Spinoza is quite familiar, of course, with the Cartesian account of striving, and he captures it accurately in his book on Descartes.[7] Further, in his account of the persistence of bodies in the interlude on bodies after 2p13s, Spinoza clearly employs this notion of striving although without, in that context, employing the term "strives."[8] Unlike Descartes (perhaps), Spinoza also gives accounts for mental power or affirmation in terms of this stripped-down notion of striving. Affirmation is nothing but the striving of a mind, and this striving is to be understood in terms of what the mind will do in certain circumstances.[9]

We saw earlier that Descartes' merely conditional account of striving commits him to the unintelligibility of the causal relation. For Descartes, striving consists in the truth of the conditional: if the body is not prevented by external causes, then, given its current state, it will be F. The fact that this conditional is true is not due to the cause and its current state alone but rather is due also to the immutable activity of God. It is for this reason that, as I noted, on the Cartesian view, the cause does not enable us to see the effect coming.

But, although Spinoza, like Descartes, has a merely conditional account of striving, Spinoza's cause does enable us to see the effect coming, does make the effect intelligible, and that is because the truth of the relevant conditional follows from – is built into – the very notion of the cause. For Spinoza, by examining the nature of the cause, we can simply see that the effect must occur, and thus, for Spinoza, the cause makes the effect intelligible.

We find Spinoza saying precisely this in my favorite axiom, 1ax4: "the knowledge of the effect depends upon and involves the knowledge of the cause." As Spinoza employs this axiom, it is clear that for him, if something is caused by a thing, then the first thing is conceived through or understood through the second (see 1p6c). Spinoza also makes clear that he accepts the converse: if one thing is conceived through a second thing, the first is caused by the second (see 1p25d). Thus, for Spinoza, conceptual relations are equivalent to causal relations.

Spinoza links his account of striving to his commitment to the intelligibility of the causal relation. He specifies that the striving of a thing is its essence (3p7), and he identifies the essence of a thing with that in terms of which the thing is conceived and understood and caused (2def2, Letter 60, *Treatise on the Emendation of the Intellect* §96). So, to understand a thing is to understand its striving, i.e., to understand what it would do under certain circumstances, i.e., to understand what conditionals are true of the thing. Therefore, the conditionals that Spinoza appeals to in his stripped-down account of striving are conceptual truths, grounded in the very concept of the object. In Descartes, by contrast, these conditionals are not conceptually true; rather they are grounded in the immutability of God which is, in some way, extraneous to the nature of the object itself.

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[7] See Spinoza, *Principles of Cartesian Philosophy* 2p17, 3d3. A similar account of striving is found in Spinoza, *Metaphysical Thoughts* I.6.

[8] See, in particular, 2lemma3c.

[9] See 2p48, 2p49 and the discussion in Della Rocca (2003a).

Thus, the causal relation for Spinoza is fully intelligible despite the fact that Spinoza shares with Descartes a stripped-down, merely conditional account of striving. Further, Spinoza, like Descartes, sees the cause as fully intelligible too. The cause is not some mysterious Aristotelian form; rather the cause is just – in the case of bodies – the perfectly intelligible state of motion or rest of the extended body itself, a state which is – for Spinoza, but not for Descartes – conceptually and thus intelligibly connected with its effect. (Similarly, for Spinoza, in the case of minds, the cause is the perfectly intelligible representational state of the mind.) Spinoza, in his uncompromisingly rationalist fashion, thus avoids Descartes' intelligibility trade-off.

We can quickly see how Spinoza avoids Leibniz's criticism of a merely conditional account of striving. Leibniz's worry is that, for Descartes, causes do not make their effects intelligible; they are, as he puts it "intrinsically indifferent" to their effects. But this intrinsic indifference stems from the fact that the relevant conditionals of striving are not grounded in the concept of the cause. Precisely because the conditionals of striving are, for Spinoza, conceptual truths, the charge of indifference has no force against him. For Spinoza, the force – the *oomph* – of striving and causation is an inherent feature of the cause: it resides in the conceptual connection between the cause and the effect.

We can also see in these terms how Spinoza would account for the kind of resistance that Leibniz thought Descartes does not accommodate. Thus, consider a case in which a rock strikes a window and yet the window does not break (perhaps because it is reinforced with steel). Even in such a case – a case of unsuccessful striving – Spinoza would see the rock as exercising causal power and, indeed, as resisting even as it fails to keep moving. But what would this causal power be? Notice first that in this case, we cannot derive the concept of the window's breaking from the concept of the rock's motion. This is simply because in this case, there is no breaking of the window. And since causation just is conceptual connection, for Spinoza, there is no causal relation between the rock's motion and the window's breaking. However, there is here a conceptual connection between the rock's motion and the rock's continuing to move *unless other things prevent it* or between the rock's motion and the rock's breaking the window *unless other things prevent it*. This is a conceptual connection between the rock's motion and what may be called a conditional state of affairs, but it is a conceptual connection nonetheless. And if, as Spinoza holds, causation just is conceptual connection, then we have in this case of unsuccessful striving a genuine causal connection between the rock's moving and not the window's breaking but the state of affairs whereby the window will break unless something prevents the rock from breaking it. It is because Spinoza reduces causal connections to conceptual connections that his merely conditional notion of striving can allow for cases in which there is genuine causal power at work even in a case of unsuccessful striving. Thus, on Spinoza's terms, we can see the rock as exercising causal power even in the unsuccessful case, and thus, we can see how Spinoza might be able to answer Leibniz's objection.[10]

[10] This paragraph was adapted from Della Rocca (2008c), 151–152.

We saw earlier that Descartes would respond to Leibniz's objection by reminding Leibniz of Descartes' denial of the intelligibility of the causal relation. Spinoza, of course, insists on the intelligibility of the causal relation, and so, he cannot invoke Descartes' response. However, the very reason that prevents Spinoza from making the Cartesian response – viz., his assimilation of causation and conceptual connection – also provides Spinoza with a different and more purely rationalist response to Leibniz's difficulty.[11]

We can see in this light that Spinoza and Descartes are both Humean, though in different ways. Hume and Descartes agree that (in one sense of causation at least) causes do not make their effects intelligible. Spinoza disagrees with Hume and Descartes on this point. However, Spinoza and Hume agree that *genuine* causal relations hold (or would hold) only in cases in which there is conceptual connection, i.e., Hume and Spinoza agree that – in one sense of causation – *if a* and *b* are causally related, *then a* and *b* must be conceptually connected. While Spinoza and Hume agree on the truth of this conditional, Descartes, of course, denies it. It is Descartes' agreement with Hume in one respect (viz., on the point that causes – in one sense of causation – do not make their effects intelligible) that enables Descartes to respond to Leibniz's objection and it is Spinoza's agreement with Hume in a different respect (viz., on the point that causes and effects – if there are to be causes and effects – must be conceptually connected) that enables Spinoza to respond to Leibniz's objection, though in a different way from the way Descartes would respond.

3 Weakening Intelligibility

Despite this progress, Spinoza's insistence on the intelligibility of the causal relation – an insistence that puts him in a position to respond to Leibniz – only leaves him open to another, perhaps more worrisome problem that has not been adequately appreciated. This is the second of the two problems for Spinoza's account of striving mentioned at the outset.

Return to the rock and the window. The moving rock – by virtue of its very concept, if Spinoza is right – has a tendency, strives, to keep moving and to somehow go through or get around obstacles such as the window. The window – also by virtue of its concept – has a tendency, strives, to remain intact and not to move and not to be shattered by objects such as the rock. At most, one of these strivings can be successful. But what is it about the winning object in virtue of which it wins and has more

[11] It is not clear to me to what extent the Spinozistic notion of force just outlined would capture Leibniz's own position with regard to striving and force. Spinoza's reduction of force to conceptual connection seems to be in keeping with Leibniz's view that a monad exhibits genuine causal power through the unfolding of perceptions in accordance with the monad's complete concept. What is not clear to me is whether Leibniz would characterize all manifestations of causal power – in particular, God's causal power – in terms of conceptual connection. For some further discussion, see Della Rocca (2012), §2.

power than the losing object? This is a question that – especially given Spinoza's insistence on the intelligibility of the causal relation – it is incumbent on him to answer. Notice that Descartes and Hume have no worries on this score. For Descartes, one object (or body) is more powerful than the other simply in virtue of the fact that God acts in a certain way that gives the victory to that body. For Hume, the winning object wins simply in virtue of the regular conjunction – a brute fact – between events or objects of certain types. These Cartesian and Humean answers to the question of what is it in virtue of which one striving is less successful than another presuppose the non-intelligibility of the causal relation. For both Descartes and Hume, there is no conceptual connection between causes and effects (at least bodily effects, in the case of Descartes). And for this reason, these answers are not available to Spinoza.

What then can he say? To begin to answer this question, notice that, for Spinoza, something is powerful if and only if it is intelligible. Something has power just in case it causes something, i.e., for Spinoza, just in case it renders something intelligible. Now take the left-right half of this biconditional:

If *x* has power, then *x* is intelligible

and let us focus on the contrapositive:

If *x* is not intelligible, then *x* has no power.

This certainly seems correct: if *x* is not intelligible, i.e., if it has no explanation, then *x* cannot explain anything: for if *x* purportedly explains *y*, then because *x* itself is inexplicable, *y,* as allegedly explained by *x*, would itself ultimately be inexplicable. So, if *x* is unintelligible, then *x* can explain nothing, and since to serve as the explanation of a thing is for Spinoza (as we have just seen) to have causal power, it follows that *x* has no power.

Now for the right-to-left half of the biconditional:

If *x* is intelligible, then *x* has power.

If *x* is intelligible, i.e., if *x* can be *conceived,* then from the concept of x, certain things necessarily follow: how could there be a concept (at least a coherent concept) from which no other thing follows or which explains no other thing? But if certain concepts follow from the concept of *x*, i.e., if the concept of *x* renders other things intelligible, then, given the equivalence of power and rendering intelligible, it follows that *x* is powerful. That Spinoza endorses this line of thought is evident from his claim in 1p16d that "the intellect infers from the given definition of any thing a number of properties that really do follow from it (i.e., from the very essence of the thing)" and from the way in which he invokes 1p16 in his argument for the conclusion that each thing has some effect, i.e., each thing is powerful (1p36).

By virtue of this equivalence, intelligibility and power rise and fall together, and *if* intelligibility comes in degrees, for Spinoza, then so too does power. Perhaps, then, given the equivalence of intelligibility and power, we can offer a Spinozistic account of what it is for one thing to lose a striving contest with another, to be less powerful than another: the loser is the thing that is less intelligible. This account

would give us something of a handle on what it is for one thing to be less powerful than another. This is good, as far as it goes, but is the appeal to degrees of intelligibility available to Spinoza?

Intelligibility might seem to be an all-or-nothing matter, and allowing for something to be not fully intelligible may seem inconsistent with a thoroughgoingly rationalist system like Spinoza's which insists on the full intelligibility of everything that exists. To say that some things are less-than-fully intelligible would seem to be to commit one to brute facts, to violations of the PSR, i.e., violations of the principle that each thing that exists has an explanation. (For Spinoza's commitment to the PSR, see, e.g., 1ax2 and 1p11d2.) To see whether Spinoza can, given the PSR, allow some things to be less-than-fully intelligible, it will be helpful to consider what Spinoza means by the term "in," as in the phrases "in itself" and "in another," and it will also be helpful to see how the in-relation applies to the passive states such as sensory perceptions and judgements.[12]

First, consider the *in*-relation. The first thing to note is that the *in*-relation that modes bear to substance is not one of spatial containment. This is evident from the fact that things such as ideas are said to be in other things although ideas as such, for Spinoza, have no spatial properties.

Traditionally, modes of a substance are simply states of the substance. For example, the roundness of the table is a state of the table; it is a mode, a way, in which the table exists. The notion of in-ness as manifested in the mode-substance relation is, I believe, a version of the traditional notion of *inherence*: modes are in substance in the sense that they inhere in that substance. And, I believe, Spinoza understands the notion of in-ness in this sense. This is precisely what his selection of the term "mode" indicates.[13]

The mode-substance relation is thus a kind of dependence relation: states of a substance depend for their existence on the substance itself. This state of roundness depends on the round table itself, though, of course, the table does not depend on this state of roundness. The general point is that modes are intelligible through that which they are in. A mode – a dependent being – is not intelligible without that on which it depends.

The connection between the notion of "in" and the notion of intelligibility becomes even more vivid when we see the way Spinoza links the notions of one thing being in another and one thing being conceived through another. Spinoza defines a mode as a thing that is in another and is conceived through that other (1def5). And he defines substance as that which is in itself and is conceived through itself

[12] The following discussion of the in-relation draws on my paper (2008c).

[13] He uses "affectio" as an equivalent term (1def5, 1p4d) which also suggests a notion of inherence. Carriero, in "On the Relationship between Mode and Substance in Spinoza's Metaphysics," develops nicely the theme that the mode-substance relation in Spinoza is one of inherence, and he carefully spells out how Spinoza's notion of mode has sources in the Aristotelian tradition's notion of accident. Melamed, "Spinoza's Metaphysics of Substance: The Substance-Mode Relation as a Relation of Inherence and Predication" is also excellent here. In treating the mode-substance relation as one of inherence, I am siding with Carriero and Melamed in their dispute with Curley on this matter. For Curley's view, see Curley (1969, 1988, 1991).

(1def3). As Spinoza stresses (1p4d), there is nothing in the world beside the one substance and modes, so it follows that a thing is in another if and only if it is conceived through that other if and only if it is understood through that other.

Spinoza also makes quite clear that another crucial notion is coextensive with these two coextensive notions: viz., the notion of causation. For Spinoza, as we saw, causation is coextensive with conceptual connection. Given the coextensiveness of being in and being conceived through, it follows that something is in a thing if and only if the first thing is caused by the second. The connection between *in* and causation is manifested in Spinoza's view that modes – i.e., things that are in God – are caused by God (see, e.g., 1p16c1).

Given the coextensiveness of being in and being intelligible and of being intelligible and being powerful, it follows that one thing is not fully powerful if and only if it is not fully intelligible if and only if it is not fully in anything. But the question we now face is a more detailed version of the question raised a bit earlier: how in a rationalist system can a thing be not fully in anything and thus not fully intelligible?

Here is where we turn to sensory perceptions for help. Consider a sensory perception such as my perception of Paul McCartney, e.g., my sensory perception that Paul is standing right next to me. What is this perceptual state in? Starting from the narrowest perspective, we can see that this sensory perception is certainly not completely in itself because the affect is not self-caused. It is not, after all, a substance. So, the sensory perception must be at least partly in other things, viz., in its causes.

So, let us consider some *finite* cause of the perception. One of these finite causes is my mind itself. As Spinoza stresses, each change in a thing is at least partly the result of the nature of the thing in question (2ax1 after 2lemma3). So, since effects are in their causes, the perception is in my mind, but not wholly in my mind, and that is because the perception – qua passive state – is caused by things external to my mind. Let us focus on one particular external cause of the perception: Paul McCartney, the object of the perception.

Given that, as we have seen, sensory perceptions are in their causes, the perception must, to some extent, be in Paul. But Paul is only a partial cause of the perception; as we have seen, I am a partial cause too. So, the perception is in Paul to some degree as well as in me to some degree. This is fine, but we still have not found what the perception is fully in. After all, the perception is caused from outside both Paul and me. Indeed, no matter how far back we go in the chain of finite causes of the perception, we will not arrive at an individual or collection of individuals that the perception is fully in. So, we have not succeeded in finding what the perception is fully in, and thus, we have not succeeded in showing how the perception is fully intelligible.

But it seems that success here is not hard to come by. The reason that the perception is not fully in any series of finite causes is that the perception seems to be caused by something infinite – in particular, it seems to be caused by God.[14]

[14] The perception also seems to be caused by certain infinite modes which follow – directly or indirectly – from God's absolute nature. But this intermediate step between God and the perception can be passed over here because the problem that I want to raise emerges more clearly from considering the apparent infinite cause, God.

Thus, the perception seems to be in God, and since the perception is certainly not caused from outside God – after all, nothing is outside God – it seems that the perception is fully in God. Here, at last, we have found it: we have found what makes the perception fully intelligible, what the perception is fully in.

Yet just when we seem to have achieved this success, we can also see that none of this can be right that the perception cannot really be in God at all. Why not? Recall that we are dealing with a *passive* state, a sensory judgement, and thus, the perception is, as such, a confused and inadequate idea. However, no idea insofar as it is in God can be confused or inadequate. Rather, ideas – insofar as they are in God – are all adequate and unconfused.[15] Inadequacy and confusion cannot be in God and cannot characterize ideas insofar as God has them because inadequacy and confusion are, for Spinoza, the result of passivity, and God is, of course, in no way passive. The fundamental point then is that precisely because a passive perceptual judgement is passive, it cannot be in God, i.e., it cannot be made intelligible through God. But, as we saw, a passive sensory idea cannot be fully in or fully intelligible in terms of anything that is not God. And so it seems that sensory judgements are not fully in anything. For Spinoza, nobody and nothing is such that a passive perception is fully in it. And because, as we have seen, for something to be intelligible, it must be in something, it follows that passive sensory judgements are not fully intelligible. The perception is partly intelligible in terms of finite causes, but not fully intelligible in terms of them or in terms of anything else.

On this way of seeing things, at most God – the only thing that is not at all passive – is fully in itself and thus fully intelligible. And individual finite things, to the extent that they are less passive, are more like God and are thus more intelligible.

But still, how can any of this be the case in a rationalist system? Does not the existence of not fully intelligible things bring with it some kind of violation of the PSR, and how can Spinoza allow that? To answer this question finally, we need to see how Spinoza is committed to a corollary of the PSR that introduces a more nuanced picture of intelligibility.

For Spinoza, existence is equivalent to intelligibility. Spinoza explicitly identifies God's essence and God's existence in 1p20. As I have argued elsewhere, since God's essence is just God's being conceived through itself (1def3, 1def6), God's existence for Spinoza is just God's conceivability, i.e., God's being intelligible through itself (Della Rocca 2003b). Further, Spinoza's naturalism and his embrace of the PSR dictate that just as God's existence is God's conceivability, so too the existence of things in general is just their conceivability or intelligibility: to think otherwise would be to see God and other things as playing by different rules.

Given this equivalence, and given that there are degrees of intelligibility (corresponding to degrees of passivity), there are also degrees of existence. Because there are degrees of existence, it is natural to offer the following corollary of the PSR. The PSR, as originally stated, is the principle that whatever exists has an explanation. But if things can exist only to some degree, and if existence goes along

[15] See, e.g., 2p36d and Joachim (1901), 114–115.

with intelligibility, then we would expect that those things would be unintelligible, inexplicable, to some degree as well. In this light, a proponent of the PSR who sees existence as equivalent to intelligibility should be willing to grant this corollary:

Things exist to the extent that those things are intelligible.

This principle allows that there may be things that are unintelligible to some degree but requires that these things do not fully exist. The principle would rule out not unintelligible things per se but rather unintelligible things that exist to exactly the degree that intelligible things do.

Here, we can see how Spinoza can allow non-intelligible things in his system: he can do so just as long as these things do not fully exist. The relatively passive, relatively unintelligible things do not violate the more subtle version of the PSR, and they are, to the extent that they are unintelligible, also less powerful. And finally, we can see how Spinoza would answer the question: what is it for something to lose a striving contest, what is it for one thing to be less powerful than another. As we saw, Descartes and Hume would answer this question by appealing to what they see as the unintelligibility of the causal relation between physical objects – for Descartes and (at least in some contexts) for Hume, there is no conceptual connection at all between causes and effects; the causal relation is simply a brute fact. However, given Spinoza's commitment to the PSR, there cannot be two things with absolutely no conceptual connection. Thus, Spinoza cannot endorse the Cartesian/Humean answer to the question about striving contests. However, given Spinoza's acceptance of degrees of intelligibility, he can offer the following answer: *x* is less powerful than *y* just in case *x* is less intelligible than *y*.

This answer to the question of what it is for one thing to lose (or win) a striving contest is different from the Cartesian/Humean answer. Whereas Descartes and Hume would appeal to a brute causal relation, Spinoza would offer a more subtle account: things are more powerful to the extent that they are more intelligible.

But how is this less-than-fully intelligible causal dependence of a relatively passive thing on another, more active thing any different from the unintelligible causal relations that, in effect, Descartes and Hume invoke? The difference is twofold: first, intelligibility comes in degrees for Spinoza (just as, as we have seen, inherence comes in degrees, for Spinoza). Relatedly, existence comes in degrees for Spinoza. By contrast, Descartes and Hume – as far as I can see – regard intelligibility and existence as all-or-nothing matters. This greater flexibility on Spinoza's part allows him to give a more nuanced answer to the question of what it is for a thing to be more powerful than another: whereas Hume's and Descartes' answers are incompatible with the PSR, Spinoza's answer is in keeping with the more subtle version of the PSR, and thus, Spinoza's answer respects rationalism in a way that Cartesian and Humean answers do not.

So, fully understanding Spinoza's notions of striving and of striving contests and, ultimately, of judgements requires us to see Spinoza's commitment to the intelligibility of all things in a new and nuanced light. It is by strengthening Descartes' commitment to intelligibility that Spinoza is able to respond to the first problem (Leibniz's problem) that I raised for his account of striving. And we can also see that

by weakening – in a principled way – the commitment to intelligibility or at least making the commitment more flexible, Spinoza is able to respond to the second problem that I raised for his account of striving. This more flexible commitment to intelligibility has deep implications for Spinoza's metaphysics, and, in particular, it leads directly to a radical form of monism according to which the best and most accurate view of the world is one according to which at most only one thing – God – is quantified over and there is no legitimate quantification over any finite things. But these are matters to explore on another occasion. We have seen enough here to be confident that Spinoza's notion of striving provides a window not only onto his psychology but also onto his fundamental metaphysical motivations.

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Part III
Kant, Neo-Kantianism, and Bolzano

Chapter 6
The Role of Wolff's Analysis of Judgements in Kant's Inaugural Dissertation

Johan Blok

The subject and its cognitive faculties play a prominent role in the *German* philosophy and logic of the eighteenth century, including Wolff and Kant. This is due to the central place they granted to the notion of judgement. Ignoring the details, they considered a judgement to be made when two concepts or representations are united *in thought*. This aspect, already present in Wolff, comes more to the fore in the philosophy of Kant. Nevertheless, both also adopted a more formal approach to judgements. This chapter shows how both Wolff and Kant seek to combine the epistemological role of the subject with a formal analysis of the notion of judgement. As a result, I defend the claim that the criticism of metaphysics Kant elaborates in his dissertation *De mundi sensibilis atque intelligibilis forma et principiis* of 1770 relies on a transformation of this formal analysis to the meta-level of the human faculties.

The outline of my chapter is as follows: Focusing on the analysis of judgements into condition (*Bedingung*) and statement (*Aussage*), I first examine Wolff's conception of judgements in his *German Logic* and textbook on mathematics. The next section describes how Meier extends Wolff's notion of condition, thus paving the way for Kant's step in the inaugural dissertation. Subsequently, I show how Kant uses the Wolffian analysis in his dissertation of 1770. Finally, I shortly clarify how Kant extends his original idea of dangerous combinations of intellectual subjects and sensitive predicates to the idea of dangerous combinations of intellectual subjects and principles or forms of sensation.

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1 *Wolff's Analysis of* Judgements

According to Wolff, we judge when we think that something does or does not agree with a thing. According to his view, this also holds for judgements about the consequences of things, for example, 'a stone tumbling down a great height might kill one'.[1] This broadens his notion of judgement to contain much more than just the attribution of properties to objects. For Wolff, a judgement is not so much an expression of a state of affairs but an expression of what they mean to us.[2] As a result, he describes judgements in rather general terms: When we judge, we either connect or disconnect two concepts.[3]

In contrast to judgements, Wolff defines a proposition (*Satz*) as the signification of these concepts by words. These words can only be understood because of their relation to concepts in thought. In the end, it is not so much the proposition as the judgement, as a connection of concepts in thought, that is fundamental, for the proposition only signifies the judgement in that it refers to representations in thought by means of arbitrary signs, namely, words.[4] By means of the proposition, we can distinguish between the subject and the predicate, as well as between an affirmative and negative connection of the subject and the predicate, yet signification as such does not allow a demonstration or analysis because this requires that we have clear and distinct concepts in thought. Thus, from an epistemological perspective, Wolff's distinction between judgement and proposition hardly matters. Accordingly, he often uses the terms interchangeably, something we also find in the work of Kant.[5]

Apart from the description of judgement in terms of a subject who unites concepts in thought, Wolff also offers a more formal analysis of 'judgement'. In his influential textbooks on logic and mathematics, Wolff distinguishes between two elements of propositions and, hence, judgements. The first part is a condition (*Bedingung*), and the second part is a statement (*Aussage*).[6] An example of Wolff's is the proposition 'the warm stone makes warmth'.[7] In this case, the condition is

[1] Wolff, *German Logic*, 156, III §1.

[2] This must not be read in a relativistic manner. The definition of judgements in terms of human faculties coincides with a high esteem of the capabilities of these faculties. Nevertheless, the focus on these faculties prepares for Kant's critical assessment of these faculties.

[3] Ibid., 156, III, §2.

[4] Contrary to Leibniz, Wolff was rather pessimistic about the advantages of a system of signs, let alone a *characteristica universalis*.

[5] References to Kant's texts employ the volume and page numbers of the Akademie edition. References to reflexions are indicated with an R and can be found in Volume XVI of the Akademie edition. The *Jäsche* logic (IX:109) and some of Kant's reflexions (R3111, R2496) seem to contain a distinction between judgements and propositions such that the latter refer to assertoric judgements whereas the former indicates problematic judgements. Yet, it does hardly play a role in his published writings, in which judgement and proposition are used almost interchangeably. Furthermore, the distinction is not so much a distinction between propositions and judgements because 'proposition' serves as a designation of a specific kind of judgements, sufficient reason to treat the distinction as it has come down to us: as a marginal note.

[6] Wolff, *German Logic*, 159, III §6; ibid., p. 23, §39.

[7] Ibid., 159, III §6.

that the stone is warm and the statement is that the stone makes warmth. Wolff concludes that the proposition can also be expressed in a hypothetical form: 'If the stone is warm, the stone makes warmth'. In fact, according to Wolff, every proposition can be analysed such that it can be transformed into a hypothetical form.

Wolff argues that this also holds true of propositions that at first sight do not seem to contain a condition, such as 'each triangle has three angles'. In this case, the statement seems to hold unconditionally. Yet even in this case, the proposition can be analysed into a conditional and an assertive part: The conditional part consists of the essential (*eigenthümliche*) properties of the object, in this case the defining characteristics of the concept 'triangle'. Since Wolff defines a triangle as a space enclosed within three lines, the condition can be made explicit by transforming the proposition into 'if a space is enclosed within three lines, it has three angles'.[8] Seen from the perspective of Wolff's *German Logic*, the difference between categorical and hypothetical judgements is a relative one; it only indicates whether the conditions are explicitly expressed in the judgement or not. Wolff's analysis effectively makes every judgement into a hypothetical one, at least from a logical perspective.[9] In *Kant and the Capacity to Judge,* Longuenesse also describes Wolff's view that every judgement can be transformed into a hypothetical judgement including its relation to demonstrations. Whereas Longuenesse argues that this is 'often useful for demonstrations', I maintain that it is essential to Wolff's concept of demonstration as such since only analysis of judgements into conditions and statements, and the transformation into a hypothetical judgement if required, allows to demonstrate the truth of the judgement. Accordingly, the transformation into hypothetical judgements is fundamental to Wolff's epistemology. For Wolff, the task of logical analysis consists in finding the conditions of judgements in order to understand when and why the statement is valid.

Logical analysis of judgements is crucial to Wolff's epistemology because the conditions provide the ground for a formal demonstration of the statement by means of syllogistic reasoning[10]:

> [T]he condition shows not only the Ground or Principle from which the demonstration is to be taken, but also the characteristics or marks, from which I may know, that in a given case the Proposition holds.[11]

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[8] Ibid., 159, III §7.

[9] In her valuable study of Kant's philosophy, Longuenesse correctly claims an ontological difference between categorical and hypothetical judgements (Longuenesse 1998, 99–101). According to Longuenesse, a logical transformation between these judgements is possible independently of this ontological difference. Nevertheless, the ontological difference has priority in that the categorical version of a hypothetical judgement, where the conditions are transferred to the subject, is a fictional categorical judgement. I would like to add that the other side of this coin is that the hypothetical version of a categorical judgement is fictional too from the ontological perspective. For the conditions are the *essentialia* of the subject. Therefore, the ontological difference is insufficient to argue for a logical distinction between categorical and hypothetical judgements in such a fundamental manner that their transformation is fictitious.

[10] Wolff, *German Logic*, 160, III, §8; Wolff, *Der Anfangs-Gründe aller mathematischen Wissenschafften*, p. 25, §42.

[11] Wolff, *German Logic*, 160, III §8.

In his textbook on mathematics, Wolff gives the following example. The statement 'the area of a triangle is equal to half the area of a parallelogram' is correct only if the base and height are the same.[12] According to Wolff, the condition tells me in which cases the statement holds. A judgement either implicitly or explicitly includes the circumstances that make the judgement true. The condition prevents an incorrect application of the statement. More importantly, the condition of a judgement functions as the ground to demonstrate the truth of the statement. The statement of a judgement follows by syllogisms from the definition of the subject, possibly combined with the definitions of additional characteristics as contained in the condition of the judgement.

In sum, Wolff held that analysis of judgements reveals the grounds for demonstrations. Despite his somewhat 'psychological' description of judgement in terms of a connection in thought, he offers an analysis of judgements as formal as possible within the logical context of his time. We will see that Meier broadens the notion of condition, thereby preparing the way for a transformation of the condition of a judgement into a meta-condition at the level of the human faculties in Kant's inaugural dissertation.

2 Meier's Notion of Condition

As we will see, Kant's dissertation involves a much broader notion of condition than Wolff's analysis of judgements into conditions and statements. The question arises whether this broader use of condition is indeed rooted in Wolff's analysis of judgements. In my view, this question can be answered positively on the basis of Meier's exposition of Wolff's analysis of judgements.

Meier wrote quite extensively about Wolff's analysis of judgements into conditions and statements in his *Vernunftlehre* on which Kant based his lectures on logic. Meier starts where Wolff's *German Logic* ends, namely, with the condition of a judgement as the ground of it:

> All true judgements have a ground and a sufficient ground for their truth. This ground is called the condition of judgements (hypothesis, conditio judicii). Consequently, from these grounds the truth and falsity of judgements can be known. Accordingly, these grounds are indications [*Kennzeichen*] and proof [*Beweisthum*] of the truth.[13]

From the very outset, Meier regards the condition of a judgement as the sufficient ground that every true judgement must have. This view places the analysis of judgements into conditions and statements at the very heart of philosophy in the 'Leibnizian' tradition.

[12] Wolff, *Der Anfangs-Gründe aller mathematischen Wissenschafften*, p. 22, §39.

[13] Meier, *Auszug aus der Vernunftlehre*, §297, XVI:642.

As Meier explicitly states, the identification of the condition of a judgement with the sufficient ground of the judgement involves a broader notion of condition than usual. He defines the notion of condition as 'everything from which one can know that a predicate applies to a subject or not, that it must be affirmed or denied on the basis of this and no other basis'.[14] Although Meyer's approach effectively does not seem to be very different from Wolff, he broadens the notion of condition to include everything which counts as a sufficient ground. Even more than Wolff, Meier emphasises the crucial role of conditions:

> We here discovered the source [*Quelle*] from which we can and must create all learned proofs; namely, one looks for the condition of judgements. If we cannot find them, we are not capable of demonstrating a truth[.][15]

All judgements can be analysed into parts. As a result of such an analysis, one does not only find its proof but also attain a correct understanding of the judgement.[16] Without any hesitation, Meier describes the conditions of judgements as the source of proofs and, hence, as the source of knowledge.[17]

As we will see in the next section, it does not seem too far-fetched to regard Meier's treatment as a preparation for one of the steps Kant made in the dissertation. Moreover, even Kant's turn towards the faculties and sources of knowledge is prominently present in many examples of Meier. Thus, Meier analyses the judgement 'human beings are capable of learned knowledge' into the condition that human beings possess the faculty of reason (*Vernunft*).[18] In this manner, Meier presents numerous examples about human faculties of knowledge. Of course, the content remains specific to these judgements and is not attached to the analysis of judgements itself. Nevertheless, the nature of the many examples is striking, especially compared to Wolff who mainly offers mathematical examples. Moreover, Meier from the very outset describes conditions as sufficient grounds, whereas for Wolff this is a cautious conclusion. This allows him to regard the analysis of judgements into conditions as the source of demonstrations and, hence, as the source of knowledge. Although Meier's contribution mainly consists in the manner of presentation and explanation, he nevertheless seems to have influenced the early Kant. During the two decades before the publication of the dissertation, Kant extensively taught logic from Meier's *Vernunftlehre.* Therefore, it is likely that Meier's examples and starting point influenced the development of Kant's thought in this manner.

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[14] Meier, *Vernunftlehre*, p. 490, §330.

[15] Ibid., p. 491, §330.

[16] Meier, *Auszug aus der Vernunftlehre*, §300, XVI:646.

[17] Meier distinguishes between several kinds of conditions and relates them to different levels of demonstration (Meier, *Vernunftlehre*, p. 491, §331). A sufficient condition, for example, leads to a demonstration of the judgement, but an insufficient condition needs additional grounds to attain full evidence. An example of the latter is the judgement 'Leibniz is a learned man' for which the possession of the understanding is not a sufficient condition because Leibniz must also have had the circumstances to attain knowledge.

[18] Meier, *Vernunftlehre*, p. 491, §330.

3 The Strategy of Kant's Dissertation

Even more than in the *Critique of Pure Reason,* a convincing interpretation of Kant's pre-critical works is impossible without a thorough understanding of the logic which was taught at the time. This holds true, especially of Kant's dissertation of 1770, in which he introduces for the first time the notions of forms of space and time, as well as the distinction between sensibility and the understanding. In line with the prize essay of 1764, the dissertation employs these distinctions to develop a criticism of traditional metaphysics. Supported by the logic of the time, these distinctions allow Kant to argue that many metaphysical claims are unfounded. While the prize essay merely distinguishes the uncertainty of metaphysics from the apodictic certainty of mathematics, the dissertation offers a more decisive attack on rationalist metaphysics. An example of such a metaphysical claim criticised in both the prize essay and the dissertation is Crusius' principle 'whatever is, is somewhere and somewhen'.[19] The dissertation criticises this principle since it assumes that time can be ascribed to being as such.

In the fifth part of his dissertation, Kant distinguishes pure philosophy (metaphysics) from other domains of knowledge by means of a distinction between a *logical* and a *real* use of the understanding. A logical use of the understanding consists of the subordination of cognitions on the basis of the principle of contradiction. Cognitions in this context amount to concepts. Accordingly, the logical use of the understanding generates a hierarchy of concepts, for example, when one regards gold as a kind of metal.

Real use, on the other hand, involves concepts that are given by the understanding itself. Meier's textbook on logic also uses the attribute 'logical' with regard to the relations between concepts.[20] Contrary to Meier, Kant transfers the qualification 'logical' from cognitions themselves to the cognitive faculties. As a result, Kant's distinction applies to the kind of use of the understanding. This allows for a methodological use of the distinction such that the understanding does not play the same role in different domains of knowledge. According to Kant, in metaphysics we make 'real' use of the understanding, whereas we make only logical use of the understanding in mathematics and natural science. In the latter, intuition, either pure or not, produces cognition (principles and concepts), but in metaphysics this is not possible or at least problematic. Contrary to mathematics and the natural sciences, in metaphysics the content providing role of intuition is replaced by a real use of the understanding. Kant considers the understanding to have an additional task in metaphysics, namely, to produce cognitions in the form of concepts and principles. In a sense, Kant's use of the term 'real' is similar to its traditional use as a qualification of definitions. Real definitions explicitly contain the content of the defined concept. In a similar manner, real use of the understanding provides pure cognitions as content.

[19] II:294, §3; II:414, §27. Translations of Kant's dissertation stem from the Cambridge Translation (Kant, *Theoretical Philosophy 1755–1770*).

[20] Meier, *Auszug aus der Vernunftlehre*, p. 292, §285; Meier, *Vernunftlehre*, p. 484, §325.

According to Kant, the role of the understanding as the source of concepts in metaphysics must carefully be distinguished from the logical use of the understanding in mathematics and natural science. Since, in the latter sciences, the concepts stem from intuition, the understanding merely has to analyse concepts resulting in a hierarchical structure. Regardless of its importance, intensity and depth, the analysis does not change the epistemological origin of the concepts. Although logical abstraction from an empirical concept might result in a rather abstract concept far removed from its empirical origin, Kant notes its epistemological nature remains the same:

> If, therefore, sensitive cognitions are given, sensitive cognitions are subordinated by the logical use of the understanding to other sensitive cognitions, as to common concepts, and phenomena are subordinated to more general laws of phenomena. But it is of the greatest importance here to have noticed that cognitions must always be treated as sensitive cognitions, no matter how extensive the logical use of the understanding may have been in relation to them.[21]

In fact, Kant distinguishes three roles of the human faculties: sensation, real use of the understanding and logical use of the understanding. In his view, the double role of the understanding easily leads to confusion of these roles, resulting in mistaken principles or axioms.[22] This especially occurs when metaphysicians make extensive logical use of the understanding upon a sensitive cognition, which leads to the illusion of an a priori cognition of the understanding. Therefore, metaphysics needs a rule to prevent it from misusing the understanding. Kant formulates his recommendation for metaphysics as follows:

> [G]reat care must be taken lest the principles which are native to sensitive cognition transgress their limits, and affect what belongs to the understanding.[23]

This rather general formulation is followed by a remarkably precise argumentation:

> For the predicate in any judgement which is asserted by the understanding, is the condition, in the absence of which, it is maintained, the subject cannot be thought; the predicate is, thus, a principle of cognising.[24]

Slight variations of the phrase 'the condition, in the absence of which the subject cannot be thought' appear again and again.[25] This phrase is crucial, since it limits

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[21] II:393, §5. Kant takes a similar stance towards the distinction between analytic and synthetic judgements. No matter how much analytical reasoning has taken place between a synthetic premise and a conclusion, the conclusion is synthetic as well (see *CPR*, B14).

[22] The term 'axiom' is used in line with the Cambridge translation. The advantage is that it allows a distinction between the meta-level of Kant's principles and the data consisting of the principles of traditional metaphysics (referred to by 'axioms') handled by these principles. The disadvantage is that it contradicts a more general custom to distinguish between mathematical unprovable judgement as axioms and metaphysical unprovable judgements as principles. This distinction becomes especially important in the *Critique of Pure Reason* (B761).

[23] II:411, §24.

[24] II:412, §24.

[25] For example, when Kant describes the three classes of subreption which I will discuss in the next section.

Table 6.1 The argumentation that underlies Kant's recommendation for metaphysics

Subject	*Predicate*	*Premise*: The predicate is a necessary epistemic condition for the subject
Sensation	Sensation	*Conclusion*: The predicate is a predicate in the absence of which the subject cannot occur
Understanding	Understanding	*Conclusion*: The predicate is a predicate in the absence of which the subject cannot occur
Understanding	Sensation	*Conclusion*: The predicate is a predicate, in the absence of which the *sensitive cognition of* the given concept cannot occur
Sensation	Understanding	*Conclusion*: The predicate is a predicate in the absence of which the subject cannot occur
Conclusion: 'Great care must be taken *lest the principles native to sensitive cognition transgress their limits and affect what belongs to the understanding.*'		

the range of predicates that can function as conditions relative to a given subject. Within the context of Kant's analysis of the dissertation, only those predicates that are essential characteristics of the subject are taken into consideration. For the subject cannot be thought if an essential characteristic is missing, at least not in a clear and distinct manner. This restriction does not come as a surprise if one is aware that Kant aims to comprehend judgements in general, rather than axioms. From the phrase in the quoted passage, it follows that the predicate functions as a principle of cognising, that is, as something that determines which cognitions of the subject are possible.[26] The epistemological nature of the predicate determines the epistemological nature of the cognition of the subject because the predicate is essential for the subject. Thus, Kant's argumentation starts from the premise that the predicate is a necessary condition for the cognition of the subject (see the first row, third column of Table 6.1).

Based on this premise, Kant draws two different conclusions for two different cases:

> If the predicate is a sensitive concept it will only be the condition of a possible sensitive cognition; and thus it will, in particular, harmonise with the subject of a judgement, the concept of which is likewise sensitive. But if the predicate were to be applied to a concept of the understanding, such a judgement would only be valid from the point of view of subjective laws. Hence, the predicate […] may be predicated *only as the condition, in the absences of which the sensitive cognition of the given concept cannot occur.*[27]

The first sentence describes the case that both the predicate and subject are concepts stemming from sensation (see the first case of Table 6.1). In this case, the

[26] Similar to the *Critique of Pure Reason* where the term principle does not necessarily refer to something that has the form of a judgement, principles in the dissertation do not necessarily refer to unprovable judgements. It can also refer to a presupposed epistemological structure or source such as the forms of space and time (II:398, §13).

[27] II:412, §24.

predicate harmonises with the subject because both are sensitive. Therefore, the predicate is indeed something in the absence of which the subject *itself* cannot occur. For example, in the absence of the concept divisible, the concept of matter cannot be thought. The second sentence argues for the case where the predicate is still sensitive, but the subject not (third case of Table 6.1). In this case, the predicate is only a condition for the *sensitive* cognition of the subject, for example, when one ascribes indivisibility to the soul. The epistemological origin of the predicate determines the cognition of the subject. In this sense, the predicate is the condition of the subject. The combination of a concept of the understanding with a sensitive predicate restricts the validity of the judgements: The predicate does not apply to the intellectual concept itself. Therefore, the predicate only determines the *sensitive* cognition of the subject. Omission of the qualification 'sensitive' results in a faulty judgement about the intellectual subject. To be more precise, a *subreption* occurs if the sensitive predicate is itself regarded as a concept of the understanding.[28] If metaphysics nevertheless regards a sensitive predicate as such, it wrongly regards the judgement as a proper metaphysical judgement, while the judgement in fact belongs to the domain of knowledge that only considers the *sensitive* cognitions of things. Thus, the overall conclusion is that one must carefully investigate the nature of predicates in order to prevent metaphysics from determining concepts of the understanding on the basis of sensitive predicates (see last row of Table 6.1).

Apart from the cases explicitly discussed by Kant in this context, the table shows an additional, at least theoretically possible, combination of subject and predicate relative to their epistemological origin (the last combination of subject and predicate, namely, as stemming from sensation and understanding, respectively, in Table 6.1).[29] The fact that Kant did not explicitly discuss this case can easily be understood. For the aims of the dissertation are a renewed criticism of one domain of a priori knowledge, namely, metaphysics, and a renewed enforcement of the apodictic status of the other domain of a priori knowledge, namely, mathematics. The recommendation is a warning for metaphysics. Accordingly, Kant only discusses judgements that involve a real use of the understanding, and the phrase 'what belongs to the understanding' contained in the recommendation undoubtedly refers to real use of the understanding.[30] Nevertheless, the systematic nature of the distinction and the logical framework underlying Kant's analysis require that the premise also yields a conclusion in the fourth and last case.[31] More importantly, as Carson indicates,

[28] For a historical study of the notion of subreption, see Birken-Bertsch (2006).

[29] Birken-Bertsch also claims that Kant treats subreption according to the logic of judgements and discusses all four cases (ibid., 79).

[30] II:412.

[31] Interestingly, systematic treatment of all cases reveals that the distinctions of the dissertation to some extent foreshadow the possible combinations of the distinctions a priori/a posteriori and analytic/synthetic judgements in the *First Critique*. From top to bottom, the rows of the table roughly correspond to a posteriori synthetic judgements, a priori analytic judgements and a priori synthetic judgements for the last two cases. Since Kant's notion of synthetic judgement does not require the predicate or subject to be of a specific kind, the two distinctions of the *First Critique* cannot account for the differences between the last two cases.

throughout the dissertation Kant uses examples that involve the attribution of concepts of the understanding to sensible concepts.[32] The most convincing one is when Kant explains the acquisition of concepts of the understanding, such as cause and necessity, by means of attention of the understanding to 'its actions on the occasion of an experience'.[33]

A critical reader might object that Kant's use of the distinction between condition and statement is opposed to that of Wolff and Meier. Whereas the subject functions as the condition in the Wolffian analysis, Kant regards the predicate as the condition. This might even raise the question whether Wolff's analysis of judgements is relevant at all to the passages on subreption in Kant's dissertation. In my view, the question can be answered affirmatively on both systematic and historical grounds.

To start with the systematic reason, the exchange of subject and predicate can be explained by Kant's focus on metaphysical axioms instead of judgements in general. Since an axiom is an unprovable judgement, the predicate of the judgement is essential to the subject. As we have seen, Kant restricts the candidates for predicates precisely to those that are essential to the subject. Without its essential characteristics, a concept cannot be thought, at least not clear and distinct, as is required for proper knowledge. In this sense, predicates, restricted to the essential properties of a concept, are conditions for the subject.

In a lecture on logic named Philippi and dated 1772, Kant also discussed the axiom of Crusius.[34] Kant's lectures were based on the textbook of Meier, and Crusius' axiom is discussed in exactly the part where Meier and, correspondingly, Kant expose the analysis of judgements into conditions and statements. Interestingly, he uses exactly the same example as in both the prize essay and the inaugural dissertation. Having discussed judgements which explicitly contain the condition, Kant claims about the opposite case:

> But if the predicate is compared to the subject by means of a condition which is contained in the subject, the condition must emerge by means of analysis. For example: 'whatever is, is somewhere and somewhen'. Here, it must emerge how the concept of being belongs under the concept of place as a condition; although this [is] not possible.[35]

This not only confirms the connection of subreption to the analysis of judgements into conditions and statements but also explains why Kant can easily switch from the subject as a condition to the predicate as a condition. Analysis of the subject, in

[32] Carson (2004), 178.

[33] II:395.

[34] XXIV:463. Further support can be found in Kant's notes on the relevant sections of Meier's textbook on logic (R3079). In these reflections Kant writes, 'The nature of the predicate. The condition.' Within the same period that Kant wrote this, he adds 'subject' after the word 'predicate'. Although these reflections are not precise enough to conclude exactly what Kant had in mind, it seems he at least recognised that the predicate can function as a condition.

[35] XXIV:463.

this case 'being', must reveal the condition contained in the subject. Because it is contained in the subject, the predicate is essential to the subject and therefore is a condition for the subject itself. In other words, the defining characteristics of the subject are conditions for the subject itself. As soon as one picks out one of these characteristics and ascribes it as a predicate to the subject, the subject is the condition for the truth of the statement expressed by the judgement, for it is because of the defining characteristics of the subject that the predicate can be ascribed to the subject. Thus, because the predicate is a defining characteristic of the subject, both the predicate and the subject can be said to function as a condition: the subject, in the case of the attribution of a (possibly non-essential) characteristic to the subject in a judgement, and the predicate, in case it is a characteristic of the subject itself. In the first case, the subject is the condition for the statement expressed in the judgement. In the second case, the predicate is the condition for the subject itself (independent from the judgement).

Only because Wolff and Meier described conditions as the ground for truth is Kant able to conclude that the predicate is a principle of cognition. Since according to the Wolffian analysis the predicate follows from the ground or condition contained in the subject, this seems counterintuitive. However, one must recognise that Kant has a more general kind of 'condition' in mind. Kant's text makes plainly clear that 'ground' is interpreted in terms of the newly introduced distinction between two sources of knowledge, namely, sensibility and the understanding. The condition is no longer contained in the subject, but in the cognitive powers. It is the predicate that determines the kind of cognitive power relevant to the judgement as a whole. Whether a predicate objectively applies to a subject depends on the epistemological source of the predicate. Accordingly, the condition consists of the predicate instead of the subject.

Thus, Kant's notion of condition becomes much broader and relatively loose compared to Wolff's understanding of the role conditions play in the demonstration of the statement, yet I propose that a sharp distinction between the meta-level of conditions in reference to the epistemic sources and the condition of a particular judgement undermines such a conclusion. Unfortunately, this and related distinctions between methodological principles, metaphysical principles and particular judgements are not very explicit in the dissertation. As we will see, awareness of these differences is especially important for the interpretation of the complex examples presented by Kant in his exposition of the three classes of subreption.

4 Three Classes of Subreption

Apart from his recommendation for metaphysics, Kant formulates the negative side of the same coin as a principle that can be used to uncover axioms as subreptic:

> If of any concept of the understanding whatsoever there is predicated generally anything which belongs to the relations of *space and time*, it must not be asserted objectively;

> it only denotes the condition, in the absence of which a given concept would not be sensitively cognisable.[36]

Whereas the intellectual predicate expresses one of the conditions without which the subject cannot be thought in the case of proper axioms of metaphysics, a subreptic axiom only contains the condition under which one can have a sensation of the subject. A proper axiom expresses properties essential to the subject, namely, those contained in its (analytic) definition, which is the final product of metaphysics according to Kant's prize essay of 1764.[37]

The first part of the quoted passage extends the notion of subreptic from the source of predicate *qua* content to any connection to space and time. Although a concept like motion is not itself an empirical concept in the sense that its content stems from experience, it nevertheless becomes meaningless without space and time. So Kant broadens the criterion that the predicate is a sensitive cognition (e.g. motion of the train from Amsterdam to Paris) to any cognition related to space and/or time in general (e.g. motion as such). This enables him to distinguish three kinds of subreption.[38] In all these cases, the sensitive predicate is wrongly taken as the condition for the possibility of the subject of the judgement.[39] Such a predicate can be a sensitive condition in three manners[40]:

1. As a condition 'under which alone the *intuition* of an object is possible'
2. As a condition 'under which alone *it is possible to compare what is given so as to form a concept* of the understanding of the object'
3. As a condition 'under which alone some *object* met with can be *subsumed under a given concept of the understanding*'

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[36] II:413, §25.

[37] In this chapter, Kant ascribes a fundamental lack of evidence to metaphysics because of the analytic nature of the definitions: One will never be sure whether the definition is adequate and complete. Additional criticism of metaphysics is presented in the dissertation where Kant criticises the elements that form the starting point of metaphysics in its process towards analytic definitions, namely, its unprovable judgements (axioms).

[38] Several authors attempt to interpret the examples in terms of the antinomies of the *First Critique* (Guyer 1987, 289; Grier 2001, 62; Zammito 2002, 267). Although this might help, the examples stand on their own, and the (technical) details of subreption seem to be rather different from the antinomies. At least the technical apparatus used by Kant to show that it is a fallacy is quite different: In the inaugural dissertation, it involves the analysis of judgements, whereas Kant uses demonstrations of contradictory theses in the case of the antinomies. The discussions of Carson and Birken-Bertsch are more informative in so far as one is interested in the dissertation as it stands on its own (Carson, 'Metaphysics, Mathematics and the Distinction Between the Sensible and the Intelligible in Kant's Inaugural Dissertation', p. 186; Birken-Bertsch 2006, 82).

[39] Kant describes all three kinds in terms of conditions. Since Kant treats predicates as conditions in his general recommendation for metaphysics (§23), I regard this as sufficient reason to describe all three kinds in terms of the logical analysis of judgements into predicates and subjects.

[40] II:413. §26. Guyer describes these three species of subreption themselves as 'subreptic axioms' (Guyer 1987, 289). In my view, this is quite misleading since Kant does not really present the three kinds of subreption as principles or axioms. Instead, Kant presents it as three manners or forms of subreption that give rise to unjustified metaphysical principles.

The first class resembles the most natural case given a distinction between (the real use of) the understanding and sensation as the source of concepts. The other two classes stem from the notion of space and time as forms or principles of sensation introduced in the dissertation. As forms or principles, time and space, and therefore sensation, can play a role other than providing the data of the concepts. In the second class, time and/or space as the principles of sensation are means for comparison, although the concepts themselves do not necessarily stem from sensation. In the third class, time and/or space help to correctly identify an object as belonging to the extension of a concept. Here, the principles of sensation take part in the very constitution of the concept.

A prominent example of an axiom in which the first kind of subreption takes place is the axiom 'whatever is, is somewhere and somewhen'. This axiom, stemming from Crusius, is of a nature almost as general as Kant's phrases that describe the first class.[41] In fact, the axiom is a general description of what according to Kant often occurs in metaphysical discussions. A concrete example mentioned by Kant is that one interprets the presence of God in terms of space.[42] In such a case, there are in fact two errors made. First of all, one takes presence as a sensible predicate. The second error, that of subreption, consists in ascribing this predicate to an intellectual concept, namely, that of God. For Kant, the sensible version of the notion of presence is only the condition under which God would be intuited if the first error was not an error.

Kant's description of the second class of subreption uses the Latin notion of *conferri* which is translated as 'to compare'. Concepts result from comparison. In the case of empirical concepts, one compares several objects, subsequently notes shared characteristics, and finally builds a concept from these characteristics. In this manner, what is given is brought together by means of a process of comparison aimed at gathering identical characteristics. The term *conferri* has exactly this connotation, namely, of bringing things together. This allows one to understand that in the case of pure concepts, the concept of time can play a similar role, for it unites what is given into a concept. This function of time depends on the notion of time as one of the principles or pure forms of sensation, as introduced by Kant in the dissertation. The second class of subreptive axioms is distinguished from the first by the fact that the sensible part is not contained in the predicate itself:

> [A]lthough it [the concept of time] is not considered to be a characteristic mark of the subject, it nonetheless serves as a means for giving form to the concept of the predicate.[43]

A relatively clear example involves two judgements about magnitudes: 'every actual multiplicity can be given numerically'; hence, 'every magnitude is finite'.[44]

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[41] Kant criticises Crusius as he did in his prize essay attacking the same principle. The dissertation continues a critical approach to metaphysics but offers more sophisticated arguments against the axiom of Crusius.

[42] II:414.

[43] II:415, §28.

[44] II:415, §28.

The formation of the concept of magnitude and multiplicity is based on the concept of time. Units are combined into a quantity or whole with the help of the concept of time, that is, with the help of successive coordination. Given its discursive nature, the understanding cannot produce such wholes and depends for this on the form of time. Such a coordination of successive units can only be completed in a finite time. Therefore, we cannot bring about an actual infinite magnitude.[45] However, the limitation of the forms of space and time to *construe* an infinite whole does not imply that 'magnitude' as *a concept of the understanding* cannot be infinite. Thus, the conclusion that every magnitude is as such finite transgresses the principles of sensation and thus affects 'magnitude' as a concept of the understanding.

An example of the third class of subreptive axioms is 'whatever exists contingently, at some time did not exist'. Again, the converse holds: 'whatever at some time was not, is contingent'. In the case of the converse, I subsume an object denoted by comparison by means of time under a concept of the understanding, namely, contingency. This only requires a logical use of the understanding on a concept stemming from the form of time (same as the converse in the second example of the second class). However, I am not allowed to make the judgement the other way around. I cannot subsume the concept of the understanding 'contingency' under the concept 'temporarily existence'. If I do so, I would determine essential properties of the concept 'contingency' by a concept stemming from the forms of intuition. A concept stemming from a real use of the understanding would be determined by principles of sensation.

The difference with the other classes of subreption is not the occurrence of subsumption as such but that the subsumption depends on the form of time. In this class the axioms transcend the limit they can play in subsumption of an object under a concept. In the first class, the axioms transcend the limits of the predicate as a condition of the intuition of an object. The axioms of the second class transcend the limits their predicates can play in the formation of a concept. If I nevertheless establish these axioms, they only have a subjective status because they only express what I can cognise under the conditions of the forms of space and time. Such a cognition does not express a truth about the subject itself.

Summarising, in his dissertation of 1770, Kant transforms Wolff's analysis of judgements by broadening the notion of condition in such a way that the sensible or intellectual nature of concepts that function as conditions becomes decisive for the validity of judgements. Kant does not focus on the conditions of particular judgements of particular domains of knowledge such as mathematics, but transfers Wolff's analysis to meta-aspects of judgements.[46] In accordance with this abstract level, the condition of a judgement does not consist of the content of a specific domain, but of the epistemological source of the concepts united by a judgement. This epistemological source, either the understanding or sensibility, determines to

[45] The impossibility of an actual infinite magnitude does not imply the impossibility of the mathematical concept of infinity in general.

[46] II:411–417, §24–20.

which extent the statement holds. Since the epistemological source of a concept does not depend on contingent circumstances of a particular judgement, Kant is able to completely reject general principles, such as 'whatever is, is somewhere and somewhen'. Contrary to his later work, Kant does not rely on an extensive treatment of the human faculties to argue against traditional metaphysics. Instead, he offers an interesting approach that combines the more formal idea that only concepts of the same kind can be united into correct judgements with the idea that concepts must be divided into classes relative to the human cognitive faculties. In fact, Kant uses a logical theory for epistemological purposes, namely, a devastating criticism of metaphysics. Logical theory and analysis impose limits on metaphysics. At the same time, the purpose forces Kant to extend the logical framework such that it covers a quite wide range of metaphysical claims. This creates such a tension within the logical framework that one is justified to doubt whether it really helps to resort to logical analysis. It might be precisely for this reason that Kant in the *First Critique* presents a transcendental logic which from the very outset is designed to deal with the human faculties.

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Chapter 7
Windelband on *Beurteilung*

Arnaud Dewalque

A main challenge for philosophers of the late nineteenth and early twentieth century was to construct judgements as acts of decision or position (thesis) rather than as acts of combination or synthesis. Let us call this the *thetic view*. Franz Brentano (1838–1917) is usually regarded as the best supporter of this view, since he takes advantage of the Kantian-Herbartian notion of 'position' (*Setzung*) to break with the traditional definition of judgement as *symplokè* (Martin 2006, 64 sq.; see Brentano 2008, 335). Generally speaking I think this usual line of interpretation is quite correct, yet it could benefit from a more detailed account of the Brentano reception. What I would like to suggest is this: At stake in Brentano's legacy is not just the rejection of the synthetic view but also the way in which the thetic dimension is itself conceived. There are, in fact, various ways of constructing judgements as thetic or positional phenomena. Brentano's notion of 'existential assertion' is not the only way to do so.

In what follows, I discuss an often-neglected version of the thetic view, namely, the theory of 'assessment' (*Beurteilung*) developed by the neo-Kantian philosopher Wilhelm Windelband (1848–1915).[1] This theory is interesting for several reasons: (1) It takes judgement as a multidimensional phenomenon that involves both a subjective (psychological or *noetical*) dimension, namely, that of our judging acts, and an objective (logical or *noematical*) dimension, namely, that of the correlative propositional contents; (2) the concept of assessment is an original attempt to capture the

[1] Windelband's theory is not discussed in recent literature on judgement, except in Stelzner/Kreiser (2004, 183–202) and in Gabriel (2007). It is not even mentioned in Wayne Martin's historical-critical reconstruction (2006). As Hans Sluga recently suggested, it is arguable that 'Martin's critical discussion of the synthetic theory of judgement would have gained a great deal if he had paid attention to Windelband and Rickert' (Sluga 2008, 119; Seron 2006).

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noetical dimension of judgement in a way that is consistent with the thetic view and that is supposed to compensate for some deficiencies in Brentano's theory; (3) this multidimensional approach has exercised a deep influence on some neo-Kantian philosophers like Heinrich Rickert ([1]1892, [6]1928), Emil Lask (1912) and Bruno Bauch (1923)—and probably on Gottlob Frege as well (see Gabriel 1986; Sluga 1996). I do not intend to provide an extended discussion of each of these points. I will rather focus on the second and, more particularly, on the Brentano-Windelband controversy. The examination of this controversy could provide us with a more promising overview of some basic issues related to the thetic view.

1 Windelband's Definition of Judgement

The bases of Windelband's theory are put forth in a short text titled *Beiträge zur Lehre vom negativen Urteile* (Contributions to the Theory of Negative Judgement). This text was published for the first time in 1884, exactly 10 years after Brentano's *Psychology from an Empirical Standpoint* (1874) and 10 years before Twardowski's Habilitation thesis on content and object (1894). As we will see, Windelband's text can be regarded as an early dissident interpretation of Brentano's theory.

Roughly speaking, the view Windelband developed is that every 'judgement' (*Urteil*) is a synthesis of ideas, which is subject to an epistemic 'assessment' (*Beurteilung*). To put it differently, judging is nothing but assessing the truth-value of a propositional content. I propose to express this idea as follows:

(Df. J)	'A judges that *x* is *F*' $=_{df}$ there is a propositional content C (= 'that *x* is *F*') *and* A assumes the axiological true-false alternative *and* A assesses the truth-value of C.

According to this definition, the production of a judgement by a knowing agent depends on three main components: (1) a given propositional content, (2) the assumption of epistemic values and (3) the assessment of the propositional content itself. If one of those three components is lacking, the agent cannot be said to have produced a judgement. The first part of the definition indicates *what* is assessed in the judicative act, namely, the propositional content, and the second part indicates *how* it is assessed, namely, under assumption of the true-false alternative. The core of the definition is the concept of assessment itself, which captures the nature of any judicative *act*. For greater convenience, let us label the propositional content as the *noematical* component and the assessment act as the *noetical* component of judgement.[2]

This approach has far-reaching implications. Before discussing some of them, let us have a look at Windelband's argumentation for (Df. J).

[2] On this terminology, see Krijnen (2001), Dewalque (2006, 2010).

2 Windelband's Three-Step Argument

According to (Df. J), judging is assessing a propositional content as bearing the epistemic values of 'true' or 'false'. Thus, it would be hopeless to provide an efficient analysis of what judging is without taking this evaluative and axiological dimension into consideration. To justify this claim, Windelband develops a three-step argument. First, he identifies a predicative concept of 'assessment' (*Beurteilung*) by means of sentences that involve evaluative predicates. These sentences contrast with other sentences he calls 'judgements' (*Urteile*) or 'merely theoretical judgements' (*rein theoretische Urteile*) in a very restrictive sense. Second, he maintains that, *as far as they are asserted*, the so-called merely theoretical judgements are combined with an epistemic, *non-predicative* assessment, which shows itself through the affirmation or negation of a combination of ideas. Third, Windelband adds that this non-predicative way of assessing cannot be turned into a second-level predication, for doing so would imply a fatal *regressus ad infinitum*. Therefore, judging as an act is not a predication at all, but an 'attitude' of consciousness towards a given predication.

1. To begin with, let us consider sentences or propositions we use in order to say something about the world. All these propositions, Windelband maintains, can be divided into two distinct classes, namely, the class of predicative assessments and the class of 'merely theoretical judgements'. For instance, let us take the following propositions into consideration (Windelband [1]1884b):

(P1)	<This thing is white.>
(P2)	<This thing is good.>

Despite the fact that (P1) and (P2) have the same grammatical structure <*S* is *p*>, Windelband maintains that they do not have the same logical structure. The reason is that they express what one might call, using a Kantian terminology, two distinct 'functions of thinking'. Whereas (P1) exemplifies a 'merely theoretical judgement', (P2) exemplifies an assessment. The difference between 'merely theoretical judgement' and assessment concretely appears in the fact that <white> and <good> are not predicates that can possibly be used in the same way. At the logical or metagrammatical level, <white> is a *determinative* predicate, that is to say a predicate that has two main characteristics: (a) It is 'independent from our consciousness', and (b) it enlarges our knowledge of the thing about which we speak (Windelband [1]1884b, 29 = [9]1924, Vol. I, 30). Conversely, the word <good> denotes a logical predicate that is *not* a determination independent of our consciousness, because it precisely expresses the way our consciousness relates to the thing, namely, by assessing it in a definite perspective. That is why, in contrast to (P1), (P2) does not provide us with an enlarged knowledge of the thing. It is not a determinative predication, but an evaluative predication.

Now in order to construct an evaluative predication, the assessing agent must have some goal (*telos*) in mind because 'every assessment presupposes a definite goal as its own criterion, and it has sense and meaning only for the one who acknowledges this goal' (Windelband [1]1884b, 30 = [9]1924, Vol. I, 31). For instance, the one who forms (P2)—that is to say the one who attaches the evaluative predicate <good> to the subject <this thing> —aims at assessing the thing in an ethical perspective, namely, under an assumption of ethical values like 'good' and 'evil'. Assessing is therefore not enlarging our knowledge of the thing about which we speak. Rather, it is establishing its conformity or nonconformity to such and such *value* we assume. In this respect, Windelband concludes, we need to draw a sharp demarcation between judgement and assessment. When we form sentences like <This thing is a body>, <It's big>, <It's hard>, <It's sweet>, <It moves>, <It's at rest>, etc., we construct 'merely theoretical judgements' in the sense that we *connect* some ideas with other ideas in order to determine the thing as it is. In contrast, when we form sentences like <This thing is pleasant or unpleasant>, <This proposition is true or false>, <This action is good or bad>, <This landscape is beautiful or ugly>, etc., we are not engaged in a determination process in the strictest sense, rather we express our assessment of something that we are supposed to have determined the nature of before we assess it (Windelband [1]1884b, 30 = [9]1924, Vol. I, 30). Accordingly, the distinction between 'merely theoretical judgement' and assessment is first and foremost about the kind of predicates we use to construct sentences about the world surrounding us.

2. Windelband's main concern, however, is not to divide all our sentences into 'merely theoretical judgements' and into assessments. This division is, so to speak, the first and not the last word of his theory. Rather his actual goal is to establish that in the concrete life of consciousness the determinative function and the evaluative function we have just distinguished are mostly *combined* and cannot be separated except by an abstraction process. (This point is what I have called the second step of his argumentative strategy.) His argument focuses on the analysis of (P1). It goes like this: If we consider the utterance of (P1) in a living context, that is to say when someone actually *asserts* that this thing (*e.g.* this flower) is white, then (P1) appears as the expression of a definite act of assessment, for asserting an affirmative sentence is the same as assessing it as a 'true' sentence. To be sure, in the case of an affirmative sentence like (P1), there is no linguistic mark expressing that assessment. Only in the case of a negative sentence like <This thing is not white> is the evaluative function expressed at the linguistic level, namely, by the negation (Windelband [1]1884b, 31 = [9]1924, Vol. I, 32). Windelband, nevertheless, does not see any reason to analyse the two cases in a different way. No matter whether the epistemic assessment is expressed by a linguistic mark or not, as soon as we take a sentence not only as a connection of ideas but also as something we *affirm* or *deny*, the 'merely theoretical' function of thinking, which consists of determining *x* as being *f* (*e.g.* this thing as being white), is already combined with the evaluative and 'practical' function, which

consists of assessing the whole proposition content <*x* is *f*> (<This thing is white>) as true or false. Windelband concludes that:

> All the propositions of knowledge involve thus already a mixed combination of judgement and assessment: they are combinations of ideas whose truth-value we decide by means of affirmation or negation. (Windelband 1884b, 31)[3]

As a result, it would be a theoretical fiction to reduce our knowledge procedures to a value-free determination process. Knowing is an axiological behaviour, and the objectivity of our epistemic procedures depends on the assumption of epistemic values. The key argument is that a mere combination of ideas without epistemic assessment will never be an actual judgement (it could be a complex idea or, at best, a question). Judging precisely is nothing but assessing the truth-value of a propositional content, no matter whether this propositional content is constructed with a determinative predicate as in (P1) or already involves an evaluative predicate as in (P2).

3. Insofar as the concept of assessment has been introduced by means of an analysis of predicative sentences, it might seem that epistemic assessments should be rendered by means of predicative second-level constructions. And indeed, every judicative act can be said to be *equivalent* to (yet not identical with) a second-level predication in which we ascribe the evaluative predicate 'true' or the evaluative predicate 'false' to the first-level predication or to the propositional content taken as a whole. Still, that does not mean at all that assessing ipso facto means constructing a second-level predication. For instance, it is possible to make the epistemic assessment involved in the assertion of (P1) more explicit by means of a second-level predication like <It is true that this thing is white> or <The proposition that this thing is white is true>. Lotze (1874, § 40 = 1989, 61) adopted just such an analysis and called these second-level predications 'adventitious judgements' (*Nebenurteile*). However, Windelband ([1]1884a = [2]1921, 170) argues that, if the word <is> expresses a mere connecting function in both occurrences, if it is a mere operator to construct some complex representational contents, then the second-level predication does not imply the slightest judicative decision. What we actually obtain is not a judgement, but rather a multilevelled propositional content, which forms at the very least the content of a new *potential* judgement or the content of a question as well: <Is the proposition 'this thing is white' true?>. To obtain a judgement, we then should perform a higher-level predication, that is to say a predication in which the second-level predication itself would receive the predicates <true> or <false>,

[3] Of course (S2) is also a propositional content, and, as such, it can be subject to an epistemic assessment as well. In that case we would have to consider two distinct assessments: on the one hand, the ethical assessment of the thing itself, which is said to be <good>, and on the other hand, the epistemological assessment of the whole propositional content <this thing is good>, which is asserted as being true.

> and so on ad infinitum. So the epistemic assessment, which is the distinctive mark of judicative phenomena, would be indefinitely postponed—which would be senseless. To avoid such complications, one has therefore to admit that judging is not a matter of predicative content. It is simply not a way of constructing propositional contents, but rather a certain attitude of consciousness towards given contents (*id.*).

3 Judgeable Content

So far I have emphasised Windelband's theory of judgement out of consideration for his link to the Brentanian theory. A prominent part of the 'Contributions to the Theory of Negative Judgement', however, is devoted to the discussion of Brentano's theses. In a decisive passage, Windelband precisely states that Brentano has anticipated (Df. J):

> The exposition Brentano has given — starting from quite other, merely psychological viewpoints — is as clarifying and convincing as Bergmann's logical reflection on the issue [see Bergmann 1879; AD], and they both lead to the same result: to demonstrate in 'judgement', besides the function of representing (*Vorstellen*) or of combination of ideas (*Vorstellungsverbindung*), the other function of (approving or disapproving) assessment. (Windelband [1]1884a = [2]1921, 172)

This reference to Brentano is not astonishing, since Brentano's theory may be regarded as the prototype of the *thetic* view. To be sure, Windelband's concept of epistemic assessment also aims at capturing the positional or thetic dimension of any judgement, since it implies that judging is not properly combining some ideas together, yet rather assessing a *given* combination of ideas or a propositional content, which has been previously formed and is available for becoming the content of a judicative act. Predication is at the very least a property of judgeable *content* (a noematic property), provided that a judgeable content is not a full judgement yet until it is assessed as 'true' or 'false'.

To some extent, this use of the act-content distinction might be considered similar to that of Brentano. Indeed, according to Brentano too, judging is simply not a way of constructing a propositional content, for propositional content is common to judgement and to question. For instance, asking or hearing the question <Is this tree red?>, Brentano remarks, I perform exactly the same connection between the representational contents <tree> and <red> as in the judgement <This tree is red>. My ability to understand the question exactly indicates that I already grasp the representational content as a whole (Brentano 2008, 228). Yet unlike judgement, question is neither true nor false. It does not imply any epistemic decision coming from the agent. In order to provide a usable theory of judgement, one has to capture precisely the difference between questioning and judging. And this requirement does not enable us to adopt, say, a noematical concept of judgement, because noematical components may be the same in judging and questioning acts. In short, both

Windelband and Brentano maintain that judging is not a matter of content, but rather a matter of act. Both of them develop a noetical concept of judgement.

Nevertheless, for this comparison with Brentano to be instructive, it is necessary to refine our approach and to distinguish at least three Brentanian theses. Let us express them as follows:

(T1)	Every judicative act is grounded on a representational act (*grounding thesis*).
(T2)	A judgeable representational content does not need to be compounded; it may be simple (*simplicity thesis*).
(T3)	Every attributive judgement is reducible to an existential judgement (*reducibility thesis*).

Just like Brentano, Windelband assumes (T1). Every judicative act surely needs a content to which it applies, and only representing acts are likely to provide us with a judgeable content. Thus, every judging act is constructed upon a representing act; it is, so to speak, an act of higher level. So we could have representations without assessment (for instance, in the case of hypotheses or questions), but we could not perform any assessment without an assessed idea. The availability of a representational content is the first condition in order to perform a successful judgement. Nevertheless, Windelband remarks it is not necessary to understand (T1) as the claim that we have a *succession* of acts in the stream of consciousness. On the contrary, it is possible that representation and judgement occur simultaneously in the same act. In Windelband's terms, the 'representing function' and the 'assessment function' can form 'two aspects (*Momente*) of one single indivisible act' (Windelband [1]1884a = [2]1921, 175). Otherwise, Windelband fully agrees with Brentano that judging is nothing but a certain way for consciousness to relate to any representational content.

Yet another question to consider is whether any representational content is a judgeable content or not. Supporting (T2), Brentano answers that question affirmatively. Whether a given representational content is simple or complex, as a representational content it is in either case available for being acknowledged-as-true or rejected-as-false. When I judge that *S* is, Brentano maintains, I just acknowledge *S*—and not the existence-of-*S*. Windelband, however, does not admit this view. Unlike Brentano, he explicitly rejects (T2): When I state an affirmative judgement, he writes, the 'acknowledgement (*Anerkennung*) is never directed towards a simple representational content, but always deals with a relation (*Beziehung*)' (Windelband [2]1907, 192). Accordingly, in order to be a judgeable content, a representational content must be complex or compounded. Even the subject of an existential judgement is not a simple idea, for 'every "judgement" involves a combination of ideas as object of its assessment' (Windelband [1]1884a = [2]1921, 182). So, though Windelband rejects the synthetic theory of judgement at the noetical level, he keeps it at the noematical level, that is to say at the level of judgeable or assessable contents. This is his first *anti-Brentanian* thesis.

Despite his rejection of (T2), Windelband agrees with (T3). Insofar as judging itself is not producing a combination of ideas by means of a predicative feature, every judicative act may be expressed in a non-predicative way. So every judgement

would be reducible to an existential assertion of the form <*Sp* is/is not> or <There is/is not *Sp*>. As Brentano remarks, this reducibility thesis applies to all forms of judgement of Aristotle's so-called logical square. Brentano's main idea is that in all the judgements we state, including the so-called universal judgements, we take something to be existent. Every epistemic decision is thus also, in Brentano's view, an *ontological* decision, for it is the same to acknowledge as true the connection between <a rose> and <red> and to judge that there is a red rose. If the affirmative judgement is correct, it has an object.[4] Using the notation introduced by Brentano's pupil, Franz Hillebrand (1891), one therefore obtains the following interpretation of Aristotle's logical square (note that the sign <–> does not have exactly the same meaning when occurring outside or inside the brackets: Outside, <–> denotes the negative existential operator, correlated to the positive existential operator <+>; inside <–> denotes a negative predication-operator)[5]:

<Every rose is a flower>=	<No rose is an animal>=
<There is no rose that is not a flower>	<There is no rose that is an animal>
– (*S-p*)	– (*Sp*)
Brentano's interpretation of Aristotle's logical square	
+ (*Sp*)	+ (*S-p*)
<Some rose is red> =	<Some rose is not red> =
<There is a red rose>	<There is a rose that is not red>

What is, then, Windelband's own position towards this interpretation? As I have said, he agrees with (T3), yet he develops a more nuanced approach. Brentano's interpretation, he points out, implies that every propositional content is likely to be *nominalised* in order to become the subject of an existential judgement. For instance, the judgement <God rules the world> may be rendered by <God's ruling of the world *is*> (Windelband [1]1884a = [2]1921, 180). According to Windelband, Brentano would be right in maintaining such a reducibility, yet he would be wrong in neglecting the various meanings of the word <is>, which result from this nominalization process (*ibid.*, 184). So, unlike Brentano, Windelband does not maintain that every judgement is equivalent to an existential position in the proper sense, for not every epistemic assessment is about the existence of actual objects.

His main argument consists of defending another anti-Brentanian thesis. Let us call it the *ambiguity-thesis* of the *existential* <is>. Consider the following propositions:

[4] This thesis has had a lot of logical and ontological implications (see Leclercq 2008). Regarding logical implications, it appears that the particular affirmative judgement is the most fundamental judicative form while the universal affirmative is the most complicated one, since it would involve a double negation.

[5] This difference has been rightly pointed out by Antonelli (2011, LXVII).

(P3)	<Freedom *is*.>
(P4)	<God *is*.>
(P5)	<Lightning is the cause of thunder.>
(P6)	<Every rose is a flower.>

(P3) and (P4) are existential propositions, yet it is already doubtful, Windelband says, whether the word <is> has exactly the same meaning in both of them. Contrasting with (P3) and (P4), (P5) and (P6) are not existential propositions. Nevertheless, if one assumes the reducibility thesis, as Brentano and Windelband do, it must be possible to translate them into existential propositions. As for (P5), Windelband proposes the following existential translation:

(P5*)	<A causal relationship between lightning and thunder *is*.>

Now, Windelband argues, the word <is> in (P5*) does not seem to have the same meaning as in (P3) and (P4), because a causal relationship cannot be said to 'exist' in the same sense as a property or an actual being is said to exist. The issue becomes clearer again with (P6). As we have seen, Brentano maintains that (P6) should be existentially rendered by means of a double negation: <A rose, which is not a flower, is not.> Obviously, this solution enables Brentano to conserve the univocal nature of the existential <is>: The word <is>, used in existential constructions, simply means <exists>. Yet Windelband's existential translation of the same judgement is quite different. According to him, one should render (P6) by:

(P6*)	<The subordination of the rose to the concept of flower *is*.>

In such an existential judgement, the word <is> obviously has a very specific meaning, for the subordination to a concept is not something that can actually exist the way that tables, chairs or roses do. In this case, Windelband claims, our judgement does not deal with any existential positing of objects. We certainly perform an epistemic assessment, yet this epistemic assessment is not equivalent to an existential position *stricto sensu*. What does it mean then? Windelband's answer is here taken from Lotze: The word <is> in (P6*) does not mean <is actually being> but rather <is valid> (*gilt*). So, Windelband concludes being (existence) and validity are two distinct meanings of the word <is> in existential constructions. They are meanings of the *relation*, which form the judgeable content. For instance, <being> is the meaning of the so-called inherence-relationship, which links a property (<red>) to a thing (<this rose>), while <validity> is the meaning of a conceptual subordination or class-inclusion. Even if those distinctions depend in each case on the assessed content (that is to say on the nominalised subject of the existential judgement), Windelband considers that they have an impact on the meaning of the assessment itself (Windelband [1]1884a = [2]1921, 184). Therefore, a fully developed

theory of judgement has to account for the so-called division of judgements from the (noematic) viewpoint of relation.[6]

4 Assessing Under Assumption of Epistemic Values

The axiological dimension of judgement is a serious point of disagreement between Windelband and Brentano. For Brentano the rejection of the traditional predicative view takes place within the framework of an 'idiogenetic' theory of judgement, that is to say a theory in which judging acts are psychic phenomena sui generis. (The appellation comes from Franz Hillebrand.)[7] This idiogenetic theory could be rendered by means of a basic idea: Judicative acts are reducible neither to representing acts nor to 'love and hate' phenomena (affections and volitions); they form a specific class of mental phenomena. All mental phenomena surely have something in common, namely, an 'intentional relationship' to any content or any 'object'. In all our mental acts, Brentano writes, there is always something represented, believed, desired and so on. Yet this intentional relationship presents various modalities, and those modalities provide Brentano with a usable criterion for a classification of mental phenomena. Each class of acts has its intentional modality as something specific (Brentano 2008, 106). For instance, when we simply grasp something in a thought without assessing it, then the act we perform can be named a representing act. Contrasting with the other classes of mental phenomena, such acts are characterised by their *neutrality*: They imply no thetic activity at all. They form, so to speak, the lowest level of intentionality. In contrast, judging is not only grasping something but also—*in addition to that*—'acknowledging as true' or 'rejecting as false' what is grasped in representing acts. Brentano speaks of a *double* intention, for judicative intention is added to the representational one (Brentano 2008, 223). It implies a new intentional modality, which is specific to judging acts, namely, a decision about truth or falsity.

Windelband, however, does not agree with this idiogenetic view, for he does not consider judicative acts as a specific kind of mental act. While Brentano defends the idea that there are three classes of mental phenomena (representations, judgements and the so-called 'love and hate' phenomena), Windelband only admits two classes,

[6] This claim is controversial. See, e.g. Hillebrand (1891, 33): 'The viewpoint Windelband defends here would lead to huge consequences. For, as soon as one begins to transfer features of *judicative material (*Urtheilsmaterie*)* to *judicative function (*Urtheilsfunction*)*, any unified explanation of the judicative function directly disappears.'

[7] Hillebrand 1891, 26–27: 'Since what is characteristic to his [= Brentano's] theory consists in the fact that he considers judging as a specific kind (*idion genos*) of psychic phenomena, meanwhile all the other theories believe that one has to regard it simply as a certain composition of psychic elements belonging to an other kind (*allo genos*), we can refer to the first one as an *idiogenetic* theory of judgement and to all the other ones as *allogenetic* theories of judgement.'

namely, that of non-axiological behaviours (representations) and that of axiological behaviours (including judgements, affections and volitions). Instead of a *tripartition*, he finds a *bipartition*.[8]

The opposition is obvious. According to Brentano, the delimitation between judging and representing is not sufficient to provide a satisfying theory of judgement. One also has to distinguish judgemental intentionality from affective intentionality. Affections and volitions have alternative characters too, just like judgements. For instance, I can refer to something as desirable or non-desirable. Therefore, there is a certain *analogy* between judgemental and affective intentionality: They are both non-neutral phenomena. But according to Brentano, judgements, despite their alternative nature, do not belong to the same class as affections and volitions. The reason is that notions such as 'desirable', 'good' and so on are gradual and often relative. Something can be more or less good ('better', 'worse' and so on), and it can be good in itself or with respect to something else. In short, Brentano maintains that gradation and relativity are distinctive properties of 'love and hate' phenomena. In contrast to such phenomena, judicative intentionality is non-gradual; it is either acknowledging-as-true or rejecting-as-false. There is no third way, because it makes no sense to say that something is more or less true than something else—nor that something is true with respect to something else (see Brentano 1930, 2008, 405 sq.).

However, Windelband defends the exactly opposite view. According to him, gradation applies both to affections and to judgements. Acknowledging and rejecting are nothing but two *poles*, between which a series of intermediate judicative decisions take place. This interpretation, he believes, is the only way to account for the concept of *probability*, insofar as probability corresponds to a gradation of certitude, which attaches to our epistemic assessments (Windelband [1]1884a = [2]1921, 148; 1913, 27). Correlatively, judicative acts would not be directed towards a world of objects but rather towards a world of values. The most important point, in this respect, is that 'true' and 'false' are values, and as such, Windelband claims, they have to be *coordinated* to other values like 'desirable', 'good' and so on. Consequently one may hardly talk about an 'idiogenetic theory' in Windelband's case because according to him judicative acts do not form a specific class of acts but rather a *subclass* within the generic class of 'practical' and axiological behaviours. Unlike representations, judgements belong to the same *genos* as affections and volitions.

[8] Windelband's pupil, Heinrich Rickert, who maintains that Brentano's theory of judgement has 'great merits' but that his classification of psychological phenomena is 'highly questionable', also endorses this criticism. See Rickert [6](1928, 169): 'No doubt, Franz Brentano, who has discussed our issue in a detailed way and has clearly shown that judging is not representing, has great merits in this respect. But the details of his psychological theory are insignificant for our epistemological problem and his classification of psychic phenomena, taken as a whole, is even highly questionable'. The reference to Brentano's theory of judgement is again treated even more negligibly in Lask 1912 and is completely absent in Bauch (1923, 156), where the name of Brentano is simply no longer mentioned. This suggests that the links connecting Windelband's theory of judgement to Brentano's were progressively broken off by his heirs.

Only the nature of the values differs: 'true' (in epistemic behaviour), 'good' (in ethic behaviour), 'beautiful' (in aesthetic behaviour), 'pleasant' (in hedonistic behaviour) and so on. Moreover Windelband's rejection of Brentano's idiogenetic theory has another consequence, namely, the introduction of a third judicative 'quality' besides affirmation and negation.

5 The Nature of Epistemic Assessment

While the concept of proposition is supposed to capture the noematic component of judgement, namely, the judgeable content, the concept of assessment is supposed to capture the noetic component, namely, the assessing *act*. As we have seen, propositional content may be nominalised in order to become the subject of an 'existential' construction <*Sp is*>, to the effect that the word <is> becomes ambiguous and is likely to admit various meanings (<is actual>, <is valid>). This is what Windelband means when he says that we need to divide judgements from the point of view of 'relation'. Yet the various forms of assessment themselves provide us with a second division principle. Generally speaking, 'relation and quality are the two essential and equally indispensable characteristics of judgement, and they determine the division according to which the doctrine of judgement must be developed in the field of pure logic' (Windelband [2]1907, 192).[9] What is then the division of judgements from a noetic or qualitative point of view?

Brentano asserts one has to admit two and only two judicative qualities, namely, affirmation and negation, because each judgement is the expression of a 'yes, it is the case' or of a 'no, it is not the case'. In other words, every judgement is either affirmation or negation, *i.e.* judging is either 'acknowledging-as-true' (*als wahr Anerkennen*) or 'rejecting-as-false' (*als falsch Verwerfen*). Windelband considers judging an *alternative* behaviour too. But the way he understands this alternative is quite different from Brentano's, for he maintains that acknowledgement and rejection are two poles, between which a lot of intermediate judicative decisions take place. Epistemic assessment, Windelband asserts, is a *gradual* phenomenon. The graduation is an 'intensity of certitude' (*Intensität der Gewissheit*) (Windelband [1]1884a = [2]1921, 187), and as such it is a characteristic of all judgements insofar as they are more or less certain. This is the reason why, besides acknowledging and rejecting, Windelband admits again another judicative form of assessment. It consists of suspending our affirmative or negative commitment. In this respect, Windelband is much closer to Lotze than to Brentano.

Windelband's argumentation runs as follows: If one gradually progresses from affirmation (or negation) to indifference, then one finally reaches the 'zero-point of

[9] Note that, unlike Kant (KrV, A70/B95) and like Christoph Sigwart ([1]1873, 170, 192, 258 = [3]1904, 216, 238–239, 311), Windelband does not recognise *quantity* and *modality* as relevant division principles: *Relation* is 'the only division principle of judgements beside quality' (Windelband 1900, 46). On Windelband's appraisal of Kant's so-called table of judgements, see Gabriel (2007).

the assessment scale'. Yet this zero-point is likely to admit two meanings. On the one hand, it may correspond to the question (for instance, <Is this tree green?>). In this case, the absence of affirmation and negation is what Windelband calls 'total indifference'. Lotze considers it as a third judicative quality, but Windelband does not, because he wants to preserve the contrast between questioning and judging. On the other hand, the so-called zero-point of the assessment scale may be understood as the expression of an actual epistemic assessment, namely, the decision to suspend our assessment, for instance when we have no sufficient reason to affirm rather than to deny. (In such a case, the result is an assertion like: <This tree is maybe green>/<maybe not>.) Unlike Sigwart and almost all the other logicians of the time, Windelband considers this sort of minimal decision as a third judicative quality and names it 'critical indifference'. (In this expression, the adjective 'critical' clearly must be understood to be from the Greek term *krinein*, meaning judging or deciding; so critical indifference is an indifference that arises in judgements: It is a kind of epistemic assessment.)

I have already mentioned the notation Hillebrand has proposed to express the Brentanian theory of affirmation and negation. This notation is not sufficient, however, to capture all the qualitative distinctions drawn by Windelband. Therefore, for greater convenience I propose to extend Hillebrand's notation as follows:

(R)	I have the representation of *S*-being-*p*	_ (*Sp*)
(Q)	I wonder whether *S* is *p*	? (*Sp*)
(J1)	I affirm that *S* is *p*	+ (*Sp*)
(J2)	I deny that *S* is *p*	– (*Sp*)
(J3)	I do not know whether *S* is *p*	= (*Sp*)

These distinctions are deserving of significant comment. I will only mention three points. First, we have seen that Windelband maintains—just as Brentano does—that the first condition to perform a judgement is to have some representations: (R) is a necessary condition for (J1), (J2) and (J3). Second, contrasting with (R), (Q) is not necessarily presupposed by (J1), because an affirmative judgement about something perceived is not necessarily an answer to a question. Yet Windelband maintains that (Q) is presupposed by (J2) because every negative judgement is an answer to a question (Windelband [1]1884a = [2]1921, 177; see already Sigwart [1]1873, 137 = [3]1904, 182). Thus, negation would be 'less original' than affirmation. For that reason, it seems that Windelband does not admit an *absolute* coordination between (J1) and (J2). Third, one of the most peculiar points of Windelband's view is that he admits (J3) as a third judicative quality. In this respect Windelband's theory of judgement is less economic in qualitative distinctions than Brentano's.

Now, if one takes a quick look at the later theories of judgement developed by Rickert and Frege, one will observe the inverse tendency, namely, the tendency to transfer indifference (Rickert), and even negation (Frege) into judgeable content. We find by them, so to speak, a *noematization* of act qualities. According to Rickert, (J3) would be reducible to an affirmative judgement with a multilevelled content like <I affirm that I do not know, whether *S* is *p*> (Rickert [6]1928, 177). In our

notation, + [= (*Sp*)], where it appears that the affirmative operator <+> does not apply to (*Sp*) itself, but to the complex content [= (*Sp*)]; it is an assessment of [= (*Sp*)] and not of (*Sp*). Moreover, Frege ([1]1879; [2]1964, § 2 sq.) argues that negation itself should be transferred into judgeable content, to the effect that even (J2) should be rendered by an assertion of the form <I affirm that it is not the case that *S* is *p*>. Here again, the judicative quality is noematised, and the result is an affirmative act with a multilevelled content: + [− (*Sp*)]. In short Frege's 'razor' is sharper than Windelband's and Rickert's: 'Acknowledgement' (*Anerkennung*) becomes through Frege the only judicative quality. That is why in his *Ideography* he considers it sufficient to admit a single judgement stroke that symbolises the same assertive force (*Behauptungskraft*) in all our judging acts. Accordingly, there are no distinctions on the noetical side.

To close this reconstruction of Windelband's theory, let us consider the unexpected logical translation Windelband proposes at the very end of his 'contributions'. This unexpected translation is about the negative proposition <No rose is an animal>. Following what we have seen, this judgement should be rendered by <The subordination of the rose to the concept of animal *is not*> (*i.e.* <…is not valid>). In this respect, the universal negative judgement would be a negative assessment about the validity of a conceptual subordination. But, paradoxically, Windelband proposes another analysis. He chooses to render such a judgement by the affirmative existential judgement <The exclusion between the concept of rose and the concept of animal *is*> (*i.e.* <…is valid>). This solution is all the more surprising because Windelband usually treats negation as a qualitative concept, namely, as mere as-false-rejecting. Here, in contrast, he proceeds to a noematization of the rejection: He transfers negation into the judgeable content and thus understands the universal negative judgement as the affirmation of a negative content, namely, of an exclusion—just as Frege does.

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Chapter 8
A Priori Knowledge in Bolzano, Conceptual Truths, and Judgements

Stefan Roski

According to Kant, a true judgement can be called *a priori* in case it can take place absolutely (*schlechterdings*) independent of experience. Propositions that are knowable in this way are called *a priori propositions* by him (Kant 1787 B, 3–4). As is well known, the class of those *a priori* propositions that are *synthetic* was particularly important for Kant. In contrast to analytic propositions, they are supposed to contain nontrivial information about the world and yet be irrefutable by experience. Not many of his critics were satisfied with Kant's way of drawing this distinction. Peter Strawson, for example, writes in his commentary on the *Critique of Pure Reason*:

> Kant nowhere gives an even moderately satisfactory theoretical account of the dichotomy between analytic and synthetic *a priori* propositions; nor can any be gleaned from his casually scattered examples. (Strawson 1966, 43)

One of Kant's most emphatic critics in the nineteenth century – Bernard Bolzano – would undoubtedly have affirmed Strawson's remark. In contrast to the latter, however, Bolzano did not rest with this conclusion but tried to *give* a satisfactory theoretical account of the notion of synthetic *a priori* proposition. Roughly speaking, he located Kant's mistake in the attempt to introduce a distinction among propositions by means of a distinction among judgements. Bolzano reversed this order and aimed instead to explicate the valid core of what Kant tried to capture in epistemic terms entirely in objective, logical ones. Mark Textor has called this approach *objective explication* (Textor 1996, 195ff.).[1]

[1] As will become explicit below, what Bolzano does is not too different from what Carnap calls "explication," namely, "the task of making more exact a vague or not quite exact concept used in everyday life or in an earlier stage of scientific or logical development, or rather replacing it by a newly constructed, more exact concept" (Carnap 1956, 7).

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Bolzano's explication has two aspects, a logical and an epistemological one. The logical aspect consists in drawing a precise and workable distinction in terms of non-epistemic notions. The epistemological aspect concerns the way in which Bolzano's suggestion might work: What is his account of how one can come to know synthetic truths *a priori*?

While there have been investigations of Bolzano's objective explication of the notion of *a priori* proposition (see Textor 1996, chapter 4), the epistemological details have never been examined in great detail.[2] The task of this chapter is thus to tell the epistemological story behind Bolzano's objective explication.

I should note right from the beginning that the aim of the chapter is descriptive and historical. Primarily, I want to make sense of what Bolzano plausibly had in mind, rather than assess its intrinsic plausibility.

1 *The* Apriori *in Bolzano*

The key notion to understand Bolzano's account of the *apriori* is the notion of a conceptual truth.[3] As it is used in contemporary philosophy, the term "conceptual truth" is often taken to be interchangeable with "analytic truth." Further, if the notion of analyticity is accepted at all, analytic truths are taken to be knowable *a priori*. Neither of these claims holds with respect to Bolzano's use of those terms. What he calls "conceptual truths" is something different from analytic truths, and the latter are not necessarily knowable *a priori*, whereas the former are. The following section will be devoted to making the concept of conceptual truth and its relation to the concept of analytic truth precise. In order to do this, I have to introduce several concepts of Bolzano's logic, in particular his concepts of proposition and of representation.

1.1 Concepts and Conceptual Truths

The concept of proposition (*Satz an sich*)[4] is one of the fundamental (undefined) concepts of Bolzano's logic. Propositions are abstract objects which are either true or false and can be the matter[5] of judgements, sentences, or utterances (much like

[2] An exception is Lapointe (2010).

[3] I use "the *apriori*" here as an umbrella term for the concepts of *cognition* or *judgement a priori* and of *a priori propositions* or *truths*.

[4] Since the translation of Bolzanian terminology is not always straightforward, I always add the German terms in brackets. When I quote longer passages from Bolzano, I also quote the German original in a footnote. Unless indicated otherwise, all translations of Bolzano's texts are my own.

[5] In this chapter, I will use the term "matter" (which is a translation of Bolzano's term "Stoff [eines Urtheils]") to refer to what is normally called the *content* of a judgement, i.e., the proposition that the person who judges holds to be true. The reason for this is that the term "content" is used by Bolzano in another sense.

Frege's *Gedanken*). Other important logical concepts are introduced essentially in mereological terms: Those *parts* of a proposition which are not themselves propositions are called *objective representations* or *representations-in-themselves* (*Vorstellungen an sich*).[6] Representations (I omit the "in itself" in the following when no confusion can arise) are either simple or complex (they may or may not have parts) and refer to one object or plurally or even not at all.[7] Thus, representations are distinguishable with respect to (a) their extension, the objects that "fall under" them; (b) with respect to their content, the parts they are composed of; but also (c) with respect to their order of composition.[8] Given this, obviously there can be (and, according to Bolzano, there are[9]) representations that are simple, that is, that do not contain any parts and only have one single object as their extension. Bolzano calls those representations *intuitions* (*Anschauungen*). Now, intuitions can be the parts of complex representations. If a complex representation consists solely of intuitions, it is called *pure intuition* (*reine Anschauung*), otherwise *mixed representation* (*gemischte Vorstellung*). Representations that neither are nor contain intuitions are called *concepts*. *Conceptual propositions* (*Begriffssätze*) are accordingly those propositions that are composed solely out of concepts (*WL*, §133). In form of a definition:

(CP) *P* is a conceptual proposition iff no constituent of *P* is or contains an intuition.

A *conceptual truth* is simply a true conceptual proposition. An example given by Bolzano is the proposition expressed by the sentence "There are propositions." Conceptual propositions are opposed to empirical propositions (*Anschauungssätze*), which always contain at least one intuition (*WL*, §133). Put again in form of a definition:

(EP) *P* is an empirical proposition iff at least one constituent of *P* is or contains an intuition.

An *empirical truth* is accordingly a true empirical proposition. An example is the proposition expressed by "This smell is sweet," in which "this" expresses, according to Bolzano, an intuition that refers to a particular mental event. In fact, Bolzano

[6] Within Bolzano (1969ff./1837) (henceforth *WL*), Bolzano was reluctant to accept this characterization as a proper definition. In a letter to Franz Exner, however, he considered it to be one (Cf. Bolzano and Exner 2004, 141f.).

[7] Note that Bolzano rejects the traditional doctrine that with increasing complexity of a representation the cardinality of its extension decreases (Cf. *WL*, §120).

[8] Two extensionally equivalent representations which are composed out of the same parts can still be different, according to Bolzano. Take, for example, the representations expressed by "2^4" and "4^2." For a criterion of identity for Bolzanian representations, see Morscher (2008), p. 50f.

[9] In Bolzano's times, this thesis was far from trivial. According to the then widespread doctrine that the cardinality of the extension of a representation decreases with increasing complexity of its content, representations whose content has the smallest possible complexity and which yet have only one object as their extension must have seemed to be an impossibility.

seems to assume that intuitions *always* refer to particular mental events (*WL* I, 331, cf. Textor 1996, 91ff.).[10]

1.2 Conceptual Truths and Judgements A Priori

The distinction between conceptual and empirical truths is used by Bolzano to explicate several concepts that are found within the work of his predecessors. The ones I will be concerned with in the following played a prominent role in the works of Kant and Leibniz, namely, those involved in

> the division of our cognitions (*Erkenntnisse*) into those we can become convinced of (as we usually say) only by experience and others that require no experience. […] [T]his division of our cognitions almost coincides with the division of propositions into conceptual and empirical propositions, since the truth of most conceptual propositions can be decided by mere thought without any experience, while propositions that include an intuition can in general be judged only on the basis of experience. (*WL* II, 36)[11,12]

By "the division of our cognitions" (*die Eintheilung unserer Erkenntnisse*), Bolzano obviously refers to the distinction between cognitions *a priori*/*a posteriori* in Kant.[13] A cognition (i.e., a true judgement) is called *a priori* by Kant iff it can take place absolutely (*Schlechterdings*) independent of experience – otherwise, it is called *a posteriori* (Kant 1787 B, 3–4). Derivatively, Kant also divides the class of propositions into those that are knowable *a priori* and those that are knowable *a posteriori*.

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[10] It has often been noticed that Bolzano's intuitions bear some resemblance with the logically proper names of the early Russell. Mark Textor has argued that with his concept of an intuition, Bolzano provides a *structural* characterization of direct reference, which is the function of Russell's logically proper names (Textor 1996, 89). Bolzano's doctrine of intuitions has some crucial consequences. Since he seems to assume that intuitions always refer to mental events, namely, sensations, it follows that each representation that refers to a single *abstract* object must be complex (cf. Textor 1996, 79). Thus, Bolzano is committed to the thesis that number terms like "four" express complex concepts. Conversely, every simple concept has to refer plurally (like the one expressed by "some") or not at all (like the ones expressed by "has" or "and"). For a critical discussion of Bolzano's theory of intuition, see also George (1999).

[11] "die […] Eintheilung unserer Erkenntnisse in solche, von denen Richtigkeit wir uns (wie man zu sagen pflegt) nur durch Erfahrung allein überzeugen können, und in andere, die keiner Erfahrung bedürfen […]. [D]iese Eintheilung unserer Erkenntnisse [fällt] mit jener der Sätze in Begriffs- und Anschauungssätze beinahe zusammen […]; indem die Wahrheit der meisten Begriffssätze durch bloßes Nachdenken ohne Erfahrung entschieden werden kann, während sich Sätze, die eine Anschauung enthalten, insgemein nur aus Erfahrungen beurteilen lassen" (*WL* II, 36).

[12] Bolzano's claim that only "most" conceptual propositions can be known without the aid of experience presumably derives from his assumption that there are conceptual propositions that are too complex for human beings to grasp. He mentions moreover that to derive some conceptual propositions, one has to rely on memory, which Bolzano classifies as a kind of experience (cf. *WL* III, 214).

[13] And also to the related distinction between truths of reason and truths of fact in Leibniz. Bolzano cites Xenophanes, Parmenides, the Eleats, Plato, Descartes, and Cudworth as other philosophers who already drew similar distinctions (*WL* III, 166).

A proposition is said to be *a priori* iff it can be the matter of a cognition *a priori*.[14] Kant thus clearly starts his analysis on the level of judgements (his cognitions are events, things that can *take place*) and uses a distinction drawn there to introduce a related one on the level of propositions.

Although Bolzano approved the high importance of the Kantian distinction, he was unsatisfied with the direction of explanation implicit in Kant's way of introducing it:

> [T]his title [i.e. "*a priori* truths", S. R.] seems to me not so appropriate, in that it is merely borrowed from the way in which we come to know those truths in most cases. (Bolzano 1975/1833–41, 61)[15]

Rather than drawing a distinction among propositions by reference to the epistemic and modal properties of the corresponding judgements,[16] Bolzano tried to explain[17] those epistemic properties by structural properties of *propositions* that are the *matter* of the respective judgements[18]:

> If the given proposition consists of pure *concepts* […] then its truth or falsity depends solely on the properties of those concepts. […] Thus, truths of this kind (pure *conceptual truths*) you come to know in virtue of the fact that you know the concepts they are constituted of. (*WL* I, 181)[19]

Bolzano's idea seems to be this: Why are conceptual propositions knowable independently of experience? *Because* the concepts of which a proposition consists and their respective properties are not determined in any way by empirical matters. Thus, they can be known independently of empirical matters: *a priori*. Analogously, *because* empirical truths contain representations that are such that they can only refer to particular empirical events – intuitions – they can only be known *a posteriori*. Accordingly, the very notions from which Kant started, that is, the notion of a *judgement a priori* and the notion of a judgement *a posteriori*, are introduced by Bolzano only later in terms of the notion of a conceptual proposition:

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[14] Cf. Kant (1787) B, 3–4, Textor (1996), 195–207.

[15] "Nun däucht mir diese Benennung [i.e. 'Wahrheiten *a priori*', S.R.] nicht so Zweckmäßig, weil sie bloß von der Art, wie wir zur *Kenntnis* solcher Wahrheiten in den meisten Fällen gelangen, entlehnt ist."

[16] They *can* take place independently of *experience*.

[17] By using the term "explain" here, I want to emphasize that Bolzano's explication is supposed to enable him to answer certain why questions, e.g., why *do these propositions have such and such epistemic properties?*

[18] Cf. Textor (1996), 207ff., Proust (1989), 52, Berg (1987), 14.

[19] "Wenn der gegebene Satz aus bloßen *Begriffen* besteht […] dann hängt die Wahrheit oder Falschheit desselben bloß von der Beschaffenheit dieser Begriffe ab. […] Wahrheiten dieser Art also (reine *Begriffswahrheiten*) erkennst du kraft dessen, daß du die Begriffe, aus welchen sie zusammengesetzt sind, kennest." Note that Frege's version of the distinction between *a priori* and *a posteriori* propositions bears some similarities to Bolzano's in the sense that Frege, too, grounds the distinction in properties of propositions, i.e., their generality. Cf. Frege (1987/1884), 4. On the relation between Kant, Bolzano, and Frege on analyticity, see also de Jong (2001).

> If the propositions from which we infer a judgement **M**, and similarly the propositions from which *they* are inferred and so on up to the immediate judgements, are entirely purely conceptual truths: one may call **M** a judgement from pure concepts, or purely a priori; in every other case it shall be called a judgement drawn from experience, or a posteriori. (*WL* III, 202)[20,21]

1.2.1 Conceptual Truths and Analytic Truths

The Bolzanian strategy of explaining why some truths are knowable *a priori* bears a superficial resemblance to the logical empiricist account of the *apriori*. This is clear, for example, in the following passage from Ayer's *Language, Truth and Logic*:

> Our knowledge that no observation can ever confute the proposition "7 + 5 = 12" depends simply on the fact that the symbolic expression "7 + 5" is synonymous with "12," just as our knowledge that every oculist is an eye-doctor depends on the fact the the symbol "eye-doctor" is synonymous with "oculist." And the same explanation holds good for every other *a priori* truth. (Ayer 1946, 85)

The direction of explanation is similar to Bolzano's. Certain sentences are knowable independently of experience *because* they have certain non-epistemic, semantic features. Did Bolzano thus anticipate logical empiricism in this respect? No. In Bolzano, the distinction between conceptual and empirical propositions must not be confused with the analytic/synthetic distinction. Bolzano conceives of both distinctions in such a way that they cut across one another such that none of the following boxes is empty (cf. *WL*, §197):

Analytic and Conceptual	Analytic and Empirical
Synthetic and Conceptual	Synthetic and Empirical

From this, it is clear that Bolzano's theory is different from Ayer's.[22] Since conceptual truths are supposed to be knowable *a priori*, Bolzano is committed to the thesis

[20] "Wenn die Sätze, aus welchen wir ein Urtheil **M** ableiten, und ebenso auch diejenigen, aus welchen wir zuvor schon jene abgeleitet und so fort bis zu den unmittelbaren Urtheilen hin, durchgängig reine Begriffssätze sind: so kann man **M** ein Urtheil aus reinen Begriffen, oder rein, a priori nennen; in jedem anderen Falle mag es ein aus der Erfahrung geschöpftes oder ein Urtheil *a posteriori* heißen." – Here and in the following, underlined text is spaced in the original.

[21] Note that this definition does not exclude that a conceptual proposition may be the matter of a judgement *a posteriori*. A conceptual truth does not *necessarily* have to be judged on the basis of other conceptual truths (cf. Sect. 4, below).

[22] Note, moreover, that in Bolzano, the analytic/synthetic distinction divides the class of propositions and not the class of sentences.

that there are synthetic truths *a priori*, whereas one of the main tenets of Ayer's empiricism was the rejection of that thesis. Moreover, Bolzano's theory also implies that there are analytic truths that can only be known *a posteriori*. The latter claim may sound even more controversial. However, this appearance is due partially to the fact that Bolzano's notion of analyticity differs significantly from the explication that has nowadays become standard, that is, "truth in virtue of meaning." To get a clearer picture of the implications of Bolzano's account, I will discuss the two controversial boxes above in the following a bit more deeply.

1.2.2 Empirical Analytic Truths

That there are analytic empirical truths may sound plainly wrong to modern ears, but Bolzano was clearly aware of the fact that *his definition* of analyticity does not exclude propositions that contain intuitions and are thus empirical propositions, which should imply, according to Bolzano's explication, that they cannot be the matter of a true judgement *a priori*. Let me explain this.

According to Bolzano, a proposition is analytic iff it contains at least one representation that may be exchanged for any other representation with a nonempty extension without a change in truth-value (*WL* II, 83). Since the talk of "exchange" here is somewhat misleading (one cannot literally change parts of unchangeable, abstract objects), one might put the definition as follows:

> **(B-A)** A proposition *P* is analytic iff there is at least one representation *i* in *P*, such that each proposition *P*', (a) whose subject and predicate parts have nonempty extensions and (b) which differs from *P* at most with respect to *i*, has the same truth-value as *P*.[23]

Every proposition that is not analytic is synthetic. This understanding of analyticity is obviously different from most modern ones in that it does not rely on the notion of meaning. It is also considerably broader. Put very roughly, Bolzano's idea is that a proposition is analytic as soon as at least one of its constituents is *inessential* to its truth-value.[24] With this notion, he tried to capture in precise terms what he thought

[23] Condition (a) is due to the fact that according to Bolzano's definition of truth, propositions whose subject is empty are always false. (B-A) is an example of an application of Bolzano's famous *method of variation*. The method consists in characterizing properties of propositions (or representations) by considering certain parts of them to be *variable*. Up to a point, variation is analogous to the familiar method of substitution with respect to linguistic objects. Bolzano uses this method not only to define analyticity but also numerous other logically important concepts – most importantly, (logical) consequence, as well as probability-theoretic and epistemological concepts (for this, see below). Note that when more parts of a given proposition are considered to be variable, one has to assure that variation is executed in a systematic way. For a detailed account, see Morscher (2008). For an extensive discussion of Bolzano's notion of analytic truth, see also Künne (2008).

[24] This is very close to what Quine calls "vacuous occurrence" in his definition of logical truth (as noted by Quine). Cf. Quine (1977a), 88 and 105 and Quine (1977b), 117ff. – the difference being that Quine defines those notions for *sentences* and not for propositions. Cf. Künne (2008), 290ff. for a discussion of this point.

to be the valid intuition underlying the definitions by Kant and others in terms of notions like conceptual containment (*WL* II, 87; cf. de Jong 2001; Lapointe 2010). According to (B-A), propositions like the one expressed by

(1) Every animal which is rational is rational

come out as analytic, since the representations expressed by "animal" and "rational" can be varied at will without a change in truth-value. Moreover, since Bolzano holds that *every* proposition has subject-predicate form, under his reading of (1), the subject part (expressed by "every animal which is rational") *literally* contains the predicate (expressed by "is rational").[25] Notably, according to (B-A), not only propositions in which the predicate is contained in the subject are analytic but also the usual examples of logical truths (or, rather, their Bolzanian subject-predicate equivalents). Moreover, one can easily define a notion of *logical analyticity* that bears close resemblance with the modern definitions of logical truth given by Quine and Tarski by restricting the representations that have to be varied to the nonlogical ones.[26]

What may sound surprising is that (B-A) does not exclude empirical propositions in Bolzano's sense, that is, propositions that contain intuitions, from being analytic. Consider the following sentence:

(2) This, which is a drake, is male.

In (2), the representation expressed by "this" (uttered in a suitable context) can be varied *salva veritate*, but since that representation is an intuition, Bolzano has to consider the proposition to be an empirical truth. And since he wants to explain the distinction between *a priori* and *a posteriori* in terms of the one between conceptual and empirical propositions, he is committed to the thesis that (2) is a proposition *a posteriori*.

Bolzano indeed gives arguments in order to show that to know (2), one has to invoke empirical knowledge. These arguments are, however, largely dependent on Bolzano's thesis that all propositions have the form "*A* has *b*"[27] (and all sentences – even logically complex ones – are reducible to that form), his theory of indexicals, and his theory of truth. According to Bolzano, a proposition of the form "*A* has *b*" is true iff there is something that falls under *A,* and everything that falls under *A* has one of the properties that fall under *b*.[28] Now, in (2), the intuition expressed by "this"

[25] Though of course this does not hold if one analyzes (1) by means of a first-order language (i.e., as "$\forall x\,(Ax \wedge Rx \rightarrow Rx)$." Although the proposition is still analytic in Bolzano's sense, one cannot literally say that its predicate is contained in its subject.

[26] Note that the class of logical notions in Bolzano comprises different elements than modern suggestions and that he had a certain pragmatic attitude toward drawing the distinction between logical and nonlogical notions (cf. Künne 2008, 259ff.).

[27] Where "*A*" is a placeholder for representations of all kind and "*b*" is a placeholder for designators of properties, e. g., "blackness." Note that *b* may also collectively refer to particularized qualities (Cf. Betti 2012).

[28] For further elaboration, see Künne (2008), 236.

is part of a complex subject expressed by "this which is a drake," and in order to find out whether the complex subject actually expresses an intuition referring to a drake, one needs to invoke empirical knowledge, that is, pay attention to the context in which the respective sentence is uttered (*WL* II, 333 and *WL* III, 454). Hence, argues Bolzano, the proposition cannot be known independently of experience.[29]

1.2.3 Synthetic Conceptual Truths

The truths contained in the other notable box are synthetic conceptual truths. These are propositions that do not contain any intuition and also do not contain any constituent that may be varied at will without risking a change in truth-value. One finds those truths among mathematical propositions such as the one expressed by

(3) The number eight is even.

Other examples include propositions such as:

(4) The concept [representation] is nonempty.
(5) Each proposition contains at least three parts (cf. WL III, 240).

No part in (3), (4), or (5) may be varied at will without risking a change in truth-value; thus, none of the sentences is (Bolzano-) analytic. Yet no part of those propositions is an intuition, and arguably no part contains one. Hence, the propositions are conceptual truths and should thus – according to Bolzano's explication – be knowable *a priori*.

1.3 How Are Synthetic Judgements A Priori Possible?

On the surface of it, Bolzano seems to be in agreement with Kant in that he argues that there are synthetic truths *a priori*. Both philosophers, however, tell vastly different epistemological stories with respect to the question that Kant conceived to be the central problem of his theoretical philosophy:

> The proper problem upon which all depends, when expressed with scholastic precision, is therefore: How are synthetic propositions a priori possible?[30]

Kant's own solution relied on the concept of *pure intuition*. Arithmetical propositions have to be justified by reliance on the pure intuition of time, geometrical ones by pure intuition of space. A great deal of Bolzano's *WL* is devoted to a detailed criticism of the Kantian approach and of the very idea of pure intuition

[29] For an in-depth discussion of this argument, see Textor (1996, 241, 2001).

[30] "Die eigentliche mit schulgerechter Präzision ausgedrückte Aufgabe, auf die alles ankommt, ist also: Wie sind synthetische Sätze a priori möglich?" (Kant 2001/1783, 41). Of course, I take Kant here to be asking not about the possibility of a certain kind of object but rather about how one could *justify* synthetic propositions *a priori* and hence *know* them.

(Cf. in particular *WL* III, 180ff.; also Rusnock 2000). I won't evaluate Bolzano's criticism of Kant in this chapter but will rather ask the following question:

> **(Q)** *How can Bolzano account for our knowledge of synthetic truths a priori in his epistemological framework?*

Surprisingly, Bolzano seems to see this as a rather trivial problem:

> Especially here, where K.[ant] envisages a difficulty, there seems nothing incomprehensible to me. "What justifies understanding to connect a subject **A** with a predicate **B** foreign to the concept of **A**?"[31] Nothing else, I say, than that the understanding *has* and *knows* the concepts **A** and **B** [daß der Verstand die Begriffe **A** und **B** beide *hat* und *kennet*]. In my opinion, from the mere fact that we have certain concepts, we must also be in a position to judge about them. For to say that someone has certain concepts **A, B, C,…** is indeed to say that he knows and differentiates them. But to say that he knows and differentiates them is again only to say that he asserts something about the one that he does not want to assert about the other; [which] means therefore to say that he judges about them. (*WL* III, 180, for similar claims see *WL* I, 180f. and 194)[32]

This passage contains two crucial claims:

> **(C1)** One can come to know a conceptual truth of the form "*A is B*" if one knows the concepts *A* and *B*.[33]

> **(C2)** To know the concepts *A* and *B* means to form judgements of a certain kind about *A* and *B*.

It is, however, not especially clear how exactly Bolzano thought this conception might work. How can knowing the concepts that a conceptual proposition contains suffice for knowing whether it is true? In the case of classical examples of *analytic* truths, the idea is more or less intuitive. To know that the concept of a man is contained in that of a bachelor – or that the latter is composed of the concept of a man and the concept of being unmarried – suffices to know that every bachelor is a man.

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[31] Bolzano is clearly alluding to a specific passage of the *Critique of Pure Reason* here, in which Kant asks how one can come to know synthetic truths *a priori*, namely, "What is the unknown = X which gives support to the understanding when it believes that it can discover outside the concept A a predicate B foreign to this concept, which it yet at the same time considers to be connected with it?" ("Was ist hier das Unbekannte = X, worauf sich der Verstand stützt, wenn er außer dem Begriff von A ein demselben fremdes Prädikat B aufzufinden glaubt, welches er gleichwohl damit verknüpft zu sein erachtet?") (Kant 1787 B, 13).

[32] "Mir will gerade hier, wo K. eine Schwierigkeit antraf, nichts Unbegreifliches erscheinen. 'Was den Verstand berechtige, einem Subjecte **A** ein Prädicat **B**, welches doch in dem Begriffe von **A** nicht lieget, beizulegen?' Nichts Anderes, sage ich, als daß der Verstand die Begriffe **A** und **B** beide *hat* und *kennet*. Bloß dadurch, daß wir gewisse Begriffe haben, müssen wir (meine ich) auch in dem Stande seyn, über sie zu urtheilen. Denn sagen, daß Jemand gewisse Begriffe **A, B, C, …** habe, heißt doch wohl sagen, daß er sie kenne und unterscheide. Sagen, daß er sie kenne und unterscheide, heißt aber wieder nur sagen, daß er von dem einen derselben etwas behaupte, was er nicht eben auch so von dem andern behaupten wollte; heißt also sagen, daß er über sie urtheile."

[33] Two remarks on this formulation are in order: First, I take it that by talking about being justified in connecting certain concepts, Bolzano wants to indicate that one has *knowledge* of the respective truth. Second, as it is immaterial to the problems I will be concerned with later on, I follow Bolzano talking about propositions of the form "*A* is *B*," even though strictly speaking one should talk about propositions of the form "*A* has *b*" in Bolzano's framework.

And indeed, Bolzano is able to *explain* this intuition via his notion of analyticity (see below, Sect. 4).[34] But in the case of synthetic truths, it is – at least *prima facie* – far from clear how knowledge of the concepts of which these truths are constituted of should suffice to know their truth-value.

Strangely enough, Bolzano himself does not address this question explicitly. His theory of knowledge contains extensive considerations on the *origination* of non-inferential, inferential, and empirical judgements, but an explicit account of knowledge of conceptual synthetic propositions is missing. In the following, I will thus try to put together the bits and pieces to be found in Bolzano's theory of knowledge that may yield to a coherent account of knowledge of synthetic conceptual truths. In particular, I will first introduce Bolzano's conception of knowledge, then give an account of what Bolzano plausibly means by "knowing a concept," and finally try to show how his theses (C1) and (C2) should be most fruitfully interpreted against this background.

2 Understanding (C1): Bolzano's Epistemology

The third part of *WL*, is called "Erkenntnislehre," which is most naturally translated as "theory of knowledge." Neither is, however, the concept of knowledge the central concept of Bolzano's epistemology nor the concept of belief or of justification or related notions that are central to modern studies in that discipline. Rather, he is generally concerned with "the conditions to which cognition of the truth is subjected, especially in us human beings"[35] (*WL* III, 3). This includes an extensive investigation of the subjective counterparts to his *An-sich* entities – *judgements* and *subjective representations* – their relations among each other, and the conditions under which they originate.[36] In this context, Bolzano devotes a great deal of attention to questions that nowadays would be squarely located within the philosophy of mind.[37] The concepts most central to the understanding of the claims (C1) and (C2) are basically the concept of knowledge and the concept of knowing a representation. I will introduce both concepts in the following. Before that, however, I have to introduce some parts of the basic conceptual framework of Bolzano's theory of judgements.

[34] To see that this claim expresses an analytic truth in the first place, one has to recognize that the proposition expressed by "Every bachelor is a man" is identical to the one expressed by "Every man which is unmarried is a man." (Cf. also de Jong 2001).

[35] "den Bedingungen […] denen die Erkenntnis der Wahrheit, besonders bei uns Menschen, unterliegt".

[36] For good overview of the conceptual framework of Bolzano's epistemology (cf. Siebel 1999).

[37] For a recent exposition of this (cf. Konzelmann Ziv 2008).

2.1 Judgements and Subjective Representations

Judgements and subjective representations are mental events.[38] As such, they take place within the spatiotemporal realm. Both subjective representations and judgements always have a *matter* (*Stoff*). The matter of a judgement is a proposition, and the matter of a subjective representation is an objective representation.[39] Judgements and subjective representations have parts, but neither are *just* collections of parts. Their parts have to stand in a mutual relationship of a certain kind. Bolzano speaks of "a special kind of reciprocation" ("eine eigenthümliche Wechselwirkung" (*WL* III, 20–1)) in the case of representations and of "a certain very special kind of connection" ("eine gewisse ganz eigenthümliche Verbindung" (*WL* III, 109f.)) in the case of judgements. He is, however, not able to shed much more light on these notions, and I won't attempt to do so either in this chapter. In any case, passing a judgement is not just *grasping* a proposition in one's mind but *holding* it to be true (cf. *WL* III, 108 and *WL* I, §34).

Being concrete events, judgements stand in causal relationships that are, according to Bolzano, governed by certain faculties and forces within the human mind.[40] On his account, judging is an activity that is not subject to direct influence of the will (a position that is nowadays often called "epistemic involuntarism"). One can concentrate deliberately on certain objects or think about certain propositions, which will often indirectly bring about the occurrence of a judgement, but one cannot *deliberately* hold a proposition to be true (*WL* III, 110).

2.2 Bolzano's Analysis of the Concept of Knowledge

A special class of judgements is constituted by *cognitions* (*Erkenntnisse*).[41] Cognitions are judgements whose matter is a true proposition. This can be expressed in form of the following definition:

(E) *S* cognizes the proposition *P* iff *S* judges that *P* and *P* is true (*WL*, §36).

According to §36 of *WL,* the German term "Erkenntnis" (cognition) can be used interchangeably with the term "Wissen" (knowledge). But (E) is hardly a plausible

[38] Bolzano actually calls judgements "actions of the mind" (*Handlungen des Geistes*) (*WL* I, 155).

[39] Cf. *WL* III, §§270ff. for subjective representations and *WL* III, §290ff. for judgements.

[40] Anita Konzelmann Ziv argues that Bolzano's picture of the human mind is in many aspects similar to a view that is nowadays known as the *modular view* of the mind (Cf. Konzelmann Ziv 2008, 4ff.).

[41] The translation of Bolzano's epistemological vocabulary is not easy. A straightforward translation of "Erkenntnis" is also "knowledge." However, to be able to differentiate both terms, I will use the somewhat odd-sounding "cognition."

explication of the concept of knowledge. Cognitions as defined by (E) are obviously a type of (mental) *events* (since judgements are), whereas knowledge is normally conceived to be a *state*. However, Bolzano is aware of this:

> If we ascribe knowledge to someone, that is a knowledge of the truth **A**: we do by no means want to say that he passes the judgement **A** at the same moment we ascribe this property of knowledge to him; rather, it suffices that he passed the judgement at some past time and presently nothing more than an external inducement is needed to repeat it.[42] (*WL* III, 27, see also *WL* III, 200 and III, 207)

What he seems to have in mind is that someone who knows *P* has judged that *P* some time before and is disposed to judge that *P* again, in the sense that she *would* judge that *P* in case the question whether *P* would arise.[43] (E) can easily be modified accordingly:

> **(E*)** *S* cognizes* the proposition *P* iff *P* is true, *S* has judged that *P* at some past time, and *S* is disposed to judge that *P* again if the questions arises.

However, if the concept of knowledge is conceived to be identical to the concept of cognition* thus defined, it is completely disconnected from the concept of justification.[44] Fortunately, Bolzano does offer a different definition that is much more in the spirit of traditional definitions according to which knowledge is *more* than just true belief (or true judgement). According to this definition, knowledge is a special case of cognition*. The concept is introduced thus:

> If, thus, the confidence we have in the judgement **M** seems to be such that it is impossible for us to annihilate it presently, then I say that the truth **M** has become knowledge for us.[45] (*WL* III, 288)

A true judgement (in the dispositional sense) constitutes knowledge iff one judges (again in the dispositional sense) *about that judgement* that one has a particularly strong *confidence* in it. Using the already defined concept of cognition*, one may make this more precise in the following way:

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[42] "[W]enn wir Jemand ein Wissen, nämlich das Wissen der Wahrheit **A** beilegen: so wollen wir damit keineswegs sagen, daß er das Urtheil **A** in eben dem Augenblicke, wo wir ihm diese Beschaffenheit des Wissens zuschreiben, fälle; sondern es genügt uns, wenn er dieß Urtheil nur schon irgend einmal gefällt hat und gegenwärtig nichts als eines äußeren Anlasses bedarf, um es zu wiederholen."

[43] The modern concept of a *belief* can thus also be defined within the Bolzanian framework. *S* believes that *P* iff *S* is disposed to judge that *P*. In the following, therefore, I will use the term "belief" in this sense. For an analysis of Bolzano's account of disposition, dispositions to belief, and belief in the dispositional sense, see Siebel (1999), 70–1 and 75ff.

[44] Although, interestingly, some contemporary epistemologists have proposed similar definitions (Cf. e.g. Sartwell 1991).

[45] "[W]enn also die Zuversicht, mit der wir dem Urtheile **M** anhängen, uns als eine solche erscheint, die zu vernichten gegenwärtig nicht mehr in unserer Macht steht, so sage ich, die Wahrheit **M** sey bei uns zu einem *Wissen* erhoben."

> **(K-B)** S knows that *P* iff (i) *S* cognizes* *P* and (ii) *S* judges that her confidence in *P* is such that it presently seems impossible for *S* to annihilate.[46]

There are two problems with this definition. *Firstly*, the condition that the confidence of *S* has to be "such that it seems presently impossible…" is rather vague. This problem concerns not only the fact that Bolzano does not give any indication as to the respective *degree* of confidence that would suffice for knowledge. Also, according to him, two judgements may have an equal degree of confidence, and yet a subject may be able to think of reasons to the contrary of only one of them (*WL* III, 289). Thus, there may be something in addition to the degree of confidence that makes the difference between knowledge and mere belief. But what exactly is this? The *second* problem is that it is not clear what the confidence someone has in a judgement of her is precisely. I will try to solve these problems (at least partially) in the following two sections, beginning with Bolzano's concept of confidence.

2.2.1 Confidence

Bolzano uses the term "confidence" not in its vague ordinary sense. Rather, it is a precisely defined technical term for – roughly speaking – the reasons that a person has, all things considered, for affirming the truth of a specific proposition. More precisely, the confidence a subject *S* has in a judgement is determined by the degree of probability of the matter of that judgement (a proposition) with respect to all propositions *S* holds true (i.e., all her other beliefs/judgements) (*WL* III, 276).[47] The idea is that the confidence *S* has in a judgement depends on the *objective* probability of the matter of that judgement relative to a collection of propositions that is determined by *S*'s (subjective) beliefs, that is, all propositions she holds true. Since the notion of probability is precisely defined by Bolzano, it is even possible to represent the degree of confidence someone has in a certain judgement by a numerical value. To discuss Bolzano's highly elaborate remarks on this topic in all their detail would lead too far astray. However, since the notion is central to his epistemology and his heuristics, I will devote a paragraph to sketch the basics.[48]

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[46] Mark Siebel argues in Siebel (1999, 83f.) that Bolzano's explanation of the definition allows for different interpretations. He holds that it is not clear whether Bolzano holds that one is supposed to judge about one's *confidence* or about possible other *reasons* that speak against the judgement that constitutes knowledge. Contrary to Siebel, I opt for the first option, since it is closer to Bolzano's words. Further, I will show below that the confidence one has in a judgement is determined by the reasons that speak against it. Thus, focusing on the reasons that speak against a judgement is a way of estimating its confidence.

[47] Bolzano also indicates that the degree of confidence is additionally determined by the respective degree of confidence *S* has in all other those beliefs/judgements, but from the way he spells out this idea, it becomes apparent that he in fact only considers the degree of probability of a given proposition with respect to all other propositions the subject holds true (cf. also *WL* III, 277).

[48] I will mostly follow Berg's reconstruction in Berg (1962, 148–150, 2003). I follow Berg also in his somewhat anachronistic use of set theoretical terminology, since it simplifies the exposition.

The fundamental concept used in defining the notion of confidence is that of probability, which is defined in terms of variation. The degree of probability of a proposition *P*, with respect to the representations *i*,*j*,… considered to be variable is the ratio of the number of cases in which admissible variants of *i*,*j*,… result in true propositions to the number of all admissible variants of *i*,*j*,....[49] The variation of the representations *i*,*j*,… has to be *uniform*, that is, each occurrence of a representation *i* has to be replaced by the same representation. Let "P_i" denote the set of all those uniform variants with respect to *i* (for simplification, I will abbreviate the list of representations to be varied by a single "*i*") and "T(P_i)" the subset of P_i containing only true propositions. Further, let "$<P,\Gamma>_i$" denote the class of all uniform variations of *P* and each member of a set of propositions Γ with respect to *i* and "T($<P,\Gamma>_i$)" the subset of $<P,\Gamma>_i$ containing only true propositions. Finally, let "#(*x*)" denote the number representing the cardinality of *x*. The probability of *P* with respect to *i* – for short, *p*(*P*,*i*) – can then be represented as follows:

$$p(P,i) = \frac{\#(T(P_i))}{\#(P_i)}$$

The probability of *P with respect to a collection of propositions* Γ is, then, the ratio of the number of all variants that make *P and* Γ true to the number of all variants that make Γ true with respect to *i*, where *P* and Γ have to be consistent in the sense that there is at least one variant with respect to *i* that makes each proposition in Γ and *P* true (cf. *WL*, §161, Berg (1962), 148–50). For short,

$$p(P,\Gamma,i) = \frac{\#(T(<P,\Gamma>_i))}{\#(T(\Gamma_i))}, \text{ where } T(\Gamma_i) \text{ is nonempty.}$$

The *degree of confidence* in a single judgement with the matter *P* (for *S*) is the surplus *P* has over the probability of ¬*P* relative to all propositions Γ that *S* holds true (hence my talk of "all things considered" above).[50] This can be expressed numerically as follows. Let "μ" denote the probability of *P* relative to Γ. Then the confidence *S* has in a judgement with the matter *P* relative to the propositions Γ she holds true is $\mu-(1-\mu)$ or $2\mu-1$, for short. By plugging in the definitions given above, the confidence of *S* in a judgement whose matter is *P* with respect to the propositions Γ *S* holds true can be represented thus:

[49] Bolzano poses several conditions on which variants are admissible. Most important for the present context is that extensionally equivalent variants may be counted only once. Without this constraint, it would be impossible to calculate the probability of any proposition, since for most representations, there are infinitely many extensionally equivalent ones. For this and other constraints, see *WL* II, 78ff. and Berg (1962), 93.

[50] This idea is not completely unproblematic. Since Γ has to be consistent, Bolzano's definition is not applicable to inconsistent sets of beliefs, which should be an extremely rare phenomenon. One might think that Γ has to be restricted to a consistent set of *S*'s beliefs, but this chapter is not the place to discuss possible adjustments of Bolzano's account.

$$c(S,P,\Gamma,i) = 2\frac{\#(T(<P,\Gamma>_i))}{\#(T(\Gamma_i))} - 1,$$

where Γ and P are (collections of) propositions that S holds true, and $T(\Gamma_i)$ is nonempty.

Note that the degree of confidence applies to the *judgement* corresponding to P and not (at least not primarily) to P itself. Yet it is determined by the objective probability P has with respect to Γ, just that Γ is determined by the beliefs *of a specific subject*. If $p(P,\Gamma,i) = \frac{1}{2}$, the confidence in a judgement with that matter is zero. If this is the case for a subject *S,* then, according to Bolzano, she will generally suspend her judgement (*WL* III, 155). A general precondition to judge at all is, thus, that the probability of the respective proposition relative to all other propositions one holds true is $> \frac{1}{2}$. Note that confidence in a judgement may also have a negative value. Bolzano interprets these cases as cases where the confidence of the negation of that judgement has the respective positive value (*WL* III, 285).

What is important to notice is that the confidence *S* has in a certain judgement not something directly determined by her will (*WL* I, 155). This is a consequence of the fact that judgements are not directly determined by the subject's will and that the subject does not necessarily have to be aware of each judgement she forms (see below, Sect. 3.2). Now, since the degree of confidence in a particular judgement is ultimately determined by the objective probability it has (with respect to the matter of all other judgements/beliefs), one will neither be always aware of it nor be able to influence it directly. The confidence in a judgement is thus constituted, one might say, by the reasons someone objectively has for it, given all one's beliefs.

2.2.2 How Much Confidence?

So, at least the term "confidence" in Bolzano's definition of knowledge can be made sufficiently precise. Still, the first problem with (B-K) is not yet solved: Bolzano does not give much of a hint of how high a subject's confidence in a judgement has to be in order to be such that "it seems to be impossible to annihilate presently."[51] Bolzano surely cannot demand that a judgement be *certain* in his sense of the term, namely, that it have the highest degree of probability with respect to one's beliefs

[51] A further problem is that, to avoid an infinite regress, Bolzano does not demand that *S* has to estimate her confidence in the given judgement in such a way that she *knows* how high it really is (*WL* III, 276). What is more is that he does not even seem to demand that one has to judge *truly* about one's confidence in a judgement. This, however, opens the possibility that *S* may judge wrongly about her confidence in *P* in which case the (B-K) is still satisfied – which resembles the problem Gettier famously pointed out with respect to the classical definition of knowledge.

(i.e., 1) and thus the highest degree of confidence (*WL* III, 264). This would imply that most propositions are unknowable for us. So the margin will lie somewhere between $\frac{1}{2}$ and 1, but where? We do not find an answer in *WL*.[52] What complicates the issue even further is that Bolzano seems to claim that two judgements can have the same degree of confidence even if only one of them constitutes knowledge. So apparently, it is not only the degree of confidence that matters but also other factors. However, Bolzano does not indicate which factors.[53] In the present context, I have to leave these questions unanswered.

Let us return to (C1). On Bolzano's account, a judgement (or a disposition to judge, a belief) constitutes knowledge for a subject, if she is aware of the fact that her confidence in it is – all her other beliefs considered – such that she cannot think of any reasons to the contrary. If we plug this into claim (C1), we get the following result:

> **(C1*)** If *S* knows the concepts *A* and *B*, *S*'s confidence in judgements that have as their matter conceptual truths composed out of those concepts will be such that – all *S*'s other beliefs/judgements considered – *S* cannot think of any reasons to the contrary.

This, in particular, is also supposed to hold for synthetic conceptual truths (as those expressed by the sentences (3), (4), and (5) mentioned in Sect. 1.2.3 above). In order to understand this claim precisely, one has to understand Bolzano's claim (C2), which says that in order to know a concept, one has to form certain judgements about it. The following section will be devoted to provide an interpretation of this claim.

3 Understanding (C2): Knowing a Concept

As (C2) says, knowing a concept means to pass certain judgements about *it*. This raises two questions: (a) What kind of judgements? and (b) on what basis can we pass judgements about abstract, mind-independent objects such as concepts in the first place? I will give Bolzano's answers to both questions in the following.

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[52] It should be mentioned that Bolzano himself has expressed some dissatisfaction with (K-B) after the publication of *WL* in a letter to Zimmermann (cf. Bolzano 1978/1848, 189). In that letter, he complains that his definition of knowledge runs against the normal use of the word "knowledge" ("ganz gegen den Sprachgebrauche"). According to Bolzano, "we say that someone *knows* something not if he assumes with perfect confidence, but if he assumes something according to *truth*." ("Denn nicht von demjenigen, was jemand mit vollkommener Zuversicht annimmt, sagt man daß er es *wisse*, sondern von dem was er der *Wahrheit* gemäß annimmt.") However, it is not clear to what extent (K-B) actually does fail to respect this aspect of the normal use of "knows," for Bolzano explicitly restricts the class of propositions that can be known to *true* propositions.

[53] One can even read the first section of *WL* §321 in which Bolzano introduces the concept of knowledge in such a way that the notion of *degree* of confidence is entirely absent from Bolzano's definition of knowledge and *only* the *kind* of confidence matters. I take this, however, not to be the best interpretative hypothesis, as virtually the *only* characteristic of the notion of confidence that Bolzano discusses is its degree.

3.1 The Correspondence Assumption

How can we judge about the properties of *abstract* objects such as propositions, representations, and, in particular, concepts? Bolzano assumes that this is possible by judging about the subjective counterparts of them. But why, in turn, should that be possible? Because, as he assumes, judgements and subjective representations share the mereological properties of their objective counterparts. If a subjective representation has a complex objective representation as its matter, the former has as many *parts* as the latter:

> If one wants to say truly that we have grasped within our mind [*in unser Gemüth aufgefasst haben*] a certain objective representation which is not simple, but composed of parts: then all parts the combination of which it consists of must have been grasped by us. For the objective one consists only of as many and of those parts as the subjective one, which we may call the appearance of the former in our soul.[54] (*WL* III, 39, see also *WL* II, 244 and *WL* I, 113)

In the following, I will call this assumption "the correspondence assumption." It allows Bolzano to claim that knowledge of relations and properties of objects of his *an-sich* realm can be derived from knowledge of one's own mind. Given the correspondence assumption, one can see *what kind* of judgements about concepts Bolzano plausibly has in mind, namely, those that assert which parts a concept has and in which order they are composed.[55]

3.2 Having a Representation, Clarity, and Distinctness

What is important for the correspondence assumption to be plausible at all is another assumption of Bolzano which I mentioned already above, namely, that

> it is not always necessary that we are distinctly conscious of everything we think, nor that we can report on it.[56] (*WL* III, 245)

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[54] "Soll man in Wahrheit sagen können, daß wir eine gewisse objective Vorstellung, welche nicht einfach, sondern aus mehreren Theilen zusammengesetzt ist, in unser Gemüth aufgefaßt hätten: so müssen alle die einzelnen Theile, in deren Verknüpfung sie besteht, von uns aufgefaßt worden sein. Denn nur aus ebenso vielen und solchen Theilen, als in unserer subjectiven Vorstellung sich finden, besteht die objective, für deren Erscheinung in unserer Seele wir jene ausgeben dürfen."

[55] In this respect, I depart from the interpretation given by Sandra Lapointe, who argues that Bolzano must have had judgements about the *essential* properties of the *objects* that fall under the respective concepts in mind: "What properties of concepts are here relevant? The significant feature of concepts Bolzano has in mind here is that we can 'infer' from them the properties of the objects to which they refer." (Lapointe 2010, 276). In the passage from which I extracted (C1) and (C2) and in similar passages (e.g. *WL* I, 180ff.), Bolzano does only mention properties of *concepts* and not (essential) properties of the *objects* that fall under the concepts the respective conceptual truths are composed of.

[56] "[es] ist […] doch nicht immer nötig, daß wir uns alles dessen, was wir uns denken deutlich bewußt sind, und es auch anzugeben vermögen."

Someone who has a certain representation does not necessarily have to be conscious of the parts it is composed of, nor of the fact that it is composed of parts at all, nor of the fact that she has it at all (see also *WL* III, 110). In an adaptation of classical terminology, Bolzano holds that one's subjective representations can be clear or confused (*klar* or *dunkel*) and allow for different degrees of distinctness (*Deutlichkeit*).[57] Before I clarify these notions, however, I have to spend a note on the notion of having a representation.

It is not always clear what Bolzano means by "having a representation." In some passages, he seems to use the phrase interchangeably with "knowing a representation." A striking example of this is the passage I cited in Sect. 1.3, where he claims that "to say that someone has certain concepts **A, B, C, …** is indeed to say that he knows and differentiates them"[58] (*WL* III, 180, see also *WL* I, 181). However, this rather strong sense of "having a representation" cannot be the *only* way in which Bolzano uses the phrase. Otherwise he would contradict what he said in the passage cited at the beginning of this section: If it is not required that we are conscious of everything we think (where thinking means having representations and forming judgements), then knowing and differentiating a representation cannot be a condition for *having* it.[59] Further, Bolzano's concepts of clarity and distinctness, which I will introduce in a minute, would hardly make sense on that assumption. I assume thus that Bolzano uses the phrase "having a representation" also in a weak sense, according to which someone *has* a representation already if it is simply present in her mind, for example, as a result of being acquainted with an object or even as a result of imagination (cf. *WL* I, 217f.).[60] To have a representation in this sense does not amount to being aware of having it.

Bolzano defines the traditional notions of clarity and distinctness with respect to subjective representations and – derivatively – also with respect to judgements. A subjective representation is clear for someone, if she is conscious of having it. More precisely,

> **(B-clarity)** A subjective representation *i* is *clear* for a subject *S* iff *S* judges truly that she has *i* and *i* is referred to by an intuition in the corresponding judgement (cf. *WL* III, 29).[61]

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[57] Both notions are also defined for judgements derivatively (*WL* III, 116–118).

[58] "Denn sagen, daß jemand gewisse Begriffe **A, B, C, …** habe, heißt doch wohl, sagen, daß er sie kenne und unterscheide."

[59] On another occasion, Bolzano even claims that having a representation is a *precondition* of knowing it (cf. *WL* III, 243).

[60] Bolzano also has a detailed account of how we acquire subjective representations. He holds that we acquire intuitions as causal effects of certain changes within our mind, caused, e.g., by external objects. The change caused by the smell of a rose causes an intuition that has that change as an object (*WL* III, 84–7). Pure concepts are acquired when we are engaged in passing a judgement and lack some necessary part of it – for example, the concept expressed by "and" (*WL* III, 86). Bolzano assumes that the human mind is somehow capable of bringing the respective missing subjective representation about.

[61] The second conjunct is needed to prevent representations that fall under representations as the one expressed by "the representation I had between 6 and 6:30 pm 2 days after I was born" from satisfying the definition (*WL* III, 28).

Even if a subjective representation is clear for one, one will not necessarily be aware of all its parts. The degree to which one *is* aware of them determines the degree of *distinctness* (*Deutlichkeit*) of the respective subjective representation. This concept is introduced in the following way:

> **[(B-distinctness)]** [I]f we are, thus, able to pass a judgement of the following form "The representation **A** consists of the representations **a, b, c, …** in this and that order," this is a kind of knowledge concerning the representation **A** that in some cases, and especially in scientific investigations, serves important purposes. To indicate that those circumstances obtain with respect to a representation, one usually calls it a *distinct* one.[62] (*WL* III, 40)

The idea is rather straightforward. To give a schematic example, say that a certain representation *i* consists of the parts *m* and *n* where *m* is simple and *n* consists of *r* and *s*, which are both simple. Then *i* is *distinct* to the degree 1 for a subject *S* iff *S* judges that *i* consists of *m* and *n*. In this case, *m* and *n* will be clear for *S* (cf. *WL* III, 41). If *S* further judges truly that *n* consists of *r* and *s and* that *m* is simple, *i* is distinct to degree 2 for *S*. If *S* finally judges that *r* and *s* are simple, *i* is distinct to the highest degree for *S*. The last point is important: If *S* has a simple representation *i* that is clear for *S*, it does not follow that *i* is also distinct. This is only the case if *S* judges truly *that i* is simple. Note further that one does not only have to know the parts of the representation but also the way in which they are composed.

With these notions at hand, one can clearly state what Bolzano plausibly means by "knowing a representation." Knowing a representation (*eine Vorstellung kennen*) is a special case of knowing an object (*einen Gegenstand kennen*). The latter notion is explained in the following way by Bolzano:

> We say, I hold, that we know an object, if we are able to pass as many true judgements about it as is humanly possible and purposeful with respect to an object of its kind.[63] (WL III, 261)

Since each judgement about an object is a claim that it has or lacks a certain property, knowing an object amounts to judging truly which properties it has. And this is indeed in accordance with what Bolzano says in the passage quoted above (i.e., *WL* III, 180).[64] However, whether this is a plausible conception of knowledge of objects may be disputed. It is rather vague, since Bolzano does not give us any hint on how he thinks one should determine how many true judgements about an object are "humanly possible," and he also does not give any hint as to how the restriction on serving a purpose is to be understood. But Bolzano's conception is not doomed.

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[62] "wenn wir sonach ein der Wahrheit gemäßes Urtheil von folgender Form: "Die Vorstellung **A** besteht aus den Vorstellungen **a, b, c, …** in dieser und jener Verbindung," zu fällen im Stande sind, so dieß eine die Vorstellung **A** betreffende Kenntniß, die uns in manchen Fällen, besonders aber bei wissenschaftlichen Untersuchungen, wichtige Dienste zu leisten vermag. Gewöhnlich pflegt man, um das Vorhandenseyn dieses Umstandes bei einer Vorstellung anzuzeigen, sie eine *deutliche* zu nennen."

[63] "Wir sagen, meine ich, daß wir einen Gegenstand kennen, wenn wir so viele, sich auf ihn beziehende, wahre Urtheile zu fällen im Stande sind, alles bei einem Gegenstande dieser Art für uns Menschen überhaupt möglich und nützlich scheint".

[64] And also on other occasions such as *WL* I, 180f.

One can view his claim as a definition of *perfect* knowledge of an object (or knowledge to the highest degree) and admit that even if someone does not know all the properties of a certain object, she nevertheless may plausibly be said to know it to a lesser degree.[65]

How can we know which properties a certain non-spatiotemporal, mind-independent object – an *objective concept* – has? The correspondence assumption allows Bolzano to explain this via the notions of clarity and distinctness that are defined for *subjective representations*. Someone (say again *S*) knows a certain objective representation *i* (perfectly) iff the corresponding subjective representation is distinct (to the highest degree) for *S*. Note that this makes also sense with respect to (C1): Bolzano claims that in order to know a synthetic conceptual truth, we have to know *the properties of the concepts* contained in it.

According to the present interpretation, Bolzano's thesis (C2) amounts to the following:

> **(C2*)** To have and to know a certain representation – in particular a certain concept – means to have it clear and distinct in one's mind; that is, to *judge* truly that it is constituted out of such and such parts in this and that order.

In virtue of the correspondence assumption, a clear and distinct grasp of a given subjective representation enables one to know the mereological properties of the respective *objective* one, that is, its constituents and their way of composition (cf. also *WL*, §399). Note that one way of attaining a clear and distinct grasp of a certain representation is thus being presented with an explicit definition (*Erklärung*). For, according to Bolzano, an explicit definition consists in a specification of the parts and the manner of composition of a representation (cf. *WL* I, 91). For the most part, we can thus also think of (C2*) as follows: To have and to know a certain representation is to know its definition.[66] It remains to be seen how we can make sense of (C1*) on this basis, that is, how this kind of knowledge can result in knowledge of synthetic conceptual truths. To this question I now turn.

4 Definitions, Proofs, and Synthetic Truths

If the interpretation given above is correct, that is, if the paraphrases (C1*) and (C2*) are correct, we can summarize Bolzano's idea as follows:

> **(C)** Knowing how the concepts contained in a conceptual truth are constituted suffices for recognizing that one is justified in judging it with a confidence high enough to constitute knowledge.

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[65] Note that we are *not* speaking about *knowledge that* in the normal sense (the German "Wissen") but about knowledge of an *object* (the German "einen Gegenstand kennen"). In the latter case, the talk of degrees of knowledge might seem to be more plausible than in the former.

[66] We will discuss some important exceptions to this below in Sect. 4.3.

As I already noted before, this idea has some initial plausibility for some *analytic* conceptual truths, namely, those in which the concept expressed by the predicate is literally contained in the subject. In some passages, Bolzano suggests that we can come to know those truths solely in virtue of their form (*Gestalt*) (*WL* III, 451f.).[67] This makes sense with respect to the present interpretation: Coming to know how an analytic proposition of that type is constituted will show that all nonlogical parts occur inessential in it. Having, for example, a clear and distinct apprehension of the concept of a bachelor enables one to see that the proposition expressed by

(6) All bachelors are unmarried

cannot be false because of its form. Or more exactly, given that its subject is non-empty, it is structurally incapable of being false, since – as becomes explicit when the concepts contained in (6) are clear and distinct – the concept expressed by "bachelor" is identical to the one expressed by "unmarried man."[68]

In case of analytic truths in which the predicate is not part of the subject (which, as we have seen in Sect. 1.2.2, do exist in the Bolzanian framework), one may have to know certain other truths (analytic or synthetic) in addition to the definitions of the concepts contained in the respective proposition. In case of the proposition expressed by

(7) $1 \cdot 0 = 0$,

For example, mathematical knowledge is needed to see that the concept expressed by "1" occurs inessentially in (7). (Cf. *WL* II, 392–3.)

However, this does not answer how (C) may be cashed out with respect to *synthetic* conceptual truths. The *form* of propositions like the above mentioned (3), (4), and (5) would not reveal their truth-value. So how should we understand Bolzano's claim in those cases?

4.1 Knowledge and Proof

To approach this problem, I will look at a particular example Bolzano gives in order to illustrate his definition of knowledge – the Pythagorean theorem:

(8) In any right triangle, the area of the square whose side is the hypotenuse is equal to the sum of the areas of the squares whose sides are the two legs.

[67] In that passage, Bolzano speaks of coming to know whether the relation of derivability obtains. But in those cases, in which we speak of logical derivability, this amounts to the same as coming to know a logically analytic (conceptual) proposition that states an implication. Now, if these kinds of propositions can be known by knowing their form, then the same surely also holds for tautological conceptual propositions. Note that it is crucial that we speak here of tautological *conceptual* propositions, since there may very well be tautological propositions in Bolzano's sense which cannot be known without experience (See *WL* III, 454 and also Sect. 1.2.2 above.).

[68] Note that I do not want to claim that the notion of knowledge in virtue of form is unproblematic; I just take it for granted for the purpose of this chapter.

Since (8) is a synthetic conceptual truth according to Bolzano, knowing the constituents of the concepts contained in it should suffice to come to know it according to (C). This is, however, not what Bolzano says explicitly in his discussion of (8). Rather, he claims that we come to know (8)

> once we have been presented with a proof of it. For now we recognize the truth of the proposition such that we are assured that even if we wanted, it would not be possible for us to convince us of its falsity. (*WL* III, 288)[69]

Finding (or being presented with) *a proof* for the proposition expressed by (8) increases one's confidence in a judgement that has (8) as its matter in such a way that one cannot convince oneself of its falsity. However, does a proof of (8) in any case have to involve knowledge of the concepts contained in it in the sense specified above (Sect. 3)? This does not seem to be the case if one reflects on what Bolzano understands by a proof:

> In general we use to call a proof of a proposition **M** any arbitrary something that is such that we imagine that it could be used to bring about the judgement **M** in the mind of a thinking being that either had not passed that judgement yet, or at least not with such a high confidence, by drawing its attention to it. (*WL* III, 457)[70]

On this very broad understanding, any object whatsoever that is capable of bringing about a judgement counts as a proof. Note that Bolzano thus presumably thinks of proofs as those *concrete* objects that are causally responsible for the judgements whose matter they prove and not as the abstract, propositional counterparts they may have (although such counterparts exist for each proof that consists of written or spoken sentences or thought propositions). Bolzano cites, for example, the blushing of a person as an example of a proof for the proposition that this person is ashamed. However, proofs of that type are *empirical proofs* (*Erfahrungsbeweise*) which are contrasted with *proofs from pure concepts* (*Beweise aus reinen Begriffen*) (*WL* III 453–6). A proof of the latter kind is based solely on the linguistic or mental counterparts of conceptual propositions and leads thus to a true *judgement a priori* according to Bolzano's explication (cf. *WL* III, 202 and Sect. 1.2 above). This suggests the following interpretation of Bolzano's claim (C): For every conceptual truth, one can (at least in principle[71]) find a conceptual proof, *provided* one knows the concepts contained in it in the sense specified in Sect. 3.

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[69] "wenn man uns mit einem Beweise desselben bekannt macht. Denn nun erkennen wir die Wahrheit dieses Satzes in der Art, daß wir versichert sind, es würde uns, selbst wenn wir wollten, nicht gelingen, uns von der Falschheit desselben zu überreden."

[70] "Wir pflegen überhaupt jedes beliebige Etwas, von dem wir uns vorstellen, daß Jemand sich desselben bedienen könnte, um durch die Lenkung der Aufmerksamkeit eines denkenden Wesens auf dasselbe in dem Gemüthe des letzteren ein Urtheil **M** zu erzeugen, das es bisher entweder noch gar nicht, oder doch nicht mit so hohem Grade der Zuversicht gefällt hatte, einen Beweis […] des Satzes **M** zu nennen."

[71] As mentioned above, some propositions in Bolzano's universe might be too complex to grasp for human beings.

4.2 Two Remaining Problems

Yet even in this more clarified version, Bolzano's thesis is far from being clear. One might ask two questions:

> **(Q-a)** Why did Bolzano assume that for every conceptual truth – in particular for every synthetic conceptual truth – one can find a proof from pure concepts?
>
> **(Q-b)** Why did Bolzano assume that knowing how the concepts contained in a given conceptual truth are defined is sufficient to find such a proof?

To prepare an answer to these questions, let me first note that, according to Bolzano's broad definition of the notion of proof, a proof of a conceptual proposition such as the one expressed by (8) *need* not be a proof from pure concepts. Thus, in order to come to know (8), it is not *necessary* to grasp every concept contained in it clearly and distinctly. Bolzano does not exclude that one can convince oneself (or others) of a proposition such as (8) by means of an empirical proof. He explicitly cites, for example, incomplete induction as a valid means to convince oneself of arithmetical propositions (cf. *WL* III, 244). As a working mathematician, Bolzano was well aware of the fact that proofs from pure concepts are often rather difficult to find even for theorems that are quite evident (cf., e.g., the foreword to Bolzano (1804)). He emphasized, moreover, that for some purposes, it would be rather cumbersome and unnecessary to prove a proposition like (8) by a proof from pure concepts (cf. *WL* IV, 192–4 and 385ff.).

Still, Bolzano also frequently emphasized that a *proper scientific presentation* should not only convince the reader *that* the propositions it contains are true but also show *why* this is the case (cf. *WL* IV, §525). Bolzano calls proofs that are able to achieve this end *demonstrations* (*Begründungen*) in contrast to (mere) *certifications* (*Gewißmachungen*). Now what makes a proof a demonstration? In order to explain this, I have to point to a fundamental assumption of Bolzano. According to Bolzano, all true propositions stand in a certain objective explanatory order structured by a relation he calls *grounding* (*Abfolge*) (cf. *WL* II, §162 and §§198–222 and Bolzano (1975/1833–41) §13). Indicating the *grounds* of a given truth shows *why* it is true – on which truths it *objectively depends*. This explanatory order is independent of the order in which knowledge is actually acquired.[72] Now, a demonstration is simply a proof that follows this objective order, that is, refers to the *grounds* of a given theorem (*WL* IV, §525).[73] When it comes to conceptual truths, Bolzano argues

[72] To give an intuitive example (cf. *WL* II, §162), coming to know the truth that a well-functioning thermometer stands higher at a certain location l_1 than at a location l_2 is a means to come to know the truth that it is warmer at l_1 than at l_2. Nonetheless, the latter proposition is the *ground* of the former according to Bolzano, i.e., the latter precedes the former in the objective order of grounding. For discussions of Bolzano's theory, consider Sebestik (2011), Betti (2010), Tatzel (2002), Buhl (1961), Lapointe (2010), and Centrone (2011).

[73] That this assumption is crucial for Bolzano's account of *a priori* knowledges seems also to be the hypothesis of Sandra Lapointe. Cf. Lapointe (2010), section 5.

that their respective grounds will always consist in conceptual truths (cf. *WL* II, 384). Hence, a conceptual truth can only be *demonstrated* by proofs from pure concepts. What is important for present purposes is that *because* Bolzano held that conceptual truths are embedded in the grounding structure, he could assume that – apart from some exceptions we will discuss below – for every conceptual truth, one can (at least in principle) find a conceptual proof. The given conceptual proof will consist in a demonstration.[74] This provides thus a first partial answer to (Q-a). Before I explain why this cannot be more than a partial answer, let me connect it to (Q-b). Why did Bolzano assume that knowing the concepts contained in a given conceptual truth suffices for finding a demonstration for it? What seems to be the most plausible answer to this question is the following: Bolzano thought that specific traits of the grounding relation among conceptual truths allow to find the grounds of a given conceptual truth by becoming aware of the concepts it is composed of. In particular, he introduces several "criteria" for determining what the grounds for conceptual truths are, the most of which are formulated in terms of mereological properties of conceptual propositions (cf. *WL*, §221; Bolzano (1975/1833–41), §17). Bolzano holds, for example, that the grounds of a conceptual truth are always at least as simple (with respect to the parts they contain) as the given truth (*WL*, II 387). More importantly, the grounds of a given conceptual truth do not contain any concept that is not itself part of it (*WL*, IV 449). These assumptions explain why Bolzano frequently emphasized that in the context of making the "objective connection among truths" explicit, it is necessary that the truths involved (and the concepts they contain) are made *clear and distinct*:

> Hence, whenever as much distinctness as possible is to be achieved in the presentation of a science, one should not fail to point out (as far as possible) of which parts propositions are composed, and, in the case of representations, either that they are simple, or of which simple representations they are composed. Not only should this give the reader an excellent exercise in thinking, but also enable one to make the most accurate judgements and give the clearest proofs *concerning the objective connection that holds among the truths* we have set out. (Bolzano 1975/1833–41, 69–70, my emphasis.)[75]

[74] It should be emphasized that not *every* purely conceptual proof needs to be a demonstration. An example for this is Bolzano's proof (or attempted proof) of the claim that there exist infinitely many truths. He says explicitly that for them to succeed, a reader does not need to know whether the concept of a truth or of a proposition is simple or complex and, if the latter, of which parts it consists. (*WL*, I 71) The only purpose of the proof is to *convince* the reader that there are infinitely many truths, not to show why this is the case. Yet, the proof does not make use of any empirical truths.

[75] "So oft es sich also in dem Vortrage einer Wissenschaft um die Erreichung einer möglichst hohen Deutlichkeit handelt; sollte man nicht unterlassen, bei jedem Satze, soviel es möglich ist, bemerklich zu machen, *aus welchen Theilen derselbe bestehe*, und bey jeder Vorstellung zu zeigen, *daß sie entweder einfach, oder aus welchen einfachen Vorstellungen sie zusammengesetzt sey*. Nicht nur, daß man durch dieses Verfahren dem Leser eine vortreffliche Uibung im Denken verschaffen würde, sondern durch Zergliederungen würde man auch in den Stand gesetzt, *den objectiven Zusammenhang*, der zwischen den aufgestellten Wahrheiten herrschet, am Richtigsten zu beurtheilen und am Deutlichsten nachzuweisen."

Consider also the following passage:

> [I]n order to judge whether a given truth is a fundamental truth or a consequence, and in the latter case from which other truths it follows, we must know the constituents not only of the truth in question but also of those truths that are advanced as its grounds. An analysis of the given proposition that extends to its simple parts, insofar as we are able to perform it, must be our first business in this problem. (*WL* III, 496)[76]

By obtaining a clear and distinct grasp of the concepts a given conceptual proposition consists of, Bolzano seems to assume, one will be able to find out on which truths it objectively depends and thus to find a conceptual proof for the proposition. This provides thus an answer to (Q-b). Taking both answers together, the following picture emerges: Conceptual truths are ordered via the relation of grounding. Knowing the concepts contained in a conceptual truth is a means to come to know its grounds and thus to come to know a conceptual proof of it, in particular a demonstration.[77]

4.3 The Case of Fundamental Truths

As I noted above, what we have established so far can only be a partial answer to questions (Q-a) and (Q-b). There are two problematic cases. First, Bolzano assumes that some conceptual truths do not have a ground. He calls those truths *fundamental truths* (*Grundwahrheiten*) (*WL* II, 375). Lacking a ground, these truths are incapable of being proven by a demonstration. Second, there are conceptual truths consisting only of *simple concepts*. With respect to them, it is not clear how obtaining a clear and distinct apprehension of the concepts they are composed of should suffice for finding a conceptual proof. Bolzano suggests that these classes of truths largely overlap, that is, simple conceptual truths form a large class of fundamental conceptual truths (cf. *WL* III, 402). In the following, I will thus restrict my attention to truths that belong to both of these classes, that is, simple conceptual truths that are fundamental.[78]

Bolzano clearly states that his thesis (C) also holds for these kinds of truths:

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[76] "[Z]ur Beurtheilung der Frage, ob eine gegebene Wahrheit Grund- oder Folgewahrheit sey, und in dem letzteren Falle, aus welchen, anderen Wahrheiten sie erfolge, [ist es sehr nothwendig (S.R.),] die einzelnen Theile zu kennen, aus welchen sie selbst sowohl, als auch die Wahrheiten, die man für ihren Grund ausgeben will, zusammengesetzt sind. Eine Zergliederung des gegebenen Satzes, die sich, sofern wir es vermögen, bis auf dessen einfache Theile erstrecket, wird also wohl unser erstes Geschäft bei dieser Aufgabe seyn müssen."

[77] This is also strongly suggested in *WL* III, 453ff.

[78] I will assume that the suggestion as to how Bolzano's claim (C) can be applied to such cases I discuss below is also valid for the case of complex fundamental truths (should there be any) and simple non-fundamental truths (should there be any).

> Since this [i.e. (C), S.R.] holds universally, it holds as well in the case in which these concepts are perfectly simple.[79]

If the thesis thus holds for *all* conceptual truths, it holds in particular for truths belonging to both of the problematic classes mentioned above. But how should Bolzano's claim be understood in those cases?

According to my interpretation of (C), obtaining a clear and distinct grasp of the concepts contained in a simple fundamental truth is a means of finding a conceptual proof for it. However, if a given proposition is a fundamental truth – and thus in particular if it is a simple fundamental truth – looking for a demonstration of it will be futile. The respective conceptual proof has thus to be different from a demonstration. What kinds of proof could Bolzano have in mind instead? Sandra Lapointe has suggested the following:

> Bolzano […] suggests that a proposition whose axiomatic status has been shown and whose logical consequences are also known to be true […] is very likely to be itself true. (Lapointe 2010, 279)

The idea can roughly be sketched as follows: Coming to know that a given proposition consists of simple concepts (by obtaining a clear and distinct apprehension of them) is a means of coming to know that the proposition satisfies criteria for being a fundamental truth (what Lapointe calls showing the "axiomatic status" of the proposition) (cf. *WL* IV, 388). Convincing oneself of the truth of such a proposition proceeds by subsequently showing that a number of well-established propositions are derivable from it and thus situating it in the grounding structure. Bolzano calls these kinds of proofs also *deductions* (*Herleitungen*) (*WL* IV, 389, Bolzano 2004/1810 II, §21).[80] Note that this way of coming to know a given conceptual truth does not proceed via infallible methods but also involves an element of "trial and error." Nonetheless, given Bolzano's broad definition of the notion of proof, convincing oneself of the truth of a proposition by means of a deduction can surely be classified as a proof. Moreover, since the method does in no way require to rely on empirical propositions, such a proof can also be qualified as a conceptual proof. These suggestive remarks undoubtedly will leave many questions open, but I have to postpone a more extended discussion of them to another occasion. In any case, it seems that Bolzano has the resources to argue for his claim (C) also with respect to truths of the two problematic classes sketched above.

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[79] "Da dieses ganz allgemein gilt, so gilt es auch in dem Falle, wenn diese Begriffe ganz einfach sind" (*WL* III, 180).

[80] Lapointe also refers to Bolzano's earlier writings in which we find this idea quite explicitly stated. To convince oneself of a fundamental truth (a *Grundsatz* in the terminology of Bolzano 2004/1810), one proceeds by finding

> some generally accepted and unmistakably clear propositions which are however basically nothing but *consequences*, and even judgements *inferred* from that axiom [*Grundsatz*, SR] which we wish to deduce. By making this connection apparent we will become convinced of the truth of the axiom itself. (Bolzano 2004/1810 II, §21, note)

For a discussion of this idea, consider Rusnock (2000), pp. 23, 43, 51ff.

So let me, by way of summary, give a final answer to questions (Q-a) and (Q-b): Since conceptual truths are embedded in the grounding structure, for each non-fundamental conceptual truth, one can (at least in principle) find a particular type of conceptual proof, namely, a demonstration. Fundamental truths, on the other hand, can be proven by deductions. Since the place of a conceptual truth in the grounding structure is determined by its mereological properties (*WL* II, §221), a sufficient condition for finding a conceptual proof of it is to acquire a clear and distinct grasp of the concepts it is composed of.

Lapointe argues, however, that this picture creates a tension in Bolzano's system. According to her, Bolzano holds that the only admissible proofs for conceptual truths are *demonstrations* (cf. Lapointe 2010, 278–9). However, given Bolzano's broad definitions of what counts as a proof and what counts as knowledge (see above, Sects. 2.2 and 4.1), I can see no reason for this conclusion. Although one cannot *demonstrate* a fundamental truth (which is almost a tautology), one can still *prove* it in Bolzano's loose sense of the term. And, further, one can even *prove it from pure concepts*, since neither showing *that* it is fundamental nor that certain known conceptual truths follow from it seems to require to invoke empirical truths.

5 Conclusion

In this chapter, I have tried to elucidate the epistemological background of Bolzano's objective explication of the notion of *a priori* truth. What I have, following Textor, called "objective explication" is Bolzano's attempt to explain the notions of *a priori truth* and *a priori judgement* in terms of the notion of conceptual truth. Bolzano claims that conceptual truths can be known by knowing the concepts contained in them. I have tried to clarify this suggestion within the framework of his epistemology. Since Bolzano holds that there is a certain correspondence between the abstract realm of propositions and representations and their psychological counterparts, he assumes that knowledge of the concepts a proposition consists of can be derived from knowledge of one's own mind. This kind of knowledge should, according to him, enable a subject to find a conceptual proof for any given conceptual proposition. I have suggested that this idea of Bolzano is rooted in his thesis that all truths are ordered by the relation of grounding and that the place a conceptual truth occupies in this order is dependent on its mereological properties. Coming to know the mereological properties of a conceptual truth by obtaining a clear and distinct apprehension of the concepts contained in it suffices therefore to find a conceptual proof for it. In case of non-fundamental truths, it will be possible (in principle) to find a *demonstration,* and in the case of fundamental conceptual truths, it will be possible to find a *deduction*.

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Part IV
Husserl, Frege and Russell

Chapter 9
Immanent and Real States of Affairs in Husserl's Early Theory of Judgement: Reflections on Manuscripts from 1893/1894 and Their Background in the Logic of Brentano and Stumpf

Robin D. Rollinger

1 Introduction

The significance of the concept of a *Sachverhalt* (usually translated as 'state of affairs') in Edmund Husserl's *Logical Investigations* has not gone unnoticed.[1] The Munich phenomenologists, the first among his contemporaries to receive this work with enthusiasm, were particularly attentive to the potential applications of this concept in various logical, phenomenological, and ontological inquiries.[2] In recent discussions Husserl's views regarding *Sachverhalte*, whether they are accepted or

[1] For the more technical usage of this term, see Husserl (1900), pp. 12–14, 16, 49 f., 88–92, 105, 190 f., 229 ff., 242, 244 f., and Husserl (1901), pp. 9, 25, 26 f., 30, 38 n., 43, 68, 78, 94, 162 f., 182, 310, 312, 347, 369, 378, 380, 385, 387, 393, 395, 401, 405, 412, 432, 416 ff., 422, 425 f., 429–431, 441–444, 446, 451, 455, 485, 559, 594, 596 ff., 610, 612 f., 615 f., 619, 625, 627–632, 644, 653 f., 663, 677 f., 681 f., 684, 686, 691 f. The term is used in a less technical sense in Husserl (1901), pp. 66, 131, 362 f. The abundance of passages in which Husserl refers to *Sachverhalte* in a technical way impresses upon us how important this concept was in the *Logical Investigations*. In English translation the term 'state of affairs' occurs in Husserl, (ed.) Moran, (trans.) Findlay (2001) I, pp. 17 f., 19, 38, 62, 64, 72, 121, 144 ff., 142, 153, 169, 184 f., 187, 195, 197 217, 225, 242, 274 f., 321 n. 4; II, pp. 148, 155 ff., 263, 265, 280 f., 313, 329, 33.

[2] The leading figures among the Munich phenomenologists were Johannes Daubert and Alexander Pfänder, both students of Theodor Lipps. The younger member of their circle, Adolf Reinach, has actually received considerable attention with regard to his usage of the concept of a *Sachverhalt*. See Mulligan (ed.) (1987), Smith (1989), and Balzer-Jaray (2010). The work of Pfänder still awaits similar treatment. See Pfänder (1963) (Pfänder, [trans.] Ferrari (2009)]), where the concept in question plays a significant role. While Daubert (who was in fact the first to draw inspiration from Husserl) did not publish anything, his views on *Sachverhalt* which were developed in manuscript are discussed at length in Schuhmann (2004), 201–217.

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rejected, are regarded as worthy of attention.[3] The occurrence of this concept in Husserl's earlier writings, however, has yet to receive attention, mainly because the relevant material has been published only recently in a volume (*Husserliana* XL) which consists of texts on the theory of judgement written as early as 1893 and as late as 1918.[4]

As is well known, Husserl took the concept of a *Sachverhalt* from Stumpf and transformed it in the *Logical Investigations*. While specialists would like to know whether this transformation can already be seen prior to this work, we are also very naturally concerned with a question of a more general interest, namely, whether something worthy of future philosophical reflection (a thesis or perhaps a problem) can be found in Husserl's early treatment of the concept under consideration. Here I would like to address these matters by examining the concept of a *Sachverhalt* in two early manuscripts which appear for the first time in the above-mentioned volume of the *Husserliana*. One of these manuscripts bears the date 1893, while the other is dated 1893/1894.[5] Unless additional manuscript material comes to light which would indicate otherwise, there is apparently no earlier usage of the concept of a *Sachverhalt* in Husserl's literary remains (leaving aside nontechnical uses of the relevant term).

The two manuscripts which are the focus of discussion here are best seen in connection with other manuscripts and publications of Husserl within this same time frame (1893/1894). While there is a temptation to reach beyond such chronological restrictions, it will prove rewarding to give special attention to this early phase of his theory of judgement. It should not be presupposed, after all, that what came later was necessarily better. Though it cannot be argued here that Husserl already reached a high point in his philosophical development in the *Logical Investigations* and perhaps even prior to that, the following can indeed be regarded as a contribution to the ongoing demonstration for such a thesis.[6] However, even if it is not seen as such a contribution and the reader is convinced that Husserl later reached a higher level of maturity (e.g. in the *Ideas Pertaining to a Pure Phenomenology and Phenomenological Philosophy*[7] or at some other stage), it is instructive to isolate the various phases of this development in order to see the intellectual struggles which

[3] See Smith (2001), Reicher (2001), Künne (2003), Correia and Mulligan (2007), Simons (2009), Textor (2009), and David (2009).

[4] Husserl, (ed.) Rollinger (2009). Most of the texts published in this volume were gathered together by Edith Stein at the request of Husserl in 1918 and are known as the *U-Blätter*. The ones which are examined here were not among the *U-Blätter*.

[5] See Texts Nr. 1 (pp. 1–30) and Nr. 2 pp. (31–50) in Husserl, (ed.) Rollinger (2009). The appendices number from I to VIII (pp. 51–67) are materials which belong together with Text Nr. 2. Here I shall discuss Text Nr. 2 first. It is indeed quite possible, though by no means definitively confirmed, that this text had been written before Text Nr. 1 had been.

[6] Previous contributions to such a demonstration are Rollinger (1999) and Schuhmann (2004) as well as other more minute studies too numerous to cite here.

[7] Husserl (1913).

led to later phases. If Husserl later arrived at a solution of the problems under consideration here, this will be all the more satisfying if we empathetically think through the cognitive situation in which they first emerge in his work.

2 Brentano and Stumpf on Contents of Judgement

It will of course be necessary for our purpose here to consider Stumpf's concept of a *Sachverhalt*.[8] Before we consider this, however, it will prove illuminating to discuss relevant aspects of the theory of judgement that had been put forwards in lectures by Franz Brentano, the mentor of both Stumpf and Husserl.

2.1 Brentano

As far as we thus know from Brentano's theory of judgement, he does not use the word *Sachverhalt* as a technical term. However, those who do research regarding Brentano have for a long time been highly restricted to what he published in his own lifetime and posthumously published materials which have been very poorly edited and are hardly reliable. His *Psychology from an Empirical Standpoint*, first published in 1874, still remains the chief source for understanding the theory of judgement which he taught to his students, first in Würzburg and then in Vienna. The view that judgements make up one of the main classes of psychical phenomena and are more particularly characterized as instances of acceptance or rejection, as this is expressed in the just-mentioned work,[9] remains central in Brentano's philosophical teachings throughout his academic career. Here, however, I shall draw upon unpublished notes from his lectures on logic.[10]

In Würzburg Brentano gave lectures on logic in the winter semesters of 1869/1870 and 1870/1871. While his lecture notes from these courses are partly preserved in his literary remains, parts of them have been replaced by materials which he later used in Vienna.[11] Some parts of these notes, however, are most definitely from

[8] Here I am restricting the discussion to what Husserl took from his mentors regarding the concept of a *Sachverhalt*. For further considerations of his relations to them, see Rollinger (1999), 13–67, 83–123 and Rollinger (2004).

[9] Brentano (1874), 266–305.

[10] When passages from unpublished materials are cited, I shall provide the reader with the passage in the original German as well as my own translation thereof into English. However, in cases where the cited passage is published (either online or in book form), I shall only give my own translation.

[11] This manuscript is to be found under the signature EL 80 in Brentano's literary remains. Brentano, (ed.) Mayer-Hillebrand (1956), which is allegedly an edition of this material, may be disregarded. A preliminary edition of EL 80 is now available online (http://gandalf.uib.no/Brentano).

Brentano's very first lecture course from the summer semester 1869/1870. The following passage, which we cite from Anton Marty's copy of these notes,[12] is of great interest to us in the present context:

> 'That a God is' is provable, the existence of God is provable. These are only distinctions in the names. For the whole sentence 'that a God is' is a name and means the same as 'the existence of God'.
>
> 'Dass ein Gott ist' ist beweisbar, die Existenz Gottes ist beweisbar. Das sind nur Unterschiede in den Namen. Denn der ganze Satz 'dass ein Gott ist' ist ein Name und besagt dasselbe wie 'die Existenz Gottes'.[13]

If these two names mean the same thing, we may ask: What sort of object or entity do they name? Apparently it is not a thing in the sense that a stone, a person, or perhaps even God is a thing, but it is also not a quality, a quantity, or a relation. Indeed, if we consider any of the Aristotelian categories, which were of considerable significance for Brentano,[14] the object or entity named in the case under consideration does not seem to fit any of them. Though Brentano does not answer this question in his lectures on logic, we shall soon see that Stumpf does.

It is of great importance to Brentano in his early logic that he can name not only things but also objects such as the existence or non-existence of things. This becomes apparent in his treatment of disjunctive and conditional judgements.[15] As is well known to anyone who has explored Brentano's philosophy, he held the view that a judgement is in all cases an acceptance or rejection of an object. The acceptance of an object is for him best formulated in a statement that asserts that the object in question exists, whereas the best formulation of rejection of an object is on his view a statement that asserts that the object in question does not exist. This means that disjunctives and conditionals must be formulated in this way and must accordingly be affirmative (to express acceptance) or negative (to express rejection). Brentano maintains that a disjunctive is best formulated as an affirmative in which the existence of existences is asserted. If one judges, for instance, that either an object A exists or an object B does not exist, this judgement is best expressed, he tells us, in the following form: 'One of the two, the existence of A and the non-existence of B, exists.' On any other view of disjunction, it is difficult to see how such a judgement can be affirmative or negative. As for conditionals, these may of course be reformulated disjunctively, though Brentano prefers to state 'If A exists, then B exists' as follows: 'The existence of A and the non-existence of B does not exist'.[16]

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[12] I thank Thomas Binder of the Forschungsstelle and Dokumnetionszentrum für österreichische Philosophie in Graz for providing me with a digital facsimile of these notes, which are catalogued under the signature Br 7. Some pages of this manuscript are copied from the notes which Carl Stumpf took during the lecture during the winter semester 1869/1870 and perhaps also from the following winter semester.

[13] Marty (Br 7): B24913.

[14] See Brentano (1862).

[15] Brentano (EL 80),13.313–13.334.

[16] A singular verb is used here because what is said not to exist is the existence of A and the non-existence of B *taken together*. Much of Brentano's logic is of course quite familiar in contemporary propositional calculus, though he of course prefers formulations in terms of existence, in keeping with his theory of judgement.

Yet another quotation from Brentano's lectures on logic is of interest to us here, this time from his own manuscript that contains material from the Würzburg period, though the passage here is taken from a part of the manuscript that was added in his Vienna period (perhaps in the summer semester of 1875 or that of 1877). The context of the passage to be quoted is one in which Brentano is wrestling with the question concerning the meaning (*Bedeutung*) of a statement (*Aussage*). As he had previously identified the content (*Inhalt*) of a presentation (*Vorstellung*) as the meaning of the name which expresses this presentation, he goes on to say the following concerning the meanings of statements:

> Like names, they [statements] have a double relation: a) to the content of a psychical phenomenon as such, and b) to possible external objects. The first of these [the content] is the meaning. The relevant [psychical] phenomenon in this case, however, is a judgement rather than a presentation. The judged as such is the meaning.
>
> Wie die Namen, haben sie [Aussagen] eine doppelte Beziehung: *a*) auf den *Inhalt* eines psychischen Phänomens als solchen, und b) auf etwaige äußere Gegenstände. Der erste ist die Bedeutung. Das betreffende [psychische] Phänomen ist aber in diesem Fall keine Vorstellung, sondern ein Urteil. Das Geurteilte als solches ist die Bedeutung.[17]

The relevance of this passage to the present discussion will again soon be made clear.

We may thus sum up the two points we have drawn from Brentano: (1) A clause of the form 'that X exists' is a *name* which means the same as 'the existence of X'. (2) The meaning of a statement is the judged as such or the content of the judgement which this statement expresses. Here, before leaving this brief discussion of Brentano, we should stress that his apparent identification or indeed confusion between content and object, as this is often seen in *Psychology from an Empirical Standpoint*,[18] is corrected in his lecture notes, where he, on the contrary, makes a very pronounced distinction between them and utilizes this distinction in his theory of meaning. For a long time now, it has been thought by researchers of the school of Brentano that this distinction was first introduced by Alois Höfler and Alexius Meinong and further elaborated on by Twardowski.[19] Brentano's manuscripts, however, reveal that this is not at all the case.

2.2 Stumpf

Now let us turn to Stumpf's lectures on logic from 1888. Though Husserl had already done his habilitation by this time, he did have a copy of the *Diktate* (the

[17] Brentano EL 80/13020. Brentano held an earlier view of contents of judgements. This and its influence on Marty are discussed in Rollinger (2009a). Brentano's mention of 'the judged as such' indicates his influence on Husserl's later theory of judgement, which lies beyond the purview of the present discussion. For an elaboration on such an influence, see Rollinger (2010b).

[18] See Brentano (1874, 115 f).

[19] See Hölfer/Meinong (1890, § 6) and Twardowski (1894).

main points) of this lecture. The following quotation from these *Diktate* is of great significance here:

> From the matter of a judgement we distinguish its content or the *Sachverhalt* expressed in the judgements. E.g., 'God is' has God as its matter, the being of God as its content. 'There is no God' has the same matter, but 'nonbeing of God' as the content.[20]

Here we encounter the word *Sachverhalt* as a technical term and can also now see the connection with the above-quoted passages from Brentano's lectures on logic. As Brentano had spoken of the content of a judgement as distinct from the object (or matter), Stumpf does so as well. This content is what he calls a *Sachverhalt*. At the same time, however, he characterizes the being of God as the content of the judgement that God is and the nonbeing of God as the content of the judgement that there is no God. We have seen that Brentano also was aware of certain names, such as 'the existence of God' or 'that a God is'. In accordance with Stumpf's recommendation, we would take such names as names of *Sachverhalte*. Here, then, is his answer to the question that we raised above in connection with Brentano's lectures on logic. Yet, we must bear in mind that, though Brentano had distinguished the content of a judgement from its object X, in his lecture notes, he does not *explicitly* identify this content as something to be named 'the being of X' or 'the nonbeing of X'.[21] The two passages that we cited from Brentano's lectures thus involve two distinct concepts which are regarded as one by Stumpf. We may thus raise the issue whether we are really dealing with a single concept here: namely, the *Sachverhalt* (1) as the being or nonbeing of the object of a judgement and (2) as the content of a judgement.

It is arguable that these cannot be forged together, as Stumpf suggests. If the judgement is true, we may indeed wish to speak of its *Sachverhalt* as one that obtains, but in this case there seems to be no *Sachverhalt* of the contradictorily opposite judgement. Suppose, for instance, that the judgement that X exists is true. In this case the judgement has the being of X as its *Sachverhalt*. At the same time, however, we may not wish to say that the contradictorily opposite judgement, i.e. that X does not exist, has the *Sachverhalt* called 'the nonbeing of X'. The latter judgement would, one might argue, have no *Sachverhalt* at all. Yet, if we speak of *Sachverhalte* as contents of judgements, we seem to have no difficulty in saying that *both* the judgements that X exists and that X does not exist have *Sachverhalte*. These are, after all, aspects of the judgements which are in some sense *contained* in them and not certain items existing externally. Hence, it is possible for a judgement to have a *Sachverhalt* in one sense, but not in the other.

In order to distinguish the two concepts of a *Sachverhalt* just indicated, the following terminology may be used here. When speaking of the content of a

[20] Stumpf, (trans.) Rollinger (1999), 313. Cf. later elaborations in Stumpf (1907), 32 ff. and comments in Rollinger (1999), 89–93.

[21] According to Stumpf (1919), 106–107, such an identification was conveyed in Brentano's logic lectures of the winter semester 1869/1870. Yet, this is indicated in neither Brentano's own notes (EL 80) nor Marty's copy of them (Br 7).

judgement as a *Sachverhalt*, we may call this 'the immanent *Sachverhalt*'. When speaking of a *Sachverhalt* as something that only a true judgement can have, we call this 'the real *Sachverhalt*'.

2.3 Excursus: Other Students of Brentano

Besides Stumpf and Husserl, other students of Brentano also came to make use of the concept of a *Sachverhalt* in their work, though sometimes without using this term or indeed any technical term at all. It is of course well known that Alexius Meinong made use of this concept under the heading 'objectives' (*Objektive*).[22] Marty later made contents of judgement and analogues in the sphere of interest focal in his ontology, even to the point of diverging from Brentano, who was later to eliminate such entities as well as all other non-things from his ontology.[23] It is noteworthy, however, that even in the nineteenth century, before Husserl, Meinong, or Marty expressed any well-defined position regarding *Sachverhalte* in their respective publications, Twardowski also asserted in his celebrated work *On the Content and Object of Presentations* that existence is the content of a judgement.[24] Another interesting case where the existence or non-existence of an object was thematized, though without a technical term, is to be found the work of yet another student of Brentano (and indeed of Meinong), namely, Christian von Ehrenfels, who asserts in his *System of Value Theory* that a desire is always directed at the existence or non-existence of an object.[25] Here it cannot be decided, however, whether Husserl prior to the publication of the *Logical Investigations* was influenced by either Ehrenfels or Twardowski in his own attempt to formulate his views on *Sachverhalt*.[26] It is in any case clear that the concept of a *Sachverhalt* was coming into prominence even before it found expression in Husserl's published work.

[22] Meinong (1902), 150–211.

[23] Marty (1908). See also See Rollinger (2010a), 84–101. The disagreement between Brentano and Marty (and also between Brentano and students of Marty) is documented in their correspondence in the twentieth century, partly documented in Brentano (ed.) Mayer-Hillebrand (1966), 100–321.

[24] Twardowski (1894), 5–9.

[25] Ehrenfels (1897), 53 f. See Smith (1994), 286. Cf. Meinong (1894), 55: 'Much is known whose existence is dear to us and which is therefore called a good, much whose existence is loathsome to us and which is therefore called an evil'. The work cited by Meinong was in fact a great inspiration to Ehrenfels.

[26] In the late 1890s, Husserl does in fact make clear that for him a wish or a hope relates to a *Sachverhalt* (Husserl, [ed.] Rollinger [2009], 94 ff., 103), though he does not refer Ehrenfels. There are in fact few references to Ehrenfels in Husserl's writings. For this reason Husserl's relation to him is not dealt with in Rollinger (1999).

3 Husserl's Theory of Judgement (1893/1894)

The texts by Husserl to be examined now were written at a crucial turning point in his thinking. After the publication of his book, the first volume of his *Philosophie der Arithmetik*, he did not in fact follow up this publication with a second volume as planned, but rather became more and more concerned with logic. A whole decade of work finally blossomed in the *Logical Investigations*, where the concept of a *Sachverhalt* becomes central.

3.1 Psychological Studies in Elementary Logic

In 1894 Husserl published two so-called Psychological Studies in Elementary Logic (*Psychological Studien zur elementaren Logik*), the first of which was concerned with the distinction between 'concrete' and 'abstract', the second with intuition (*Anschauung*) and representation (*Repräsentation*).[27] Both of these, as well as the lengthy manuscript that Husserl wrote in 1893 on intuition and representation,[28] are of great relevance to the early manuscripts on the theory of judgement to be discussed and should accordingly not be overlooked here.

Husserl's distinction between concrete and abstract sets out from the distinction between independent or separable contents and dependent or inseparable ones that had already been adopted by Carl Stumpf.[29] Abstract contents are for Husserl the ones that cannot be separated from others, such as the extension of a coloured patch, whereas the concrete contents can be separated.

While Husserl's distinction between intuition and representations is reminiscent of the distinction between proper and improper presentations, as we find this in Brentano's work as well as the work of other students of his,[30] Husserl insists, contrary to the teachings of Brentano, that the two types of mental acts under consideration in his second 'Psychological Study' do not even belong to the same species. Moreover, he expands the notion of intuition to include not only the instantaneous but also the continuous, e.g. melodies. Of course he describes consciousness as an interweaving of intuitions with representations.

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[27] These studies are published in Husserl, (ed.) Rang (1979), 92–100, 101–123.

[28] The manuscript is published in Husserl, (ed.) Rang (1979), 270–302. A page from the manuscript was missing from the convolute used by the editor. Hence, on page 292 there is a break in the text. The missing page has been discovered in recent years and appears in Husserl, (eds.) Giuliani and Vongehr (2004), 442 f.

[29] In Husserl, (ed.) Rang (1979), 92 reference is explicitly made to Stumpf (1873), 109, though this distinction is ultimately taken from Brentano's theory of parts and wholes. There was no published text from Brentano on this topic to which Husserl could refer, though he had abundant copies of notes from Brentano's lectures. Most of these copies have unfortunately been lost.

[30] See the quotation from EL 80/13057 in Rollinger (2008b), 34 n. f. Cf. the discussion concerning Meinong's theory of relations and Brentano's reaction to it in Rollinger (2008b), 160 ff.

3.2 Proper and Improper Judgements

The first manuscript of Husserl on the theory of judgement I want to consider here is published as Text Nr. 2 of *Husserliana* XL.[31] Its main title, given to it by the editor, is 'Proper and Improper Judgements' (*Eigentliche und uneigentliche Urteile*). The original, which is dated 1893/1894, is written in Gabelsberger shorthand and apparently not at all ready for publication. Husserl's thought expressed in this manuscript seems so closely related to the 'Psychological Studies' and the early manuscript on intuition and representation that it may well have been intended as a rough draft of the third 'Psychological Study'. The gist of Husserl's thought in 'Proper and Improper Judgements' is that, just as there is a distinction between intuitions and representations in the domain of presentations, there is also an analogous distinction between proper and improper judgements. The proper judgements are about intuitive contents, whereas the improper ones concern what is merely represented. In the course of developing this line of thought and criticizing Brentano's theory of judgement along the way,[32] Husserl encounters the concept of a *Sachverhalt*.

While the concept of a real *Sachverhalt* is certainly used in passing in the manuscript now under discussion, Husserl is mainly concerned here with the immanent *Sachverhalt*. This comes to light when he raises the question whether the *Sachverhalt*, more particularly the judgeable (*beurteilbarer*) *Sachverhalt*, is a presented judgement (*vorgestelltes Urteil*).[33] While this question may strike us as odd, the notion of a presented judgement was thematized in writings by such prominent philosophers as Christoph Sigwart and Anton Marty.[34] If, for instance, I do not judge that God exists and also do not judge that God does not exist, I may leave it as a problem whether or not God exists or not. While Kant classified such a mental act as a judgement, others in the late nineteenth century, such as Sigwart and Marty, were inclined to say that this is a presentation of a judgement or simply a presented judgement. In view of the fact that this act is concerned with the existence of God or what is also named 'that God exists', Husserl's question whether the *Sachverhalt* is a presented judgement makes perfect sense. Whenever we present something that can be formulated by a 'that' clause, it may very well be asked whether the object of presentation here is in fact a judgement, though perhaps with less emphasis on its act-character than in other cases.

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[31] The fact that this manuscript is closely tied to the 'Psychological Studies' and the other one (Text Nr. 2) bears traces of the influence of Bolzano suggests that Text Nr. 1 was in fact written earlier, though there is no hard evidence for this conclusion.

[32] Husserl, (ed.) Rollinger (2009), 42 ff. While Husserl still conceives of judgments as acts of consciousness here, he can only accept the view that when this act occurs, it is founded on yet another act of consciousness, namely, presenting.

[33] Ibid., 45. See also n. 2 on this page, where it is indicated that Husserl initially answers the question under consideration affirmatively.

[34] Sigwart (1889), 303. Marty, (ed.) Eisenmeier (1916), 58.

Though Husserl initially answers this question affirmatively in 'Proper and Improper Judgements', he scratches out this answer and attempts to characterize *Sachverhalte* in another way. This is indicated in the following passage:

> If I have the *Sachverhalt* in believing, I have it as the foundation of believing. If I have it in mere presenting, <I have it> as the foundation of presenting. Hence '*Sachverhalt*' is, properly speaking, an *abstractum*. It is something common to many phenomena.[35]

As we have already seen, the abstract for Husserl at the time under consideration is to be found among the dependent contents of consciousness. Accordingly when he characterizes a *Sachverhalt* as an *abstractum*, this means that it is a part of something without which it cannot exist. Thus, if I judge that X exists, the *Sachverhalt* is a part of the phenomenon of judging. This part could not exist independently of the judgement or of some other phenomenon, such as presenting, just as the colour red could not exist independently from a circle, a square, or some other extended whole. If we consider any statement as an expression of a judgement, formulated as 'I judge that X is the case', this can be taken as a whole that breaks down into two parts, one of which is indicated by 'I judge' and the other by 'that X is the case'. This second part, according to Husserl, is a *Sachverhalt* which could also be part of some other phenomenon, such as a presentation, but would in all cases have to be a part of some mental act. Just as the 'that' clause of the sentence is dependent on 'I judge' and the like, what the dependent clause represents is likewise dependent on the relevant mental act.

Is the *Sachverhalt*, as it is ultimately characterized in 'Proper and Improper Judgements', an immanent or a real *Sachverhalt*? Since an abstract part of a phenomenon is in all cases immanent to that phenomenon, it is abundantly clear that a *Sachverhalt* is here conceived of as immanent rather than real. Now let us consider Husserl's usage of the concept *Sachverhalt* in the other early manuscript to be discussed.

3.3 Essay on the Origin of the Concepts 'Necessity' and 'Necessary Consequence', on Hypothetical and Causal Judgement

The text now under consideration, which is published as Text Nr. 1 in *Husserliana* XL and bears a title taken from Husserl's manuscript (*Versuch über den Ursprung der Begriffe 'Notwendigkeit' und 'notwendige Folge', über hypothetisches und kausales Urteil*), was written in 1893. Like the 1894 manuscript on intentional objects,[36]

[35] Husserl, (ed.) Rollinger (2009), 49.

[36] This manuscript was first published in Husserl, (ed.) Rang (1979), 303–348 and again in an improved edition in Schuhmann (ed.) (1990/91). The latter edition is translated in Husserl, (trans.) Rollinger (1999), 251–284.

it was written in longhand and sent to Meinong for his perusal in 1902 and immediately returned to Husserl (with the remark that Husserl was hypersensitive about his intellectual property).[37]

Though the concept of a *Sachverhalt* here is not singled out for thematic treatment as it is in 'Proper and Improper Judgements', the term occurs not only with great frequency throughout the manuscript but also in very important assertions. Here I shall quote some of these occurrences of the term in question and comment on the quotations as they relate to the issue of whether it is an immanent or a real *Sachverhalt* that concerns Husserl.

First of all, Husserl says the following about modalities, such as necessity and possibility:

> Turning to the *Sachverhalt* in judgement, we do not as a rule reflect on our judging ego. Thus the *Sachverhalt* itself appears as a necessary or impossible one; the latter resists the yes and compels the no, and what is intended now appears as necessary. And again within a categorically or hypothetically structured *Sachverhalt* of the kind under consideration here the predicate appears as necessary and impossible with a relation to the subject, the consequent in relation to the antecedent.[38]

In short, the *Sachverhalt* is the bearer of modalities. Since Husserl here speaks of turning to the *Sachverhalt* and not reflecting on the ego, it is suggested that it cannot be the immanent *Sachverhalt* which he means. For the *Sachverhalt* in this sense would be an object of reflection if indeed it were an object at all. Does Husserl therefore mean the real *Sachverhalt*? If this is so, there is a difficulty, namely, that he speaks of the predicate of impossibility being ascribed to the *Sachverhalt*. If the *Sachverhalt* is real, how could it be impossible? Or should we identify yet a third concept of a *Sachverhalt* that is at stake here?

The same question rises in connection with the following two quotations:

> In the case of a (sufficiently) firm conviction the attempt to accept the contradictory opposite *Sachverhalt* or one that is in conflict in some way we will be unsuccessful; we rather feel forced to reject this and to accept the original *Sachverhalt*.[39]
>
> If we speak of facts, we mean the objective *Sachverhalt*, not the subjective one, the psychological one…[40]

Husserl is here speaking of the *objective*, not the real *Sachverhalt*. Yet, if the objective *Sachverhalt* is a fact, this would suggest that this is only a terminological preference. One could, after all, quite comfortably characterize facts as real *Sachverhalte*. However, Husserl also speaks of the contradictory opposite *Sachverhalt*. Such a way of speaking seems impermissible if we are considering real *Sachverhalte*.

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[37] This somewhat unfriendly exchange between Meinong and Husserl was prompted by the publication of Meinong (1902), which Husserl regarded as a work that in many respects merely echoed his own publication from the previous year. An examination of Husserl's reaction to Meinong (1902) can be found in Rollinger (1996), which is revised in Rollinger (1999), 186–199.

[38] Husserl, (ed.) Rollinger (2009), 5.

[39] Ibid., 6.

[40] Ibid., 12.

If the being of X is real, it simply has no contradictorily opposite. This cannot be the immanent *Sachverhalt* in the sense of the *content* of the judgement either, for Husserl is clearly regarding the *Sachverhalt* as the *object*, i.e. as that which is accepted or rejected, not the content of a judgement.

Nor does this have to be an object of a judgement. It can also be the object of mental acts of other kinds, as the following passages indicates:

> Hypothetical positing, 'assuming', is a peculiar psychical conduct towards the objectively posed ('presented') *Sachverhalt* and of the same irreducibility as accepting, rejecting, surmising, doubting, etc.[41]

Here we see Husserl making use of the concept of 'assuming' as an additional type of act that is correlated with *Sachverhalte*, and thus, it is obvious why he sent the 1893 manuscript to Meinong immediately after the publication of *On Assumptions* (1902). The suggestion that various types of acts can have *Sachverhalte* had indeed already been suggested in 'Proper and Improper Judgements', but there Husserl did not thematize the notion of assumptions. Nor did he characterize the *Sachverhalt* of these different types of acts as something 'objectively posed' (*objektiv hingestellt*), as he does in the 1893 manuscript.[42]

3.4 Intentional Objects

From the above-cited passages from the 1893 manuscript, it appears that what Husserl has in mind there is not the immanent *Sachverhalt* which had been under consideration in the other early manuscript we have discussed. While it is also problematic to say that this is the real *Sachverhalt*, the only remaining option seems to be that it is the objective *Sachverhalt*, i.e. the object of either a true or false judgement, as well as the object of assumptions, doubts, surmises, etc. If, however, this is what Husserl has in mind here, he faces an enormous problem that he had faced in the other manuscript that he sent to Meinong in 1902,[43] namely, the 1894 manuscript on intentional objects, where he argued that strictly and properly not every presentation, including a *Sachverhaltsvorstellung*, has an object. In those cases

[41] Ibid., 18. Cf. ibid., 19: 'Certainly not ever presentation of a *Sachverhalt* presupposes corresponding judgement. That Gold is green I can present without presenting the judgement.' It cannot be decided here whether Husserl regards assumptions simply as presentations of *Sachverhalte* or not. As already noted, he already diverges from Brentano in his concept of a presentation in 1893/1894, though he has not yet arrived at the characterization of presentations as 'objectifying acts' (Husserl [1901], 445 ff.).

[42] Husserl, (ed.) Rollinger (2009), 18.

[43] This manuscript was first published in Husserl, (ed.) Rang (1979), 303–348. This edition, however, includes some of Husserl's later revisions and is incomplete. A later edition, which is complete and strictly adheres to the text of 1894, is published in Schuhmann (ed.) (1990/91) and translated in Husserl, (trans.) Rollinger (1999), 251–284. Concerning Husserl's reaction to Meinong (1902), see Rollinger (1999), 186–200.

where we speak of the object of a presentation, while there is no object in reality, we mean only an object under assumption or under hypothesis.[44] Accordingly, it would be modified speech when Husserl speaks of an impossible *Sachverhalt* or one that is correlated with a false judgement.[45] An impossible *Sachverhalt*, for instance, is not real. Nor is it immanent, for impossible or contradictory objects cannot exist in the mind any more than they can exist outside of the mind.[46] It need not be said, however, that an impossible *Sachverhalt* is objective (though not real), for if we follow the line that is taken in the 1894 manuscript on intentional objects, we may say that it is not a *Sachverhalt* at all, any more than a dead man is a man or a cancelled lecture is a lecture. Accordingly, in the time frame under consideration, Husserl has the tools to avoid any allowance for objective *Sachverhalte*, which are neither real nor immanent.

3.5 Excursus: The Question of the Influence of Bolzano

Nothing has thus far been said about Bernard Bolzano's influence on Husserl's thought for the simple reason that Husserl does not refer to Bolzano in either of the texts on the theory of judgement under consideration.[47] As is well known, in the *Logical Investigations* and related writings, Husserl adopted the notion of a 'proposition' (*Satz*) as an ideal object of logic. This notion was taken from the Bolzanian concept of a 'proposition in itself' (*Satz an sich*), which is distinguished from both the linguistic expression and the judgement (the judgement being described as a 'mental action').[48] Of course, some philosophers do not make a clear-cut distinction between a proposition and a *Sachverhalt*, whereas Husserl was to arrive at the view which he already expresses in manuscripts from the late 1890s:

> Truth is not a property of the *Sachverhalt*, but rather of the proposition.[49]
>
> Truth and *Sachverhalt* are not the same. The truth '$4=2\times2$' is different from the truth '$2\times2=4$', as the propositions are different. Sometimes we also say that it is in essence the same proposition, in essence the same truth. But 'in essence'. The addition, even when it is omitted, must be implicitly thought. The essence is simply the same *Sachverhalt*.[50]

[44] See especially Husserl, (trans.) Rollinger (1999), 261–269.

[45] The notion of modifying predicates and adjectives is one that Husserl took from Brentano. See Brentano (1874), 286 ff.

[46] Cf. Sigwart (1889), 123, as translated in Sigwart, (trans.) Dendy (1895) I, 98: 'The contradictory is impossible in my thoughts, as well as in the reality which is independent of me.'

[47] Concerning Bolzano's relation to Husserl, see Rollinger (1999), 69–82.

[48] Bolzano (1837) I, § 19.

[49] Husserl, (ed.) Rollinger (2009), 73. Cf. also Husserl (1900), 243 ff., where propositions are assigned to semantic categories (*Bedeutungskategorien*) and states of affairs to object-categories.

[50] Husserl, (ed.) Rollinger (2009), 83.

From Brentano and Stumpf, he took the concept of a *Sachverhalt* and from Bolzano that of a proposition as the bearer of truth.[51] While the Bolzanian influence came more pronounced as Husserl worked his way more and more towards the *Logical Investigations*,[52] there are hints of a Bolzanian influence in Husserl's 1893 manuscript. The most explicit one occurs in Husserl's mention of 'the judgement in the logical sense, of the "judgement in itself"'.[53] The term *Urteil an sich* is of course conspicuously close to Bolzano's *Satz an sich*. The suggestion may be made that a judgement in this sense can take on the role of the state of affairs that is abstracted from the act of judgement. Such a move would of course be palatable from the standpoint of Husserl's later rejection of psychologism. This, however, would involve another concept of the abstract (in later terms the 'specific' or the 'ideal') which is not present in the manuscripts on the theory of judgement from 1893/1894[54] and at best only hinted at in the manuscript on intentional objects.[55] In short, in 1893/1894 Husserl does not yet have an ontology of the ideal, as this is familiar to the readers of his later writings. In the first half of the 1890s, he is still for the most part working within the ontological horizons of Brentano and Stumpf, which were indeed quite 'nominalistic' (in the contemporary sense of the term)[56] and accordingly not conducive to purely logical entities of the Bolzanian variety.

4 Concluding Remarks

To sum things up here, we have identified at least two concepts of a *Sachverhalt* in the logic lectures of Brentano and Stumpf. The terms 'immanent' and 'real' have been used to distinguish them. In the early manuscripts of Husserl, we can clearly identify the concept of an immanent *Sachverhalt* in one of them, though the other suggests something that might be called the objective *Sachverhalt* rather than the

[51] See Bolzano (1837) II, § 125. Truth-bearers for Brentano and Stumpf (also for Marty) are judgments.

[52] See Husserl, (ed.) Rollinger (2009), 86, 93, 101, 138 f. See also the crucial lectures on logic that Husserl delivered in 1896 in Husserl, (ed.) Schuhmann (2001) where Bolzano's influence is most pervasive, as discussed in Rollinger (2008b), 87–108.

[53] Husserl, (ed.) Rollinger (2009), 29. In a revision of the 1893 manuscript, Husserl did add an explicit reference to Bolzano (ibid., 12 n. 4).

[54] In the 1893 manuscript, term 'ideal' is used in the 'normative sense' (Husserl, [ed.] Rollinger [2009], 9, 29 f.) which Husserl later clearly distinguishes from 'ideal' in the ontological sense. See Husserl (1901), pp. 101 f. The term is not used at all in 'Proper and Improper Judgement' or related material.

[55] Schuhmann (ed.) (1990/91), 148, 158, 171 (Husserl, (trans.) (1999), 257, 267, 281).

[56] As they understood nominalism, they were opposed to it. See Stumpf, (trans.) Rollinger (1999), 314: 'Nominalism wrongly teaches that general names always designate only a sum of individuals. They rather designate one or several abstract features which are already fully contained in a single individual, but also occur elsewhere in the same manner'. The question arises whether these features of different individual things can be numerically or only qualitatively identical. This question was a vexing one for Meinong and Husserl, as discussed in Rollinger (1993).

real one. If, however, Husserl wishes to introduce this new concept here, he may be – in accordance with his 1894 manuscript on intentional objects – speaking in a modified way. This would leave him the option of speaking of real *Sachverhalte* when he is speaking in the strict and proper sense (unless of course he is speaking of immanent *Sachverhalte*). Be that as it may, we see that there are at least two concepts of a *Sachverhalt* in Husserl's early manuscripts. There is in addition no apparent attempt at disambiguation. Hence, at this stage of Husserlian thought, we are left with a terminologically untidy situation to say the least, but maybe a much deeper problem. Before closing, I wish briefly to make a case for the latter conclusion.

This whole issue was to receive further attention in Husserl's subsequent attempts to work out a theory of judgement, as we find such an effort in the *Logical Investigations*, but also in many of the other texts published in *Husserliana* XL and in others which have been published posthumously. The distinction that he later made between the *Sachverhalt* and the *Sachlage* is certainly relevant in this regard,[57] the former being closely allied with the immanent *Sachverhalt* and the latter being allied with the real *Sachverhalt*, though of course with revisions which cannot be discussed here. In light of such a terminological innovation, it may be suggested that it is only a verbal issue at stake here. Yet, in view of the fact that Husserl did not rigorously adhere to this terminological distinction and later entered into intellectual struggles which very much reflect the early ones discussed here,[58] it hardly seems satisfactory to suggest that the matter is dealt with so easily as to introduce two technical terms.

In order to appreciate the depth of the problem, it may help here to indicate what motivates Husserl to retain the notion of an immanent *Sachverhalt*, even after he had rejected the notion of intentional objects. Such motivation particularly requires historical empathy, since those contemporaries who do take *Sachverhalte* seriously are only concerned with real or objective ones. Any suggestion of an immanent *Sachverhalt* is likely to be jettisoned as a crude psychologism that has been definitively laid to rest long ago. Here one should consider, however, that if we asked what we are judging, we may say without hesitation, and with as much certainty as we can possibly have, *what* this is, regardless whether it be true or false.[59] If, for instance, I am judging that $2 \times 2 = 5$, I know not only *that* I am judging but also that $2 \times 2 = 5$ is *what* I am judging. There is in this case no real or objective *Sachverhalt* corresponding to my judgement, and yet a 'that' clause is used to describe what is being judged. It is not implausible in such cases (and indeed in all cases of judgement) to say that a *Sachverhalt* is immanent to the judgement, for what we designate by the 'that' clause appears to be an inwardly perceived component of our judgements. At the same time, we obviously speak of actual facts by using 'that' clauses. Husserl's example from the *Logical Investigations*, 'That rain has set in at

[57] See Husserl, (ed.) Panzer (1987).

[58] See, for instances, Texts Nr. 8 (141–162) and Nr. 9 (176–207), both of which were written in 1911, in Husserl, (ed.) Rollinger (2009).

[59] Ibid., 141 ff.

last will delight the farmers',[60] is a case in point. Here it is clear that one is referring to something in the world and not in any sense something immanent to consciousness. Thus, there appears to be deep ambiguity in the usage of 'that' clauses to be sorted out through further inquiry. Since Aristotle it has of course been known that ambiguities are often systematic and philosophically interesting in nature, as in the case where we call not only a dog or a cat but also the picture of a dog or a cat an 'animal'. It is such ambiguity which Husserl keeps in mind and motivates his investigations for decades regarding the issue of immanent and real *Sachverhalte*.

From a purely ontological standpoint, we may wish only to concern ourselves with real *Sachverhalte* and forgo any considerations of the immanent ones. Husserl, however, was not merely an ontologist, a logician, or a philosopher of logic, as such terms are widely understood at present. He concerned himself with the ambiguity in the concept of a *Sachverhalt* first and foremost because the theory of judgement that he was attempting to formulate was to be the result of *phenomenological* investigations. From this standpoint the correlate of a false judgement, indeed even an absurd judgement, is of no less interest than the correlate of a true one. Yet, only the correlate of a true judgement can be real. For this reason immanent *Sachverhalte* remains or should remain a matter of concern for phenomenologists. Phenomenology, however, emerged from descriptive psychology, whereas the concern with ontological and logical matters totally void of any considerations that might be construed as psychological has been the high philosophical fashion for a long time. From the standpoint of those who are steeped in this fashion, it is understandable why immanent *Sachverhalte* is shunned without further ado. As a phenomenologist Husserl was unable to shun them. The material in *Husserliana* XL will hopefully enhance the appreciation of Husserl in this regard and perhaps even of phenomenology itself as a legitimate philosophical endeavour.

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[60] Husserl (1901), 440 (Husserl, [ed.] Moran [trans.] Findlay 2001] II, 155).

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Chapter 10
Frege and Russell on Assertion

Jeremy J. Kelly

Russell claims in *Principles of Mathematics* (1903) that grammar brings us closer to a correct logic than do the opinions of philosophers. In chapter IV of that book, he sets out to treat the nature of assertion, propositions, and the terms of grammar, prefacing his investigations with the remark, "in what follows, grammar, though not our master, will be taken as our guide" (*PoM*, §46, p. 42). Some may find this remark curious, for in so much of Russell's early work, there is more in common with the views of Frege and of other mathematically oriented logicians (e.g., Cantor, E. Schröder and Peano) than there is with the work of logicians whose conception of logic cleaves closely to the categories of traditional grammatical analysis. Representative of this latter group is the early twentieth-century logician W. E. Johnson, from whom Russell claims in the preface to *Principles* to have received "many useful hints" (*ibid.*, p. xviii).

In what follows, I advance several arguments which are derived from a theory of assertion first presented in a series of articles in the mid-1970s by the African logician Kwasi Wiredu.[1] The first argument concerns an interpretation of Frege's early theory of assertion, according to which:

(1) "The circumstance that unlike poles attract"
(2) "Unlike poles attract"

are formally expressible as

(1a) —A
(2a) ⊢ A

On this construal of both the vertical (judgement) and horizontal (content) strokes, I follow Frege. It may be said that Frege accomplishes precisely what he

[1] Cf. Wiredu (1975); also Wiredu (1973), 31–55.

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intended to accomplish in the first few pages of *Begriffsschrift*, namely, to show that for any judgement (1) is the judgeable content of (2) and that (2) shows (1) as having been asserted. On this interpretation (hereafter the syntactical interpretation), the semantic role of the expression to which the judgement stroke is attached is altered by the addition of the judgement stroke to the content stroke in (2a) in a way which requires us to either convert the sentence to a complex noun phrase or represent the alteration by converting the principal verb from its *finite* to its nonfinite form (W. E. Johnson).[2] In Frege's example (above), we have something like a transformation to a noun phrase with the use of indirect speech, but there are cases, as in Russell's example ("Caesar died"), that may be treated in accordance with (2). In taking the following pair of expressions:

(3) "Caesar's being dead" or "Caesar's death"
(4) "Caesar died,"

we have a suitable grammatical expression of the previous distinction in (1a) and (2a),

(3a) "—A"
(4a) "⊢ A"

Accordingly, the judgement stroke is construed here as an operator yielding transformations from incomplete functional expressions (typically descriptions) to complete expressions which standardly take the form of declarative sentences. In claiming that the function of the judgement stroke is syntactical, I am suggesting an interpretation according to which "⊢" is taken as a kind of operator, that is, an operator which derives chiefly from the notion of a grammatical functor (in this case, a functor which yields transformation from noun phrases to declarative sentences).

Recognition of the merits of the syntactical account among interpreters of Frege's *Begriffsschrift* has not, it seems, been forthcoming. Often, Frege's *own*, albeit tentative, argument for this account is simply dismissed by commentators, which is odd, since textual evidence in support of it is not very difficult to find. Many philosophers have tended to either downplay or fail to acknowledge altogether the logical significance of the syntactical interpretation, prominent among whom have been Dummett (1973, 314–315), Church (1956, p. 24), Geach (1965, 449–450), Anscombe (1959, 114), and Dudman (1972, 62–63). Indeed, to appreciate its significance, we need not look further than Frege's early logical writings. Witness Frege in the Begriffsschrift of 1879:

> If we omit the little vertical stroke at the left end of the horizontal stroke, then the judgment is to be transformed into *a mere complex of ideas*; the author is not expressing his recognition or non-recognition of the truth of this. Thus, let ĐA mean the judgment: "unlike magnetic poles attract one another." In that case —A will not express this judgment; it will be intended just to produce in the reader the idea of mutual attraction of unlike magnetic poles …In this case we *qualify* the expression with the words "the circumstance that" or "the proposition that".[3]

I argue that, given the plausibility of the syntactical interpretation, we can find a remedy to the difficulties that plagued Russell in the early part of *Principles*. One of

[2] Russell calls this construction in *PoM* the *verbal noun*.

[3] See *Begriffsschrift*, (trans.) Geach and Black (1960, p. 2)

these problems is to account for the difference between the finite and nonfinite forms of verbs in a way that accords with what Russell calls the "ultimate notion of assertion" (*PoM,* p. 48). It is at this point in his discussion (in §§51–53) that the guide of grammar, together with the syntactical interpretation of assertion, affords Russell a way out of his difficulties.

In the argument of §52, Russell holds that "every constituent of every proposition must, on pain of self-contradiction, be capable of being made a logical subject." He goes on to say that "[By] transforming the verb, as it occurs in a proposition, into a verbal noun, the whole proposition can be turned into a single logical subject, no longer asserted, and no longer containing in itself truth or falsehood."[4] What Russell finds puzzling about this transformation is that there is no ostensible difference between the proposition *as asserted* and the corresponding logical subject, or rather, if there is a difference, we seem unable to say what it is. Following Russell, if we go on to ask what is asserted in the proposition "Caesar died," we should say that the "the death of Caesar is asserted" or "Caesar's death." In this case, it is "Caesar's death" which is true or false, but it is equally obvious that, on Russell's analysis, truth and falsity cannot *belong* to a logical subject. So, the logical subject *qua* verbal noun must be the same as the complete proposition *qua* declarative sentence (what is asserted in "Caesar died" is "Caesar's death"), yet it is plain that they cannot be the same, for there is what Russell calls an internal occurrence of truth in the one and not the other.

Here, I think we are entitled to ask how Russell arrives at the conclusion that it is proper to say "Caesar's death" *is true*. If we take the logical subject to be an *incomplete* entity, then, while observing the distinction in (3) and (4), we should conclude that "Caesar's death" is not the sort of thing that could be true or false. The complex denoted by the verbal noun "Caesar's death" is not the kind of entity of which truth may be predicated, since this would mean that we could say with linguistic propriety – placing the predicate at the beginning – "It is true that Caesar's death" or "It is true that Caesar's being dead." Grammatically, this is odd, and assuming that Russell would have seen it as odd, it is quite imaginable that he took it as revealing something peculiar about the English language. What I am suggesting is that he would have then been mistaken – that the expression reveals something beyond a mere idiosyncrasy of English usage. The first step in avoiding the difficulty may be to say that "Caesar's death," taken as an expression on a par with the participial expression "Caesar's being dead," *transforms* to a declarative sentence once it is pressed into assertoric use. It is then possible to say that a sentence, being the result of this transformation, *is* the sort of thing of which we predicate "is true" in the usual ways.[5]

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[4] *PoM,* p. 48.

[5] George Pitcher has expressed some apprehension that a kind of schizophrenia here begins to set in to the theory of truth in this connection. Given that what I've said so far implies the possibility of two kinds of truth-bearers, Pitcher is perhaps right. (However, what I shall argue is that if it is indeed a schizophrenia, it is nonetheless a virtuous one). Below I distinguish between primary and secondary forms of truth attribution and argue that it is *sentences*, not utterances, that are bearers of secondary truth attribution. However, for the purpose of distinguishing between primary and secondary (the latter being what I have called "predicative"), it is not necessary to distinguish here between sentences and their use. See Pitcher (ed.) (1964).

In what follows, I argue that the Fregean syntactical distinction, along with the adoption of the suggested phraseology, affords a solution to Russell's problem of the "ultimate notion of assertion" and several kindred problems. I will presently consider one of them – namely, the double-aspect problem – and its bearing on the problem of assertion.

The double-aspect problem surfaces frequently in many of Russell's writings from his early realist period, specifically in connection with his attempt to address in *Principles* the so-called problem of complexes. That problem may be put in the form of a dilemma. It is assumed without question that we are capable of marking or stating any distinction that is thinkable. This we may attribute to an inheritance from F. H. Bradley's logic. There is a thinkable distinction between concepts ("as such") – that is, denoted as *meanings* on the one hand and concepts denoted as *terms* on the other. Following Nicholas Griffin (1993), we might use ***/a/*** to represent the concept "one" denoted as *meaning* and use ***/A/*** to represent the concept "one" denoted as *term*. "One," taken in its adjectival form, is to be understood as the concept *qua* meaning; "one" – taken as a substantive – is to be understood accordingly as the concept *qua* term. Given this symbolization, there is no reason why we should not expect the intended distinction to be at the very least stateable. But it is not. Russell shows that any attempt to state what the difference is, or even to state *that* there is a difference, lands one immediately in self-contradiction. On the other hand, if we take the concept denoted as meaning and the concept denoted as term to be one and the same concept – if ***/a /*** and ***/A/*** are identical – then we are destined to be "enveloped in inextricable difficulties" (*PoM*, §49, p. 45).

Russell's solution is then to say that the difference between ***/a/*** and ***/A/*** consists "solely in external relations." This is a consequence of recognizing that the difference cannot be intrinsic to the nature of the terms themselves, for in merely stating that ***/a/*** differs from ***/A/***, the term ***/a/*** is, *ipso facto*, converted to ***/A/***, hence the self-contradiction. It then follows that any proposition *about* the difference is necessarily false. As Russell notes, this is an unacceptable state of affairs. And the difficulties which lie on the other side of this double-aspect problem remain: it is impossible to state the distinction with the needed precision, for we cannot hold that there is a concept denoted as meaning that is not *also* a concept denoted as term. Yet, that there is a distinction is undeniable.

It is said that the contradiction involved in stating that /a/ differs from /A/ is avoidable if we take the difference to be one that is not internal to the terms which constitute the proposition but external to them. Is it obvious that we *must* not speak of the concept denoted as meaning and the concept denoted as term as one and the same concept? Since there is, I think, good reason to suppose that we may speak of the concept "as meaning" and the concept "as term" as the same concept, the inextricable difficulties into which we might otherwise be led in not following Russell's prescription may be shown to be avoidable.

On Russell's view, whenever a proposition expressed by a declarative sentence ("proposition proper"[6]) occurs as the logical subject of another proposition, that

[6] This is distinguished from what Russell calls a "propositional concept," which is expressed typically by a noun phrase – or verbal noun.

proposition is *un*asserted (e.g., "'Caesar died' is a proposition"). But if Russell is correct in claiming that asserted propositions have an *internal relation* to truth, then we have the following difficulty. On his view, propositions to which truth is internal may occur as the logical subject of a proposition. Thus, truth is a *constituent term* of that proposition:

> By transforming the verb as it occurs in a proposition, into a verbal noun, the whole proposition can be turned into a single logical subject, no longer asserted, and no longer containing in itself truth or falsehood…If we ask: What is asserted in the proposition "Caesar died"? the answer must be "the death of Caesar." In that case, it would seem, it is the death of Caesar which is true or false; and yet neither truth not falsity belongs to a mere logical subject. The answer here seems to be that the death of Caesar has an external relation to truth or falsehood (as the case may be), whereas "Caesar died" in some way or other contains its own truth or falsehood as an element. (*PoM*, p. 48)

The difficulty here is that because the assertive force of any proposition must be withdrawn once it is made the logical subject of another proposition (when the proposition is mentioned – e.g., "'Caesar died' is a proposition"), we cannot say that truth is *internally* related to that proposition. In saying "'Caesar died' is a proposition," we do not imply, much less assert, that it is true that Caesar died. Similarly, the "ultimate notion of assertion, *given by the verb*…is lost as soon as we substitute a verbal noun" (*PoM*, p. 48). In respect of the number of terms, the two complexes "Caesar died" and "the death of Caesar" clearly differ. This follows given that the verbal noun "the death of Caesar" lacks a constituent term possessed by the proposition proper "Caesar died" – that is, the term that is given by the *finite* form of the verb. It is interesting to note that although duly registering grammatical difference here is sufficient for the purposes of instituting the desired distinction, Russell seems to instruct us, rather casually, to disregard it as having no logical significance (*PoM*, pp. 35, 48).

To compound these difficulties, a proposition of which a truth-value is a constituent may go unasserted, in particular where the declarative form figures in truth-functional contexts and certain complex propositions.[7] Russell's requirement that such a proposition must be *unasserted*, and so must be distinct from an asserted proposition of which a truth (a truth-value) *is* a constituent, combines with still a second requirement, leading to further logical difficulties: one and the same proposition must occur in the antecedent of a conditional statement as that which occurs in the second premise of *modus ponens* (if it is to be a valid schema). But how could it be a valid schema, if the antecedent loses one term in virtue of having suspended its assertive force and the proposition in the second premise retains the term given by the finite form of the verb? The mistaken assumption here – it would seem – is to think that we cannot grammatically express the proposition *qua* logical subject of a proposition by means of a verbal noun or participial expression.

[7] This is Griffin's observation. That Russell did not wish to assert this is perhaps arguable, but it seems an unlikely possibility.

Before pursuing this diagnosis, I would like to return to the idea that truth could be internal to a proposition. Let us consider an earlier assumption of Russell's from *Principles*, namely, that the distinguishing feature of a complex is that it is the kind of thing that bears a truth-value, of which we may predicate "is true" or "is false." On Russell's theory, propositions are a kind of complex, so propositions naturally suggest themselves as candidates for truth-bearers. When we come to consider "the death of Caesar," however, we are quite reluctant to say that it could have a truth-value, even though the verbal expression (here, the verbal noun) succeeds in denoting a complex. The proposition "Caesar died" plainly *does* have a truth-value, but it is also a complex to which truth or falsity may, as in any referential discourse, be predicated in the ordinary ways; hence, the verbal constructions "that Caesar died" is true, and "it is true that Caesar died." As the doctrine of internal and external relations here suggests, there is *prima facie* reason to distinguish between *two* kinds of truth attribution in addition to distinguishing two forms of the same complex. Russell does not pursue this matter further, however. That aside, I wish to suggest that there is good reason to regard both "Caesar died" and "the death of Caesar" as the *same* complex. One reason is that if we suppose the proposition containing the finite form of the principal verb, for example, that expressed by the sentence "Caesar died" differs from the same proposition in which the verbal noun is substituted for the finite verb, then it is difficult to see, at least from the point of view of the theory of terms, how *modus ponens* could be a valid schema.

As Nicholas Griffin (1993) notes, three interrelated problems emerge from the double-aspect problem: (*a*) how can an *un*asserted proposition contain a truth-value (externally or otherwise), (*b*) how it is that an unasserted proposition can ever be the logical subject of a proposition, and (*c*) the problem of maintaining the theory of terms while respecting the validity of the inference schema *modus ponens*. We might first note – taking the second problem first – that a recurring mistake in Russell's analysis is the belief that we sometimes *do not need* to express the logical subject of a proposition as the verbal noun. This policy has the unwanted consequence of making it impossible to express a concept both as meaning *and* unasserted, to refer to propositional concepts (as opposed to what Russell called "propositions proper") *as* unasserted. Again, what motivates this reasoning is the belief that, insofar as Russell's early theory of complexes is concerned, (*i*) $\vdash$ A ("Caesar died") and (*ii*) —A ("the death of Caesar") differ in respect of the *verb* "to die," and so, they must differ in respect of a *term*. Hence, (*i*) and (*ii*) denote distinct complexes, and this is why we cannot say that they are the same. And so begin the difficulties with *modus ponens*.

A promising approach to solving the three problems emerging from the double-aspect of terms is to deny Russell's premise: let us suppose that (*i*) and (*ii*) above do *not* differ in respect of a term. In doing this, we stand to gain clarity in following W. E. Johnson (1921, Part I) and Kwasi Wiredu (1975) in construing the verbal noun (*ii*) "The death of Caesar" as grammatically equivalent to the participial form (*ii**) —A ("*Caesar's* being *dead*"). Occurring in the participial form is what W. E. Johnson calls the "latent formal element." On this approach, the verbal element is a constituent in *both* asserted and unasserted propositions; the verb contained in (*i*) is fully

inflected, whereas it takes the nonfinite form in (*ii**). We should therefore expect to find a corresponding difference in the proposition where there is a difference in the *form* of the verb. What unites the substantive and adjective is, to borrow Johnson's terminology, the "characterizing tie." The finite form of the principal verb (in this instance, the finite form of "to be") – "is" – marks the presence of another relation, that which Johnson calls the "assertive tie." This marks the addition of assertive force, but the force is added to the already existing characterizing relation.[8] Johnson's distinction provides the key to escaping the difficulties involved in maintaining that, as Russell had, "the difference between an asserted and an unasserted proposition is psychological" while admitting that "there is another sense of assertion very difficult to bring clearly before the mind, and yet quite undeniable, in which only true propositions are asserted" (*PoM*, p. 49). Wiredu's idea is to adopt Johnson's formal distinction between the characterizing and assertive ties, but to conceive of the participial rendering of the verbal noun as a unique kind of truth-bearer, to be distinguished from a secondary kind of truth-bearer (in the form of a declarative sentence). The suggestion, then, is that the characterizing and assertive ties be taken as having the same logical and semantic function which the content stroke and vertical (judgement) stroke have for Frege in the context of *Begriffsschrift*. With this distinction in place, Wiredu argues for the necessity of two kinds of truth-value (or truth-valuation), one occurring in the transition from the nonfinite to the finite form of the principal verb and a second – definable, in part, in terms of the first – occurring in the form of predicating "is true" of a declarative sentence (or to the proposition expressed).

Herein, then, lies the crucial difference between Russell's view of assertion and the modified Fregean view which I outlined at the outset: what corresponds to this added syntactic element is, to use Wiredu's term, the *primary* determination of a truth-value.[9] This determination (or attribution) is primary in the sense that an

[8] I leave out of discussion the "relational tie." It is of the utmost importance for Johnson's conception of propositions, but introducing it here would seem to needlessly complicate matters. For further discussion of Johnson's use of the relational tie, see J. Gibson (1921) 448–455. While Gibson regards Johnson's notions of the characterizing and relational ties to be of "first-rate importance," he finds the third – the assertive tie – less clear. Gibson's difficulties seem to lie in reconciling the logical function of the assertive tie with that of the characterizing tie. He believes that the assertive tie would appear to supply a *further* tie to the already existing characterizing tie. This presents for Gibson two understandable – but I do not think insuperable – difficulties. If we accept Johnson's view that "the specific difference between one kind of tie and another is determined by the logical nature of the constituents tied," then introducing the assertive tie would be at the least *ad hoc*. The second problem is that the assertive tie appears to be merely epistemic and to lack the constitutive element which both relational and characterizing ties possess. On Gibson's interpretation, the assertive tie is left to signify a "subjective attitude" toward the proposition. This seems to overlook the logical and semantic contribution made by the finite form of the verb – in particular the transition from the nonfinite to the finite form – which Johnson himself seems to recognize. It is hoped that the discussion (p.5) of Russell's problems with the internal occurrence of truth in a proposition sheds some light on my reasons for insisting on a logical – and so not *merely* subjective and epistemic – conception of assertive force.

[9] Wiredu (1975) distinguishes between this kind of determination (of truth-value) and that involved in the sort of attribution of truth by which the truth predicate is employed.

asserted indicative sentence bears a truth-value (either true or false) as a constituent term in something like the sense Russell intends in his theory of terms. It is necessary to note further that because the verbal element is a constituent of both (*i*) and (*ii*), we should not conclude that " A" and "—A" are therefore logically, or semantically, the same. The point of registering the addition of the assertive tie is to say that they are *not* logically the same. Thus, given the syntactical interpretation of Frege's judgement stroke, we are entitled to say that the propositional *content* ("judgeable content" in the language of *Begriffsschrifft*) is identical in each case, but to glibly regard the expressions "Caesar died" and "Caesar's being dead" themselves as identical would be to confuse the act of assertion with what is asserted.

It seems that any attempt to conceive the judgement stroke otherwise comes at the price of compromising the viability of *modus ponens*: the occurrence of the second premise contains an element (that supplied by the assertive tie) not contained in the occurrence of "Caesar died" in the hypothetical proposition "If Caesar died, then he died on the Ides of March," since the occurrence of the proposition "Caesar died" in a hypothetical proposition is *unasserted* and so a distinct complex from the proposition "Caesar died" as it occurs alone in the second premise. But this is a mistake. It would seem that we are misled along the way into thinking that "Caesar Died" *must* possess the element supplied by the judgement-stroke even when the proposition occurs alone in the context of an argument schema. Only by neglecting the syntactical distinction, however, would we fall into this mistake as concerns the function of the judgemement-stroke.

The proposition, so far as it may be said to possess a kind of unity, involves, as Johnson has shown, a purely formal relation between its terms. It is in virtue of this fact that a proposition may go unasserted and that the infinitive verbal element used to represent this relation ought not to have counted as a constituent term in Russell's analysis. Thus, "Caesar died," as it occurs in the conditional premise of the *modus ponens* schema, is, properly speaking, better represented by the participial expression "Caesar's being dead." Here, it is clear that the complex is destitute of the constituent term which it would have had given the use of the declarative form "Caesar died."

One may be inclined to speculate that such rendering of the proposition was seen by Russell, and after him Wittgenstein, to be unnecessary given that the assertive force belonging to the sentence "Caesar died" should be suspended in both truth-functional contexts and where it occurs in subordinate clauses. Of course, assertive force *should be* suspended in these contexts, but – crucially – no device was ever employed to mark the transformation.

Provided that these remarks are accurate in describing how Russell viewed the matter, he cannot be said to have been entirely consistent: any occurrence of "Caesar died" – where the finite form of the principal verb signifies the presence of the *assertive* tie – is an asserted proposition, which is to say, it expresses an assertion that occurs, as does any intentional act, at a specific time and place. It occurs as an episode in the cognitive history of a unique individual. Russell did not – it is fair

to assume – wish to suggest that in speaking of *modus ponens* as an inference schema, we ordinarily refer to the sentences which constitute its premises as particular dateable speech acts, which might be then relativized to specific speakers and contexts. Thus, "Caesar died," as it occurs in the second premise of *modus ponens,* is more properly construed as a *proposition* in Johnson's and Wiredu's sense, to be rendered as "Caesar's being dead" so as to indicate a judgeable content. That content plainly corresponds to a proposition whose unity is only formally specified by means of the characterizing tie. Symbolically – incorporating the assertion sign – *modus ponens* would be written as follows:

$\vdash p \rightarrow q$
$\vdash p$

———
$\vdash$ Therefore, q

To bring out the participial character of the propositional variable, we may symbolize the schema as follows, inserting the horizontal in such a way as to show that it goes with the variable:

$\vdash(\text{—}p \rightarrow \text{—}q)$
$\vdash(\text{—}p)$

———
$\vdash$Therefore, q

In any instance of this schema, the antecedent of the conditional premise must be identical to the proposition that occurs alone in the second premise; otherwise the inference will not carry. In the second schema (above), p in the first premise is identical to its occurrence in the second premise. Both occurrences may be rendered as "—p," to which the vertical judgement stroke may then be added in order to indicate that assertive force has been supplied. Thus, the proposition "If Caesar died, then he died on March 15th," should be expressed in the more perspicuous phraseology; "Caesars' being dead" implies "Caesar's dying on March 15th." On this interpretation, which is, in essentials, Wiredu's (1975), the propositional variables themselves are seen to be identical, and so, we avoid Russell's difficulties with *modus ponens*. Rather than construe p of the conditional premise as an asserted proposition ("Caesar died"), it should be construed as a participial form; the same holds for the p of the second premise. Russell's view renders p as "Caesar died" even where it occurs in a hypothetical proposition. In contexts involving the subordinate clause and in truth-functional contexts generally, assertive force is suspended. Ordinarily, we are not inclined to consider the proposition as it is expressed linguistically in its participial form when it occurs as the antecedent of a hypothetical proposition. But this is unfortunate, as it gives the *appearance* that the proposition possesses a constituent term – that which Russell noted is supplied by the finite form of the verb – which it may only properly be said to possess given the addition of the judgement stroke; otherwise the proposition does not possess the term. On account of this semblance

of a constituent term, propositional variables are incorrectly taken to represent, at once, both the proposition as asserted (in the form of the declarative sentence "Caesar died") *and* the proposition whose assertive force has been withdrawn ("Caesar's death" or "Caesar's being dead"). Assertive force of a proposition is thus suspended in virtue of the proposition's role in more complex propositions. Direct inspection of the uses of hypothetical statements confirms this fact. But here, the double aspect problem arises once again *unless* we construe propositions qua unasserted as having participial standing.

Russell's view may be seen to be especially problematic when we consider propositional variables that stand alone. A proposition that stands alone (e.g., in the second premise of *modus ponens*) is, on Russell's view, one that possesses assertive force. Not only do we wish to be able to assert that *p,* we also wish to say that *p* is of the *form* of an assertion (viz., the declarative form). But in following Russell even this far, we have again threatened to undermine *modus ponens*: just as assertive force is supplied in the second premise, the assertive force is withdrawn from the antecedent of the conditional premise, thus giving us different propositions on account of each containing a different number of terms. This difficulty vanishes if we construe *p* as it occurs alone as a participial construction.

I remarked earlier that if we must speak in terms of complexes, we should recognize what it seems Russell did not, namely, that an added element is given by the *finite* form of the verb. One reason for this seems to be, as I noted earlier, that a proposition occurring as a logical subject *need not* be rendered grammatically as a verbal noun or a participial construction. The reason for this, in turn, would seem to be that a proposition which inherits the structure of a declarative sentence can do double duty for a proposition that is either asserted or unasserted. A plausible explanation for this phenomenon is that the notion of assertion – of judgement – was for Russell, as it was for the later but not earlier Frege, primarily a *psychological* one. Of course, on neither the view I have proposed nor the later Frege and Russell view is a sentence true merely by our thinking it to be true (either it is the case that Caesar died or not). Still, what Frege and Russell perhaps should have said is that the "judgeable content" cannot be true merely by our thinking it true.

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