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Objects and Modalities

A Study in the Semantics of Modal Logic

Springer

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ISSN 2214-9775 ISSN 2214-9783 (electronic)
Logic, Epistemology, and the Unity of Science
ISBN 978-3-319-53118-2 ISBN 978-3-319-53119-9 (eBook)
DOI 10.1007/978-3-319-53119-9

Library of Congress Control Number: 2016963659

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

Among the grand themes at the intersection of logic and metaphysics, we find the problem of cross-world identity—the question of what it means to speak of one and the same individual relative to a variety of situations. If the semantics of modal expressions and expressions for propositional attitudes is analyzed in terms of possible worlds, as is commonplace in contemporary philosophy, the problem of cross-world identity is seen to be related to a number of further questions: the attribution of modal properties (the behavior of actual objects in counterfactual situations), *de re* ascriptions of propositional attitudes (beliefs concerning a specific existing object), and the possibility to think and talk about non-existent objects. Influential and widely accepted accounts of modal semantics, notably Kripke's theory of rigid designation, take the notion of 'same individual' to be unproblematic in modal settings. This book is motivated by my discontentment with views that partially or entirely underestimate the conceptual intricacies of cross-context identity.

I argue that the only unproblematic notion of identity applied to individuals is the notion of extensional identity, the notion of 'same individual' within one and the same world. Here, the relevant notion of world is a semantic notion. When speaking of 'worlds', I mean scenarios whose potential internal structure can be ignored for the purposes of semantic evaluation. When logically analyzing modal discourse, there are, then, two notions of identity to be distinguished—extensional identity and cross-world identity. The much-discussed problems of quantification into modal contexts show, in fact, that the notion of cross-world identity has non-trivial conceptual presuppositions and, therefore, cannot be considered simple and unproblematic. My positive account is in terms of 'world lines'—links between world-bound 'local objects'. I argue that the semantics of cross-world statements involves systematically two types of components: worlds and world lines. I take worlds and world lines to be mutually independent. World lines are not Lewisian counterpart relations supervenient on worlds. Even if one describes worlds in the minutest detail, one will not have even touched the question of which world lines are defined over those worlds. I take it to be a precondition of our speaking and thinking about individuals in many-world settings that they are construed as world lines.

My work has grown out of my attempts to make sense of the interpretation of quantified modal logic that Jaakko Hintikka propagated from the late 1960s onward. Terminologically, I follow Hintikka in referring to links between domains of worlds as world lines. Hintikka's proposal suffers from interpretational problems: at times, he motivates his proposal epistemologically (world lines as codifications of our means of recognizing an object in various worlds), while at other times, he opts for a 'transcendental interpretation' (it being taken as a precondition of our modal talk that individuals are thought of as world lines). I have discussed and attempted to develop the idea of using world lines in the semantics of quantified modal logic in a number of articles: 'Remarks on Individuals in Modal Contexts' [118], 'Cross-World Identity, Temporal Quantifiers, and the Question of Tensed Contents' [119], and 'Worlds, Times and Selves Revisited' [120]. In the present book, I formulate my account in detail, discuss its philosophical motivation, and spell out its theoretical consequences.

My analysis offers a surprising generalization of possible world semantics. It construes individuals and worlds as things of the same general type—as correlations between local objects. One way of partitioning the totality of local objects gives rise to worlds, while another way of partitioning them yields individuals. The resulting *world line semantics* provides a technically detailed and philosophically motivated novel interpretation of quantified modal logic. My semantic theory leads to a new account of object-directed intentional states and to a uniform treatment of physical objects and objects of thought in modal semantics—both are analyzed as world lines. At the same time, my framework allows clarifying formal similarities and dissimilarities between objects of these two types. I illustrate the usefulness of my theory by relating it to a variety of phenomena of philosophical interest, including the analysis of singular contents and the semantics of intensional transitive verbs.

In contemporary discussions of modal logic and modal metaphysics, one typically denies that there is any need for distinguishing cross-contextual identity and extensional identity and assumes that the notion of identity is simple and unproblematic. Insofar as one finds any need for clarificatory comments on what it means to speak of one and the same thing in distinct scenarios, such clarifications are attempted in terms of qualities or descriptions or in terms of causal continuity. The option of studying cross-world identity as a primitive relation in its own right is not systematically explored. There are, however, precedents to the approach that Hintikka strived to formulate and that I develop in detail in this book. In the history of analytic philosophy, the first attempt to clearly articulate the distinction between extensional identity and cross-world identity appears to be the distinction that Carnap makes in his 1928 book *Der logische Aufbau der Welt* between 'logische Identität' (logical identity) and 'Genidentität' (genidentity) [11, Sects. 128, 159]. The term 'genidentity' was coined by Kurt Lewin in his habilitation thesis *Der Begriff der Genese in Physik, Biologie und Entwicklungsgeschichte* (1920, published in 1922). Lewin understood genidentity as a primitive relation that must be presupposed in the analysis of any physical process [73, 74]. For Carnap, the criterion of logical identity of objects *b* and *c* is that *b* and *c* satisfy the same predicates.[1] Genidentity, again, is a relation that prevails among 'thing-states' of one and the same 'thing'. If *b* is a thing-state, there are numerous things-states *c* to which *b* bears the relation of genidentity but only one thing-state—namely, *b* itself—to which *b* stands in the relation of logical identity. Crucially, Carnap takes the two identity relations to be mutually independent. In particular, genidentity is not reducible to logical identity. Incidentally, Carnap analyzes things and thing-states in terms of what he calls world points ('Weltpunkte') and world lines ('Weltlinien'). His world lines consist of mutually genidentical world points. World lines are grouped together as bundles. Each thing is a class of world points belonging to one of the world lines of a given bundle, and a thing-state is a suitable subclass of a thing. In Carnap's analysis, the relation of genidentity can prevail not only among world points (those that belong to the same world line), but also among *classes* of world points (namely, among thing-states that belong to the same thing).

Unfortunately, by the time Carnap got interested in modal logic, he had abandoned the distinction between genidentity and logical identity, or, in any event, he makes no use of this distinction in his 1947 book *Meaning and Necessity* [12]. What Hintikka in effect did, albeit with less than full clarity, is that he refused to analyze the relations of identity and nonidentity among inhabitants of *distinct* worlds in terms of the identity relation applicable in connection with world-internal comparisons. Hintikka mobilized the notion of world line to account for cross-world identity. However, he never succeeded in articulating his vision in a satisfactory manner, because he never got clear about the precise role of world lines—viewing them at times epistemically, as means of recognition of an individual, and at other times conceptually, as an explication of what individuals are in modal settings.

It is, actually, not far-fetched to surmise that Hintikka's ideas on objects of perceptual experience, in particular, were inspired by Carnap's work in the *Aufbau*. In his 1975 essay *The Intentions of Intentionality* [46, Chap. 10], Hintikka says that even the most basic sensory experience is 'already experience *of* certain objects, their properties, their interrelations, etc.' so that 'one's unedited sense-impressions are already structured categorically' and 'the most primitive layer of sensation we can reflectively behold is already…organized so as to be *of* definite objects'. Hintikka takes this to mean that one 'does not perceive a hemispheric surface, and expect it to go together with the rest of a soccer ball because one recalls past experiences of it'. Instead, one 'literally perceives a soccer ball, period', and the 'backside of a tree one sees is not brought in by apperception, but is already part

[1] Carnap discusses logical identity in a linguistic setting, by speaking of conditions under which two singular terms designate the same object. However, for him, logical identity is not a relation among linguistic expressions—he does not equate it with synonymy or co-reference. Instead, it is a relation among (non-linguistic) objects. It is not a relation among 'names' but among things named. As for Carnap's criterion, unless it is supposed that numerically distinct objects can be logically identical in Carnap's sense, Carnap is committed to a version of the principle of *identity of indiscernibles*: for any pair of distinct objects, there is a predicate that one of the objects satisfies but the other does not.

and parcel of one's unedited perceptions'. According to this view, there is a conceptualizing element in all perception—but not so that this conceptualization acts on independently given sense-impressions. Instead, conceptualization is taken to be 'built into these unreconstructed sense-impressions themselves'.[2] Hintikka's comments echo what Carnap [11, Sect. 100] means when stressing that '[t]he "given" is never found in consciousness as mere raw material, but always in more or less complicated connections and formations' and when maintaining that '[i]n looking at a house, we perceive it immediately and intuitively as a corporeal object; we imagine its unperceived back side [and] its continued existence while we are not looking at it'. Both thinkers agree, then, that the language we need for describing immediate experience has a realist character. It is of the same type as the language we need for talking about physical objects. It is *not* a phenomenal language limited to describing sense data or sensory impressions in virtue of which we could at best indirectly perceive physical objects.

The objects that have epistemic primacy appear to us, then, as full-blown objects. While such objects are experienced as undivided unities, this does not preclude that through abstraction we can view them as having constituents or as being otherwise analyzable (cf. [ibid. Sects. 54, 67]). Indeed, Carnap analyzes visual things ('Sehdinge') as consisting of thing-states, which are relative to a point of view ('Ausblickpunkt'). Every thing-state is a set of simultaneous world points. These world points can be divided into those that are seen and those that are unseen from a given point of view. In the case of veridical perceptual experience, the world points that make up a thing-state can be thought of as points on the surfaces of bodies [ibid. Sects. 126–128]. Hintikka [46, pp. 201–3], too, arrives at an analysis of objects of perceptual experience but through a different type of reasoning. If contents of perceptual experience are naturally described in realist terms, as involving realistically conceived objects, then perceptual experience yields information about the world—information that may more or less accurately describe the world in which the agent is situated. Information, again, is a modal notion: a given body of information allows us to partition a totality of possible worlds into those that are compatible with the information and those that are incompatible with it. Normally, one's perceptual experience leaves open a host of alternatives: there are innumerable worlds compatible with what one experiences in given circumstances. Due to such indeterminacy, the analysis of what one experiences always involves a large number of worlds. Consequently, when the content of perceptual experience is analyzed in terms of information, the objects of experience must be considered in relation to a set of worlds—among which there may but need not be the world w_0 in which the agent is located.[3] Since Hintikka construes cross-world identity in terms of world lines, and since he is systematically led to consider objects of perceptual experience in many-world settings, it follows that in his analysis, objects of

[2]For these quotes, see [46, pp. 200–2].

[3]If, in w_0, Alice hallucinates that there are flying pigs, then w_0 is not one of the worlds compatible with Alice's perceptual experience—unless in w_0 there are, in effect, flying pigs.

experience come out as world lines considered in relation to the set of worlds compatible with the agent's perceptual experience.

This book is organized into six chapters, which can be briefly described as follows. In Chap. 1, I describe schematically the semantic framework that I will develop. As already indicated, my starting point is the view that the only unproblematic notion of identity is the notion of extensional identity—identity within one and the same world. I take cross-world identity to be in need of explication. It will be explicated in terms of the notion of world line. World lines are links between world-bound 'local objects'. World lines themselves are not local objects. By contrast, their *realizations* are local objects. The *modal margin* of a world line is the set of worlds in which it is realized. A world line need not be realized in all worlds. I take values of first-order quantifiers to be world lines. Those world lines that are possible values of quantifiers in a world w are said to be *available* in w. A world line may be available in w even if it is not realized in w and may be realized in w without being available in w. If w_1 and w_2 are distinct worlds, b belongs to w_1, and c belongs to w_2, then I judge both claims 'b is identical to c' and 'b is numerically distinct from c' to be meaningless. What may happen is that there is a world line $\mathbf{I}$ such that the realization of $\mathbf{I}$ in w_1 is b and the realization of $\mathbf{I}$ in w_2 is c. I show that Hintikka mixes up two ways of interpreting world lines (the 'transcendental' and 'epistemic' interpretations referred to above). Therefore, while Hintikka uses world lines to formulate the semantics of quantified modal logic, the motivational basis of his view is globally incoherent.

In Chap. 2, I discuss, first, the nature of the proposal put forward by the transcendental interpretation. I then formulate a formal semantics of a quantified modal language L_0 in which quantifiers range over world lines. I discern a general notion of *content* and show that both worlds and world lines can be seen as 'modal unities'. Contents are structures of interrelated modal unities. I close the chapter by clarifying how my world line framework is related to competing semantic and metaphysical views, notably those developed by Kripke, Lewis, and Fine.

In Chap. 3, I discern two *modes of individuation*: the physical and the intentional. I take physical objects to be physically individuated world lines. Intentional objects are viewed as intentionally individuated world lines, defined on worlds compatible with an agent's intentional state. A generalized modal language L is introduced. In it, there are two types of quantifiers, differing in the types of world lines they range over: physical and intentional quantifiers. I discuss in detail the distinction between *availability* and *realization*. In particular, an intentionally individuated world line may be available in a world w (that is, it may be a value of an intentional quantifier in w) without being realized in w: we can speak of intentional objects that do not exist. I relate my analysis to Hintikka's epistemically motivated distinction between the public and perspectival modes of identification and to Williamson's necessitism. While I reject necessitism about intentional objects, I do not claim to provide a knockdown argument against necessitism about physical objects. I suggest, though, that the necessitist's 'static' view of reality may well be the result of confusing a useful mathematical model of the reality with the reality itself. I close the chapter by rejecting Meinongianism and comparing my analysis of intentionally individuated world lines with Priest's Meinongian account of objects of thought.

In Chap. 4, I discern two senses of predication that are needed when discussing the two modes of individuation. I discuss the three distinctive features of intentional objects—namely, description-sensitivity, indeterminacy, and existence-independence—and show how they can be characterized in terms of world lines. These features are directly related to three questions we can ask about a given world line. First, how can predicates be used to describe the world line? Second, how is its modal margin determined? Third, how are the worlds in which the world line is available related to those in which it is realized? I take up the issue of how contents of intentional states can be analyzed in my framework. Finally, I discuss conditions that must be met in order for an intentional object to be a representation of a physical object (the problem of relational representations). Against the view that contents of intentional states are always propositional, I present a general model of contents as structures consisting of a set of worlds equipped with a sequence of intentionally individuated world lines. Propositional contents correspond to the special case in which the number of world lines is zero.

The consequences of my semantic framework to strictly logical questions are explored in Chap. 5. I discern two notions of validity (model-theoretic vs. schematic validity) that are equivalent in standardly interpreted first-order modal logic but not equivalent under world line semantics. I point out that my modal language L lacks a well-behaved notion of logical form. I explain that this is not a reason to dismiss L as a logical language worthy of study and that L has a natural extension in which the notion of logical form behaves in the expected way. I show that L is translatable into first-order logic. This result is somewhat surprising, given the apparent higher-order character of L: values of quantifiers are world lines, which can be semantically modeled as partial functions over worlds. I conclude the chapter by explaining that anomalous semantic properties of L stem from features of the subject matter discussed—in particular, from the fact that the simplest properties considered are existence-entailing. The anomalies of the language do not tell against my framework. The language used for talking about a subject matter must, evidently, be designed so as to make the relevant features of the subject matter expressible. Any formal properties of the resulting language must be tolerated as long as the language serves its purpose.

In Chap. 6, I discuss general theoretical consequences of world line semantics. I indicate how mental states involving different types of objects of thought can be uniformly represented in my semantic framework: propositional thoughts, plural thoughts, thoughts with an indeterminate object, singular thoughts, and thoughts representing specific physical objects. I define singular contents as world-relative contents involving a single intentionally individuated world line. I compare my account of singular contents to Recanati's theory of singular thought and spell out similarities and differences between my semantic analysis of intentional contents and Crane's work on intentionality. What is more, I point out that in the context of my modal language L, variables can be viewed as formulas. Syntactically, variables are singular terms, but semantically, they have satisfaction conditions. This double role of variables opens up a way of representing certain intensional transitive verbs in my semantic framework. I discern a semantic criterion that an intensional transitive verb must satisfy to be thus analyzable and refer to the relevant class of verbs as *robust intensional verbs*. Finally, I compare my logical analysis of these verbs with Moltmann's linguistically driven account of what she calls intentional—as opposed to intensional—verbs.

I wish to express my gratitude to the French National Center for Scientific Research (CNRS) for awarding me a research leave—*accueil en délégation*—for the academic year 2014–2015. Most of the work for this book was carried out during this period. The work was completed in the course of the academic year 2015–2016. I am indebted to the anonymous referees for their comments and criticisms.

This book is dedicated to the memory of my father.

Lille, France Tero Tulenheimo

Contents

Chapter 1
Individuals and Cross-World Identity

1.1 Introduction: The Notion of 'Same Object'

Many important ideas discussed in analytic philosophy since the mid-20th century have been phrased in terms of possible worlds understood as mutually incompatible but intrinsically possible alternative scenarios. Such worlds involve a number of objects that enjoy various properties and are interrelated in different ways. Further, they provide circumstances of evaluation of suitable declarative sentences, allowing one to determine such sentences as true or false. A great variety of phenomena lend themselves to a *modal* analysis. In fact, the analysis of any phenomenon that can be understood only by taking into account a number of alternative eventualities is in the relevant sense modal by nature. This is so independently of whether those 'eventualities' are times or instants; spatiotemporal perspectives; scenarios compatible with what someone sees, believes, or knows; alternative states of a physical system; or alternative physical universes. In the semantics of modal logic, such eventualities are abstractly termed 'possible worlds'. An instant of time and a spatial region of 4 square meters considered over an interval of 10 minutes are examples of worlds when the term is thus understood. Not all worlds are entire possible universes.[1]

When abstractly construed, possible worlds can have possible worlds as constituents. This is what happens with scenarios whose internal spatial and/or temporal structure must be explicitly taken into account. However, unless otherwise indicated, when speaking of 'worlds' in this book, I mean *relatively simple worlds*—i.e., possible worlds whose potential internal structure can be ignored. Occasionally, I make use of the expressions 'scenario' and 'context' to designate worlds in this sense. When I expressly wish to speak of worlds with an internal structure, I refer to them as *structured worlds*. Temporally structured worlds are composed of *temporal phases*,

[1]The notion of possible world is abstract, in the sense of admitting many interpretations. This is not to say that possible worlds are abstract. Of the various things that qualify as worlds, instants are, for example, more likely to count as abstract than the actual physical universe. For *not* understanding worlds as entire possible world histories as Lewis [76] does, see, e.g., Hintikka [47], [49, pp. 22–3].

T. Tulenheimo, *Objects and Modalities*, Logic, Epistemology, and the Unity of Science 41, DOI 10.1007/978-3-319-53119-9_1

which themselves are 'possible worlds' in the abstract sense. Unless, for example, the internal spatial structure of these temporal phases happens to interest us, they are, in particular, 'relatively simple worlds' in the above sense.

When discussing the counterfactual behavior of physical objects, not only do we need to speak of one and the same individual at different times, we also need to consider the individual as appearing in circumstances that are not realized at any time in the current structured world.[2] What does it mean, then, to speak of an individual as existing at two instants in the same structured world or in two logically possible alternative scenarios? How should the interrelationship between possible worlds and individuals be articulated? The *form* of my answer to such questions constitutes the common thread running through this book.

My starting point is the view that possible worlds and individuals are mutually independent. Positively, I will analyze both worlds and individuals in terms of what I call *local objects*. My proposal is, then, that individuals cannot be reduced to worlds and worlds cannot be reduced to individuals, but both worlds and individuals can be explicated in terms of local objects. Concerning local objects, we can make extensional claims, but there are no meaningful counterfactual claims about local objects—we cannot meaningfully consider a local object as being present in several possible worlds. Both worlds and individuals can be seen as correlations between local objects. Any world has a number of local objects as components: those that are 'compresent' in that world. Any individual, too, has a number of local objects as components: those that serve to 'manifest' this individual—those that are its 'realizations'. No two worlds have local objects in common: local objects are world-bound. Individuals are not subject to an analogous restriction: mereological considerations might lead us to consider two individuals that share a manifestation. Concerning interrelations of worlds and individuals, it is supposed that any given individual $\mathbf{I}$ and any given world w may have at most one local object in common. If a local object b belongs to both $\mathbf{I}$ and w, then b is the unique manifestation or realization of $\mathbf{I}$ in w. It may also happen that an individual simply is not manifested in a given world. The described understanding of individuals leads us to view them as *world lines* that link together local objects, each of which is bound to a specific world.

First and foremost, the notion of local object is a semantic notion that is relative to the type of discourse being analyzed.[3] Those statements that can be evaluated

[2] *Individuals* do not normally count as worlds in the sense discussed: declarative sentences are not normally taken to be true or false with respect to individuals. (In Prior's egocentric logic, however, individuals are precisely viewed as contexts of evaluation of certain types of sentences; cf. Sect. 6.5.) I follow this standard view in this book and do not consider individuals as worlds.

[3] Different metaphysical interpretations of the notion of local object are possible. If we are interested in temporally extended worlds, then local objects could be metaphysically interpreted as world-bound temporal parts or time-slices of individuals. A sense-datum theorist of perceptual experience could view sense data as local objects that in suitable circumstances serve as signs of spatiotemporally extended material objects; in this vein, Russell [106, pp. 115–6] defines physical things as those series of aspects (sense-data, appearances) that obey the laws of physics. A proponent of neutral monism could interpret local objects as portions of neutral stuff out of which physical and mental entities are constructed.

relative to a single possible world concern local objects. If we never wanted to make any claims that require considering an individual in distinct possible worlds, then we might just as well not distinguish between individuals and local objects. However, according to the present proposal, as soon as we actually wish to speak of individuals in several possible worlds, the distinction must be made. The mutual independence of worlds and individuals amounts to the fact that we can be fully informed about the local objects that constitute an individual without having any information about possible worlds, and we can be fully informed about qualities and interrelations of local objects within a world without having any information about individuals.

When logically analyzing modal discourse, there are, at least on the face of it, two notions of identity to be distinguished—extensional identity and cross-world identity.[4] The notion of extensional identity applies in connection with local objects belonging to one and the same world. By contrast, in order to make sense of cross-world identity, we need the distinction between local objects versus individuals that are understood as world lines. Extensional identity is a binary relation whose terms are local objects. Every local object bears this relation to itself and to itself only. More precisely, it is a categorial presupposition of asking whether b and c are extensionally identical that b and c are local objects belonging to the same relatively simple world. If this condition is not met, the question of whether b and c are extensionally identical is devoid of sense.[5] Any meaningful identity claims that concern local objects present in distinct worlds are claims of cross-world identity, and such claims amount to affirming that these local objects are manifestations of one and the same world line.

I take it that the only unproblematic notion of identity is the notion of extensional identity—local identity, identity within one and the same relatively simple world. This is the notion of identity we encounter in first-order logic, giving rise to such validities as

$$\forall x\ x = x,\ \forall x \exists y\ x = y \text{ and } \forall x \forall y (x = y \to \neg\neg x = y).$$

Regarding local identity, I am happy to subscribe to David Lewis's words, according to which '[i]dentity is utterly simple and unproblematic' since '[e]verything is identical to itself' and 'nothing is ever identical to anything else except itself' [76,

[4]As noted in the Preface, this is essentially the distinction to which Carnap [11] referred by speaking of logical identity and genidentity.

[5]Various philosophers have found it perplexing to consider identity as a binary relation; for a discussion, see [125]. For my part, I assume the standard mathematical understanding of relations (relations-in-extension). Thus, whenever A is a set, the set $\{\langle a, a\rangle : a \in A\}$ is a binary relation—namely, the identity relation on A. In Sect. 2.3, I point out that in the modal language I introduce, the semantic value of a formula of the form $x = y$ can be taken to be a ternary relation whose terms are a pair of world lines and a world. Yet, this fact does not make the relation of extensional identity disappear: precisely when the mentioned ternary relation prevails among world lines $\mathbf{I}$ and $\mathbf{J}$ and a world w, the realization of $\mathbf{I}$ in w and the realization of $\mathbf{J}$ in w are extensionally identical local objects of the world w.

p. 192]. While these turns of phrase have an undeniable rhetorical force, they do not point to an argument to the effect that *cross-world identity* is unproblematic.[6]

First, these phrases utilize the quantifiers ‘everything’, ‘nothing’, and ‘anything’ and presuppose that identity is a sort of relation that can prevail between possible values of quantifiers. In extensional settings, this is so, but in cross-world settings, this is not evident: for example, one could take values of quantifiers to be temporally extended four-dimensional objects while construing identity as a relation that holds between momentary stages of those objects and, at best, in a derivative sense between the objects themselves. Unless the mentioned presupposition is accepted, we have two notions of identity to worry about—cross-world identity and local identity—and no convictions that one may have concerning the latter can be automatically transferred so as to apply to the former. Second, the quoted phrases make no obvious reference to several worlds. If so, they are beside the point and cannot count as comments on cross-world identity. On a more charitable reading, we may construe ‘ever’ as a temporal quantifier having a narrow scope with respect to ‘nothing’, whence the semantic analysis of the phrase ‘nothing is ever identical to anything else except itself’ indeed involves the shifting of a world. This concession leaves us with a couple of interpretational possibilities concerning the logical form of the phrase. The precise content of each reading will depend on how the interplay of quantifiers and modal expressions is understood. I distinguish the following readings, the first being weaker than the second:[7]

[6]The semantic notions of extensional identity and local object are understood with reference to relatively simple worlds. It will depend on the type of language that is considered which sorts of contexts count as relatively simple worlds: their ‘simplicity’ need not be absolute in any sense. For example, having started with a language L_1 used for talking about temporally unanalyzed worlds, we might turn our attention to the instantaneous phases of those scenarios and formulate a language L_2 suitable for making claims about temporally structured worlds. Entities that are considered as local objects (not as individuals) in connection with L_1 are seen as temporally extended individuals in connection with L_2, linking together local objects, each of which is bound to a specific temporal phase. In L_2, the only unproblematic notion of identity is identity relative to temporal phases. By contrast, the world-relative notion of local identity that was unproblematic in L_1 becomes a notion of cross-temporal identity in L_2. It may be noted that the notion of ‘individual’ in the semantics of first-order logic is formally analogous to the notion of local object in the following sense. Formulas of first-order logic are evaluated relative to a domain whose elements are referred to as ‘individuals’. We can consider entities of *any internal complexity* as values of first-order variables (as individuals), provided that we are merely interested in talking about those entities themselves and not of their potential constitution. Among such individuals, there could be, for example, *sets* of natural numbers (that is, elements of the power set of $\mathbb{N}$). Should we wish to talk not only about such sets but also about their elements, we would need to leave aside first-order logic evaluated relative to the power set of $\mathbb{N}$ and turn attention to second-order logic evaluated relative to the set $\mathbb{N}$ itself, so that first-order variables would take natural numbers as values, while second-order variables would take as values sets of natural numbers.

[7]The difficulty in settling on a specific reading is due to the specification ‘except itself’. The meaning of the binary connective ‘except if’ is given by the equivalence (p except if q) $\Leftrightarrow$ $(\neg p \leftrightarrow q)$. Consequently, ($x$ is not identical to y except if y is x itself) has the form $(\neg\neg x = y \leftrightarrow x = y)$. While $\forall x \Box \forall y(\neg\neg x = y \leftrightarrow x = y)$ is a possible rendering of the phrase ‘nothing is ever identical to anything else except itself’, it is weaker than what is presumably intended, since the condition

a. $\forall x \Box[\forall y(x = y \rightarrow \neg\neg x = y) \;\wedge\; x = x]$
b. $\forall x \Box[\forall y(x = y \rightarrow \neg\neg x = y) \;\wedge\; \exists y \; x = y]$.

According to both readings, nothing is ever identical to anything else. The first reading adds that everything is always *self-identical*, leaving open the question of whether among the possible values of y there is the value of x. The strong interpretation claims that everything is always *identical to something*, which has the consequence that the value of x in the initial context of evaluation must indeed lie in the range of quantifiers in an arbitrary alternative context.

Now, the phrase 'nothing is ever identical to anything else except itself'— whether it is analyzed in accordance with (a) or (b)—can only serve to reassure one's sentiment of the unproblematic nature of identity in modal settings if one or two extra premises are independently motivated. Both readings require that the values of the quantifier 'nothing' be available to be talked about not only in the initial context of evaluation w_1 of the phrase but also in any context w_2 to which 'ever' may move our attention. In (a), the variable x occurs free in the subformula $\Box[\forall y(x = y \rightarrow \neg\neg x = y) \;\wedge\; x = x]$ in the scope of $\Box$; so, we must be able to talk about the value of x in the context w_2 functioning as a semantic value of the operator $\Box$. The strong reading further requires that among the possible values of the quantifier 'anything' in w_2 there be the value assigned to 'nothing' in w_1. Namely, in order for the subformula $\exists y \; x = y$ of (b) to be satisfied in w_2, it must be possible to select as the value of y the same value that has been assigned to x when interpreting $\forall x$ in w_1.

When calling into question the utter simplicity of identity in many-world settings, I am not suggesting that there might be things that sometimes fail to be self-identical. Instead, I wish to point out that the precise way in which formulas such as (a) and (b) are to be construed depends on a good number of non-trivial assumptions concerning the interplay of quantifiers and modal expressions. Suggesting that such a formula expresses something utterly unproblematic amounts to holding that these assumptions do not rule out viable conceptual possibilities and that no interpretational issues are at stake. Supposing at the outset that one's preferred assumptions are harmless begs the question of whether one is justified in thinking of identity in many-world settings on the model of extensional identity.

1.2 Intricacies of Cross-World Identity

If a language allows the combining of quantifiers and modal operators, the models relative to which it is evaluated must, overtly or covertly, provide answers to the following questions about any possible context of evaluation w:

- Which objects can function as subjects of predicates in w?

(Footnote 7 continued)
$(\neg\neg x = y \leftrightarrow x = y)$ holds vacuously for any values of x and y—also in the special case that no value of x is identical to any value of y.

- Which objects are available to be quantified over in w?
- Which objects exist in w?

As I use the term 'subject of a predicate' here, a non-linguistic object o is a subject of a linguistic predicate P if it makes sense to ask whether the predicate P is true of o. It is not required that P indeed be true of o. That is, subjects of predicates in a world w are those objects to which predicates can be ascribed in w. The answers to the three questions above specify three sets, to which I will refer as $subj(w)$, $range(w)$, and $ex(w)$, respectively. Different overall views on modal semantics lead to different ways of understanding how these sets are interrelated.

Here is a summary of the most pressing questions one must ask before a definite sense can be assigned to a phrase involving modal expressions and quantifiers:

1. Are subjects of predicates and values of variables objects of the same type?
2. Suppose a formula is being evaluated and, in the course of evaluation, a value is assigned to a variable x relative to a world w. What does it mean, and on what conditions, if any, is it possible to evaluate in a world distinct from w an atomic subformula containing occurrences of x? This is the so-called problem of 'quantifying in'.
3. Is the range of quantifiers sensitive to the context of evaluation? That is, should we adopt 'varying domains of quantification' or rather a 'constant domain of quantification'?
4. Given a world w, how are the sets $subj(w)$, $range(w)$, and $ex(w)$ interrelated?

Let us consider these questions in turn. Those philosophers who insist on the simplicity of identity and attempt to resist the need for distinguishing local from cross-world identity must hold that values of variables are themselves subjects of predicates—in particular, subjects of the identity predicate. They must respond to Question 1 in the affirmative. I already mentioned one possible reason why one could think otherwise. If in a world w, we can only evaluate identity claims pertaining to stages of four-dimensional entities present in w but the semantics assigns cross-temporal hybrids as values of variables, then we lack means in our language to state that cross-temporal hybrids (as opposed to their stages) are self-identical. If the value of x is the four-dimensional individual $\mathbf{I}$, affirming $x = x$ in w means, according to this view, that the *stage* of $\mathbf{I}$ realized in w is self-identical, not that $\mathbf{I}$ itself is self-identical. Certainly from a metatheoretic viewpoint, we can say that four-dimensional entities themselves are self-identical, too, but what is crucial is that the mentioned interpretive possibility yields a distinction between local identity and cross-world identity and has the consequence that we have no direct means of speaking of the latter in our language.

Question 2 is an expression of the much-discussed problem of quantification into modal contexts. Taking either of the formulas (a) or (b) to be self-evidently valid is to deny this problem as worthy of discussion. To see this, it suffices to consider the formula $\forall x \Box x = x$, which is by any reasonable criteria entailed by both (a) and (b).[8]

[8]Because at present we are discussing how to interpret formulas of quantified modal logic, we have as yet no precise semantics relative to which we could rigorously speak of entailment.

Assuming that the defender of the simplicity of identity has already given a positive answer to Question 1, denying the problem of quantifying into modal contexts means holding that an individual **I**, chosen as the value of x in world w_1, can itself function as a subject of a predicate in another world w_2: the sets $range(w_1)$ and $subj(w_2)$ have a non-empty intersection. The most convenient stance for the identity simplicist is to suppose that we have

$$(*) \qquad subj(w) = \bigcup_{v \in W} range(v) \text{ for all } w \in W,$$

where W is the totality of all worlds considered. That is, if a tuple $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ belongs to the interpretation of an n-ary predicate Q in w, then every $\mathbf{I}_j$ belongs to $range(w_{i_j})$ for some world w_{i_j}, but it is by no means required that $w_{i_j} = w$. According to this assumption, then, the same objects are available as subjects of predicates in all worlds, and these objects comprise all and only objects belonging to the range of quantifiers of some world. This is a partial response to Question 4.

For the champion of simplicity of identity who takes the formula (b) to be valid, it does not suffice to assume $(*)$. In fact, unlike (a), formula (b) cannot be true in w_1 unless for any possible value w_2 of $\Box$, the sets $range(w_1)$ and $range(w_2)$ can have elements in common. If there are values w_2 of $\Box$ distinct from w_1, the truth of formula (b) in w_1 leads to the adoption of the possibility of cross-world identity: it must be possible for the same object to appear in domains of quantification of distinct worlds. To see this, note that under any reasonable semantics, the formula $\forall x \Box \exists y\ x = y$ will be a logical consequence of (b), and this formula can be true in w_1 only if all objects in $range(w_1)$ lie in the range of quantifiers of w_2, for any world w_2 that can be the semantic value of $\Box$. Namely, in order for the subformula $\exists y\ x = y$ to be satisfied in w_2 by the assignment $x := \mathbf{I}$ with $\mathbf{I} \in range(w_1)$, it must be possible to assign **I** as the value of y in w_2, but this is so only if **I** belongs to $range(w_2)$. All that is guaranteed by the assumption $(*)$ is that $\mathbf{I} \in subj(w_2)$, which is compatible with **I** lying outside $range(w_2)$. The problem is dissolved if we assume unrestricted quantification—i.e., suppose that the range of quantifiers is entirely independent of the context of evaluation. This means responding to Question 3 in the negative. Combined with the assumption $(*)$, this trivializes the distinction between ranges of quantifiers and sets of subjects of predicates. The two assumptions together yield the consequence that $subj(w) = range(w) = range(w') = subj(w')$ for all worlds w and w', thereby giving a sharpened partial answer to Question 4.

Nothing has been said of existence thus far. Considering formulas (a) and (b), what if the value of x is an individual **I** that 'exists' in world w_1, but w_2 is, say, a world temporally following **I**'s going out of existence? In connection with (a), this does not pose a particular problem: given that $\mathbf{I} \in subj(w_2)$, the assignment $x := \mathbf{I}$ satisfies the identity $x = x$ even posthumously. But, with (b), there appears to be a problem. The possibility of finding a value **J** of y in w_2 so that the identity $x = y$ is satisfied in w_2 would seem to be undermined by the fact that **I** lies, by the hypothesis, outside the set $ex(w_2)$ of objects existing in w_2. In reality it all depends of course on how the notion of 'existing' in a world w is articulated in relation to the notions of being in the range of quantifiers in w and being a possible subject of a predicate in w.

In particular it depends on whether 'existing' in w is a necessary condition for lying in the range of w.

The defender of the simplicity of identity could, for example, first deny any connection between the range of quantification $range(w_2)$ and the set $ex(w_2)$, insisting that an individual can indeed lie in the range of quantification of the world w_2, even if it does not exist in w_2. This viewpoint could be motivated by the fact that we seem to be capable of speaking in general terms of non-existent things (e.g., fictional objects and persons who no longer exist). Second, the simplicist could maintain that the set $subj(w_2)$ need not be included in $ex(w_2)$, so that while $\mathbf{I}$ does not exist in w_2, we can still ascribe predicates to it in this world. Again, natural language might seem to support this view. When saying that someone is deceased or that Pegasus is non-existent or that Pegasus is a mythical horse, we ascribe certain sorts of characteristics to non-existent things.[9] Proceeding in this way would, however, run the risk of making the range of quantifiers in a world w and the set of objects to which we may ascribe predicates in w totally independent of what exists in w. In that case, our language would lose the capacity to speak of what exists locally in a given world: the sets $ex(w)$ would become semantically superfluous. In order to allow our language to make any use of these sets, its expressive resources should be increased by something like an existence predicate.[10] Alternatively, it could be insisted that some, though not all, predicates are 'existence-entailing'—i.e., they can only be true of objects belonging to $ex(w)$.[11] However, it is questionable whether it is conceptually viable to divorce the notion of existence from the notion of quantification. A more reasonable move for the identity simplicist might be to propose defining the set $ex(w)$ in terms of the sets $range(w)$ and $subj(w)$, thereby readdressing Question 4.

As examples of how the four questions have been answered in the literature, we may consider the views of Saul Kripke [67] and Timothy Williamson [127], neither of whom takes quantification into modal contexts or cross-world identity to constitute a serious conceptual challenge. The two authors agree in their answers to the first two questions. To Question 1, the answer is 'yes'. The answer to Question 2 is that it is unproblematically possible to evaluate a formula like $P(x)$ in w_2 with the value of x having been fixed in w_1. This is because interpretations of predicates in a world are taken to be subsets of $\bigcup_{v \in W} range(v)$. Kripke's reply to Question 3 is 'yes', with Williamson's reply being 'no'. Finally, the two authors' replies to Question 4 are as follows:

- *Kripke:* If w is a world, then $ex(w) = range(w)$ and $subj(w) = \bigcup_{v \in W} range(v)$. The set $ex(w)$ is the 'domain' of w, and not all worlds have the same domain. Subjects of predicates in w are not restricted to elements of $ex(w)$ [67, pp. 65–7].

[9] In Crane's terminology, these are pleonastic properties [21, Sects. 3.4 and 5.5]; cf. Sects. 3.5 and 4.2 of the present book. In his view, there are true statements, such as 'Pegasus is a mythical horse', that ascribe a metaphysically neutral property to a non-existent object.

[10] Meinongians reason in terms of a stock of objects of which some exist while others are non-existent. Objects in the former class satisfy the existence predicate; those in the latter class do not. For a discussion on Meinongianism, see Sect. 3.8.

[11] For existence-entailing predicates, see Priest [95, Sect. 3.3], Crane [21, Sect. 3.3].

- *Williamson:* For all worlds w and w', we have $ex(w) = range(w) = subj(w) = subj(w') = range(w') = ex(w')$. Williamson defends *necessitism*: necessarily everything necessarily exists [127, pp. 2, 18]. The worlds have a common 'domain'. In any world, we can quantify over all elements of this domain (unrestricted quantification), and any object can function as a subject of a predicate anywhere.

Note that Kripke rejects what Williamson [127, p. 122] calls the *domain constraint*, according to which $subj(w)$ coincides with $ex(w)$, so that the interpretation of a unary predicate in w is a subset of $ex(w)$—and generally, interpretations of n-ary predicates are subsets of the n-th Cartesian power of $ex(w)$.

The idea that quantifiers have ontological commitments suggests that $range(w) = ex(w)$. Strictly speaking, it only suggests that $range(w) \subseteq ex(w)$, but unless we have a specific reason to think that *there are* things of which we cannot, even implicitly, speak in our language—i.e., things whose existence does not affect the truth of generalizations expressible in terms of our object-language quantifiers—the inclusion must be taken to hold also in the other direction. Both Kripke and Williamson assume that the sets $range(w)$ and $ex(w)$ coincide, with the difference being that in Kripke's approach, ontological commitments remain local, while Williamson's necessitism has the consequence that no world-dependent distinctions are made in terms of existence. Williamson's semantics validates both (a) and (b), while Kripke semantics validates (a) but refutes (b). The last-mentioned negative fact does not mean that Kripke would consider cross-world identity to constitue a conceptual problem. Under Kripke semantics, a formula such as $\exists x \Diamond \exists y\ x = y$ is not valid but neither is it contradictory or meaningless. There are Kripke models in which it is true, and in some such models, it is true because the ranges of quantifiers of distinct worlds overlap; one and the same object exists in two worlds.

All I wish to have conveyed by the considerations above is that there are conceptual decisions to be made before we are in a position to pronounce on the behavior of identity in many-world settings. What is of interest here is not that the identity simplicists may find answers to the four questions above suitable for serving their purposes. What is important is that those answers depend on premises that are not self-evident and that there is nothing intrinsically absurd or immediately self-defeating in providing diverging answers. The notion of cross-world identity would be utterly simple only if it did not rely on non-trivial premises. Since it indeed relies on non-trivial premises, this notion is not utterly simple and unproblematic.

1.3 The Proposal

In this book, I develop a framework in which cross-world statements are seen as systematically involving two types of components: worlds and links between certain world-bound objects. This view is inspired by Jaakko Hintikka's interpretation of quantified modal logic, and I follow him in referring to such links as *world lines* (see, e.g., [45, p. 385], [46, p. 209]). According to this view, quantifiers range over world lines. Each world has its associated set of 'local objects'. An individual may or may not be realized in a world. Realizations of individuals are local objects. Individuals themselves are links between local objects.[12] More generally, whenever we are given entity types X and Y, we must ask how to make sense of speaking of the *same* entity of type X relative to distinct entities of type Y. For example, we can ask what it means to say that the *same* individual exists in two worlds or at two times in a given structured world, that the *same* world incorporates two individuals, or that something happens at the *same* time in two worlds. In many cases of philosophical interest, when such talk is meaningful, we must understand—this is what I propose—entities of type X as *modal unities* over Y. In the case of worlds and individuals, both are modal unities that can be seen as sets of local objects. Worlds slice up a totality of local objects in one way, and individuals slice up the same totality in another way. Regarding local objects belonging to the same world, we can pose questions of numerical identity and distinctness in the extensional sense. Local objects lying in distinct worlds but linked by a suitable world line are realizations of the same individual: world lines enable talking of cross-world identity.

Some further words of clarification concerning the notion of local object may be in order. Local objects are world-bound, like individuals are, according to Lewis. But, local objects *are* not individuals. In fact, as was explained in Sect. 1.1, my notion of local object is a semantic notion—it is not a metaphysical notion to begin with. By contrast, Lewis's notion of individual is *metaphysical*. One must pay attention to the different theoretical roles of the two notions when wishing to compare them. What one can ask is whether local objects admit being metaphysically interpreted as individuals in Lewis's sense. Because both sorts of things are world-bound, such an interpretation is by itself possible. However, in my framework, it would be incorrect to classify local objects as individuals. What is relevant here is the *semantic* notion of individual. In first-order logic and in first-order modal logic, we call 'individuals' those things that can be values of first-order variables. In connection with the quantified modal language I am describing, values of quantified variables are world lines and, therefore, not local objects. The notion of world line is a semantic notion that admits several interpretations; different forms of such interpretations will be discussed in Sects. 1.5, 1.6, and 2.7. Whereas Lewis accounts for quantification into modal contexts in terms of counterparts of world-bound individuals, I analyze quantification into modal contexts in terms of world lines. Realizations of world lines are local objects; world lines themselves—that is, individuals in the semantic sense—are not.[13]

In this book, I show how world lines provide an analysis not only of physical objects but also of objects of thought (intentional objects). Intentional objects are taken to be world lines considered over the set of worlds compatible with an agent's intentional state. I will discuss the nature of world lines in Chap. 2. In Chaps. 3 and 4, I discern two types of world lines, to be referred to as *physically individuated* and *intentionally individuated* world lines. I use the distinction to account for the

[12] Hintikka fails to make a clear distinction between local objects and individuals; cf. Sect. 1.5 below.

[13] See Sect. 2.7.2 for a comparison between Lewis's view and my proposal.

differences and interconnections between physical and intentional objects and to clarify the semantics of a language capable of talking about physical and intentional objects. Syntactically, the difference between the two types of world lines will be reflected in the distinction between two types of quantifiers. *Physical quantifiers* range over physically individuated world lines, whereas *intentional quantifiers* range over intentionally individuated world lines. There will, then, be two types of values of quantified variables, referred to as 'physical objects' and 'intentional objects'. I will continue to use the term 'individual' schematically, so that it stands for both types of values of quantified variables. In my analysis, individuals are, then, always world lines, but there are world lines of two types to be considered. In the present chapter, I offer a general picture of the conceptualizations on which I will base the semantics of quantified modal logic.

Internally to each world, everything works semantically as one would expect on the basis of first-order logic. Every world w is associated with a set $dom(w)$, the *domain* of w. Elements of domains are termed *local objects*. Atomic predicates are interpreted locally. No cross-world predications are allowed. The interpretation of an n-ary predicate in w is a subset of the Cartesian product $dom(w)^n$. This means that subjects of atomic predicates are local objects. In modal settings—when we need to speak of values of quantified variables in relation to several worlds—individuals are understood as world lines. For any individual (world line), there is a partial function $\mathbf{I}$ that is undefined on those worlds in which the individual does not 'exist' and that assigns to each world w in which the individual 'exists' a local object $\mathbf{I}(w) \in dom(w)$, the realization or manifestation of the individual in w.[14] Such a function $\mathbf{I}$ is uniquely determined by the individual. World lines *are* not functions, but they determine partial functions in the way explained. For purely semantic purposes, the only relevant aspect of world lines is the fact that they exemplify a functional dependency between worlds and local objects. Such a functional dependency can be represented in terms of a function. For terminological simplicity and without risk of serious confusion, I will extend the use of the words 'individual' and 'world line' to these partial functions. By abuse of language, even such partial functions are 'world lines'; even they are 'individuals'.[15] Claims of cross-world identity rely on world lines. If w_1 and w_2 are distinct worlds, a belongs to $dom(w_1)$, and b belongs to $dom(w_2)$, then both claims '*a*

[14] Saying that a world line (an individual) 'exists' in a world means that it is realized therein. Quantificational locutions such as 'there is', again, will be understood in terms of world lines available as values of quantified variables. The important distinction between *availability* and *realization* is discussed in detail in Sect. 3.3.

[15] The described relation between world lines and the corresponding partial functions can be compared to the relation between variable embodiments and principles of variable embodiment in Kit Fine's metaphysics. (For a discussion, see Sect. 2.7.3.) Observe that the partial functions $\mathbf{I}$ as described above are *not* 'individual concepts', if individual concepts are taken to be functions whose values are individuals (possible values of quantified variables). Values of partial functions induced by world lines are local objects, not world lines—not individuals, not entities of the sort that function as values of quantified variables. For a discussion of what world lines and their corresponding partial functions are not, cf. Sect. 2.7.4.

is identical to b' and 'a is numerically distinct from b' must be judged meaningless.[16] What may but need not happen is that there is a world line $\mathbf{I}$ such that $a = \mathbf{I}(w_1)$ and $b = \mathbf{I}(w_2)$. No local object of one world can be transported to another world. Local objects are world-bound. It is not a part of the *form* of a local object, so to speak, that it could occur elsewhere from where it does. It cannot. By contrast, it is a part of the form of an individual that it can be realized in many worlds. I call the set of worlds in which an individual is realized its *modal margin*.

I take the distinction between local and cross-world identity seriously. According to the proposed framework, we cannot ignore world lines as a separate dimension in our semantic analysis. They are cross-world links irreducible to the world-internal properties of their world-bound realizations, not supervenient on world-internal facts. Even if one describes worlds in the minutest detail, one will not have even touched the question of which world lines are defined over those worlds. To have a useful image, consider Fig. 1.1, in which four worlds w_1, w_2, w_3, and w_4 are represented, each with its own domain. Also, two individuals are depicted, one represented by the unbroken line ($\mathbf{I}_1$) and the other by the dotted line ($\mathbf{I}_2$). The former is realized in all worlds, the latter in all worlds except w_4. Manifestly, such individuals are 'transcendent' in the sense that they are not to be found in the domains of worlds: plainly, neither the unbroken nor the dotted line itself lies in any of the domains.[17] By contrast, the lines cross several domains. The individual $\mathbf{I}_1$ cuts across each domain, whereas $\mathbf{I}_2$ crosses all domains except that of w_4. The only way an individual can get manifested is locally, by being realized. For example, $\mathbf{I}_1(w_1) = \diamond$ and $\mathbf{I}_1(w_2) = \star$ (that is, $\mathbf{I}_1$ is realized in w_1 as the object $\diamond$ and in w_2 as the object $\star$), whereas $\mathbf{I}_2(w_1) = \bullet$ and $\mathbf{I}_2(w_2) = \circ$. This does not mean that individuals are reduced to their manifestations. It hardly needs to be stressed that with no cross-world information at one's disposal, one cannot reconstruct the individuals represented in Fig. 1.1 from the ingredients portrayed in Fig. 1.2.

Given the information provided by Fig. 1.2, we can reflect on different conceivable correlations between elements of the four domains. Combinatorially, there are

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[16]For this way of understanding cross-world identity, see Tulenheimo [118, p. 384]. Note that denying the meaningfulness of asking whether local objects of distinct worlds are extensionally identical in no way compromises the notion of partial function whose arguments are worlds and whose values are world-bound local objects. A partial function f is well defined as soon as there is a set A and a family $\{B_i : i \in A\}$ of sets indexed by the elements of A such that set-theoretically speaking, f is a set of pairs $\langle a, b\rangle$, where $a \in A$ and $b \in B_a$. What counts is that for every $a \in A$, it is clear whether f is defined on a, and if indeed f is defined on a, it must be clear which element of the set B_a is being associated with a. In order for there to exist such a function, it is absolutely irrelevant whether comparisons in terms of identity and numerical distinctness can be made between elements of sets B_i and B_j with $i \neq j$. In typical mathematical cases, such comparisons will be possible, but this is by no means essential for the definition of the notion of (partial) function.

[17]This is a general analysis of individuals, meant to apply however worlds are interpreted from a substantial viewpoint. Thus, there are no grounds for saying categorically that these individuals are transcendent in the sense 'passing beyond all experience'. If worlds are entire universes, then yes; if they are spatial perspectives or instants within a structured world, then no. But, under all interpretations, they *are* transcendent in the sense of not residing in a single world, when 'world' is understood in the abstract semantic sense explained in Sect. 1.1.

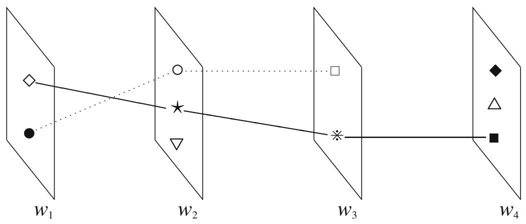

Fig. 1.1 Four worlds, two individuals

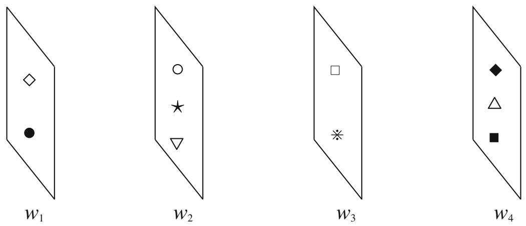

Fig. 1.2 Four worlds, no individuals

$3 \cdot 4 \cdot 3 \cdot 4 - 1 = 143$ non-empty partial functions of the relevant type. Whatever grounds we could have for constructing such correlations, constructing them is one thing, and having them as fixed components of a structure is quite another. Figure 1.2 represents no individuals in my sense, while Fig. 1.1 represents exactly two individuals. Some metaphysicians—for example, David Lewis—might still claim that world lines *supervene* on local properties of worlds. Lewis takes counterpart relations among inhabitants of distinct worlds to supervene on characteristics internal to those worlds.[18] This would mean in our example that the four worlds of Fig. 1.2 determine exactly one selection out of the 143 possible correlations between their local objects—while there are 2^{143} combinatorially possible selections of the relevant type if no specific conditions on the interrelations of such correlations are imposed. *Logically*, world lines are independent of worlds; it would be a remarkable metaphysical coincidence if in practice, however, worlds would always *determine* the world lines.[19]

[18]The supervenience of counterpart relations on local properties of worlds is a part of Lewis's thesis of Humean supervenience. Namely, this thesis leads to the adoption of what Lewis calls *anti-haecceitism*, according to which facts about any given world supervene on the distribution of qualitative properties and relations within worlds. Modal facts about Lewis's world-bound individuals (representations *de re*) are articulated in terms of counterpart relations. These counterpart relations must, then, supervene on local features of the worlds. Otherwise modal facts would be independent of the distribution of qualitative properties and relations within worlds, contrary to anti-haecceitism. For a discussion, see Sect. 2.7.2; cf. [76, Sect. 4.4], [124]. It should be noted that one can defend 'haecceitism' without postulating 'haecceities' and that world lines are not haecceities, cf. footnotes 28 and 29 in Sect. 2.7.2.

[19]Lewis admits that at best, the principle of Humean supervenience holds contingently; see [77, p. x], [78, pp. 474–5]. The supposed contingent truth of Humean supervenience in 'worlds like ours' would not render the mentioned coincidence much less remarkable. It would mean that whenever our actual world is among the worlds considered, these worlds determine a corresponding set of world lines.

What should one say about the relative conceptual priority between local objects and world lines? Once both types of entities have been distinguished, one must say that local objects have conceptual priority: unless there were local objects, there could be no realizations of world lines and therefore no world lines, either, while the mere fact that domains of worlds consist of world-bound local objects by no means entails that some local objects in distinct worlds are linked together by a world line. This said, it is possible to view world lines as being *epistemically* prior to local objects in the sense that the physical objects we experience and learn to reason about appear to us as being temporally extended and as having modal properties—that is, they appear to us as world lines. We can think of them without expressly thinking of them as links between local objects. Epistemically speaking, we arrive at the notion of local object through analysis. Such an analysis can, besides, be carried out in different ways, depending on what sorts of scenarios are considered as ‘worlds’—e.g., we may construe worlds as temporally unanalyzed logically alternative states or as temporal phases of temporally structured scenarios.[20] Similar remarks apply to intentional objects understood as world lines: psychologically, our objects of thought certainly do not present themselves to us as correlations between local objects of distinct worlds. Nevertheless, it will be argued that objects of thought can be analyzed as being such correlations.

My answers to the four questions posed in Sect. 1.2 are as follows:

1. No. Subjects of predicates are local objects, whereas values of variables are world lines with local objects as their realizations. World lines are second-order entities when compared with local objects.
2. If the world line $\mathbf{I}$ has been assigned as the value of x in w_1, then the evaluation of an atomic formula, say $Q(x)$, in a distinct world w_2 depends on whether $\mathbf{I}$ is realized in w_2. If it is not, the assignment does not satisfy $Q(x)$ in w_2. If it is, the assignment satisfies $Q(x)$ in w_2 iff the realization $\mathbf{I}(w_2)$ of $\mathbf{I}$ in w_2 belongs to the interpretation of Q in w_2.
3. Generally, yes. For each world w, $range(w)$ is a (possibly empty) set of world lines, and it may but need not happen that for distinct worlds w_1 and w_2, the sets $range(w_1)$ and $range(w_2)$ are distinct.
4. For all worlds w, all and only local objects of w are subjects of predicates in w. That is, $subj(w) = dom(w)$. If $\mathbf{I}$ is an element of $range(w)$, it may but need not have a realization belonging to $subj(w)$. In any world w' in which $\mathbf{I}$ is realized, its realization belongs to $subj(w')$. Saying that a world line exists in w is taken to *mean* that it is realized in w. That is, $\mathbf{I}$ belongs to $ex(w)$ iff $\mathbf{I}$ has realization belonging to $subj(w)$. For this to happen, it is neither sufficient nor necessary that $\mathbf{I}$ belongs to $range(w)$. That is, neither of the sets $range(w)$ and $ex(w)$ need be included in the other: generally, there may be a world line in $range(w)$ not having a realization belonging to $subj(w)$, and there may be a word line having

[20] When discussing objects of experience, Carnap [11, Sect. 128] takes visual things to be conceptually prior with respect to states of these things but notes that ‘it might be more appropriate to construct first the states-of-things’ and only afterwards the things as classes of genidentical states-of-things. In my exposition, I opt for this latter type of procedure.

a realization in $subj(w)$ not belonging to $range(w)$. This means that a world line may be available to be quantified over in w without being realized in w and realized in w without being a possible value of a quantified variable in w.

These answers require explanatory comments. First, I take it that we can ascribe atomic predicates to values of variables in a world by ascribing atomic predicates to their *realizations* in that world. Furthermore, I take this to be the only way in which we can ascribe atomic predicates to values of variables in terms of the language I am discussing. This means that the satisfaction of *atomic formulas* is a local matter. Whether $P(x)$ is satisfied in a world w under an assignment $x := \mathbf{I}$ depends only on the realization, if any, of the world line $\mathbf{I}$ in w. If both $\mathbf{I}$ and $\mathbf{J}$ are world lines realized in w and they meet the condition $\mathbf{I}(w) = \mathbf{J}(w)$, then $x := \mathbf{I}$ satisfies $P(x)$ in w iff $x := \mathbf{J}$ satisfies $P(x)$ in w. The way for a world line to have, in a world w, a property describable by an atomic predicate is for its realization to have that property in w. The relevant properties are properties that a value of a variable has in a world, depending on how it is in that world. In temporal settings, such characteristics are termed *pro tem* properties; in general modal settings, we may refer to them as *pro mundo* properties or, simply, local properties. While the semantics of atomic formulas is local, the semantics of arbitrary *complex formulas* is not. Generally, the question of whether a formula is satisfied in a world w by an assignment of world lines to its free variables depends on those world lines, not merely on their realizations (if any) in w. I do not wish to suggest that unanalyzed and therefore atomic predicates for some reason *must* ascribe local properties. However, for my purposes in this book, it suffices to confine attention to atomic predicates whose semantics can be formulated in terms of realizations of world lines. (For a discussion on this restriction, see Sect. 2.4.)

Second, it was mentioned in Sect. 1.2 that Kripke rejects the domain constraint—i.e., he does not hold that interpretations of n-ary predicates are subsets of the set $ex(w)^n$. My requirement, according to which interpretations of n-ary predicates are subsets of the set $dom(w)^n$, is in spirit similar to the domain constraint. However, my 'domain constraint' differs in an important respect from Kripke's domain constraint. In Kripke's analysis, predicates are ascribed to individuals. In my analysis, they are *not* ascribed to individuals but to entities categorically different in kind from individuals: to local objects. Local objects are not world lines—they are not possible values of bound variables: they are not individuals. If we suppose that every local object is the realization of an individual, then my requirement means that atomic predicates can be ascribed only to *realizations* of individuals.[21] Yet, they are not directly ascribable to individuals. Third, the sets $ex(w)$ and $subj(w)$ are disjoint—for trivial reasons. All elements of $ex(w)$ are world lines, whereas no element of $subj(w)$ is one. Fourth, let us say that world lines belonging to $range(w)$ are *available* in w. In accordance with what was noted in response to Question 4 above, there need not be any particular connection between world lines available in w and those realized in w—i.e., between the sets $range(w)$ and $ex(w)$. I will discuss this phenomenon systematically in Sect. 3.3. As mentioned earlier in the present section,

[21] In this book, I will indeed make this assumption. See Sect. 3.4 and cf. Sect. 4.6.

I will distinguish between two types of world lines, to be referred to as *physically individuated* world lines and *intentionally individuated* world lines. For reasons to be spelled out subsequently, I take physically individuated world lines to be available in a world iff they are realized in it, whereas in connection with intentionally individuated world lines, the two conditions are independent of each other.

Fifth, my affirmative answer to Question 3 means that I allow varying domains of quantification: the set of world lines that are possible values of quantifiers in one world can be distinct from the set of world lines that are possible values of quantifiers in another world. It should be noted that this is *not* a consequence of taking local objects to be world-bound. No single local object would be an inhabitant of two worlds even if we assumed that the domain of quantification is the same for all worlds. Adopting a constant domain approach would mean that the same *world lines* are available to be quantified over in all worlds. If w and w' are distinct worlds, a world line $\mathbf{I}$ might belong to this constant domain of quantification and yet fail to be realized in the worlds w and w'. (Generally, availability does not entail realization.) Further, the world line $\mathbf{I}$ could be realized in w, without thereby being realized in w', as well. Finally, if $\mathbf{I}$ was indeed realized in both worlds w and w', the world-bound local objects $\mathbf{I}(w)$ and $\mathbf{I}(w')$ could not possibly be identical. Actually, we cannot even meaningfully pose the question of whether they are or are not identical, since a presupposition for posing such a question concerning local objects is that they lie in the domain of the same world.

Sixth, in connection with usual formulations of the semantics of first-order modal logic, the varying domains approach is sometimes qualified as *actualist*, as opposed to the constant domain approach, which is qualified as *possibilist*—the idea being that a world-relative domain consists of things that 'exist' in the world in question, while a constant domain consists of things that 'exist' in one possible world or another. (Cf, e.g., [27, p. 94], [32].) As already mentioned, I will distinguish between two types of quantifiers according to whether their values are physically or intentionally individuated world lines: physical quantifiers and intentional quantifiers. Since availability and realization are mutually independent features of world lines, my approach is, generally, neither 'actualist' nor 'possibilist'. I take actualist quantifiers to be quantifiers whose values cannot be available in a world w without being realized therein. Physical quantifiers will be actualist, since their values are physically individuated world lines, which, as indicated above, are taken to be realized in any world in which they are available. Intentional quantifiers are *not* actualist. We can quantify over non-existent intentional objects: values of intentional quantifiers are intentionally individuated world lines, whose availability in a world precisely does not guarantee their being realized in it. Yet, intentional quantifiers are *not* for this reason possibilist. Among the values of an intentional quantifier in a world w, there need not be any physically individuated world lines, and if w' is a world distinct from w, it could happen that some or all of the intentionally individuated world lines available in w' fail to be available in w.

My semantic framework incorporates as separate components worlds and world lines. The world line analysis is motivated by the problem of what it means to speak of one and the same object in modal settings. It will allow us to account for the difference in behavior between physical and intentional objects and leads to a novel way of semantically modeling contents of thought in connection with sentences ascribing intentional states to agents. The goal of this book is to argue for this framework and to illustrate its theoretical consequences.

1.4 The Semantic Role of Cross-World Links

Since the notion of world line used for formulating the semantics of quantified modal logic stems from Hintikka, it is appropriate to critically discuss the role he has given to this notion and the reasons why he has introduced it. I discern two mutually incompatible ways in which the idea of a world line can be understood. They both are—so I claim—operative in Hintikka's writings, thereby rendering the motivational basis of his view globally incoherent. I dub the two ways of construing world lines the 'transcendental interpretation' and the 'epistemic interpretation'. The qualifier 'transcendental' is meant to convey the idea that it is a meaningfulness condition (a transcendental precondition) for our speaking and thinking about individuals in many-world settings that individuals are understood as world lines. The epistemic interpretation, again, construes world lines as codifications of methods of recognizing or reidentifying individuals in different situations. I take up these two interpretations in Sects. 1.5 and 1.6, respectively. I comment on the associated notion of transcendental precondition in more detail in Sect. 2.2.

Most philosophers and logicians discussing the conceptual problems of modalities have ignored the way in which Hintikka deals with modal individuals. Those who explicitly see themselves as being inspired by Hintikka have understood his proposal in accordance with the epistemic interpretation. Examples are Kraut [64, 65], Niiniluoto [92], Saarinen [107], and Kirjavainen [62]. In newer literature, world lines have been taken up in technical settings by Schurz [109], as well as Kracht and Kutz [66]. Priest [95] and Belnap and Müller [4] discuss closely related ideas. I comment on Priest's functional notion of object in Sect. 3.8. Belnap and Müller develop a framework initially formulated by Bressan [6]. Hintikka concentrates on the philosophical uses of his notion of world line but says very little about the intended formal setup of his proposal. This may explain why in fact he is not clear about the distinction between objects of domains (local objects) and world lines (values of variables) and why he interprets world lines occasionally as constitutive of modal individuals and at other times as means of recognizing an already available individual. Bressan, again, develops a detailed formal framework motivated by questions in the foundations of physics. He does not address the issues in the philosophy of language, formal semantics, and epistemology that function as a driving force for Hintikka's work.

In two papers published in 1969, 'On the Logic of Perception' [42] and 'Semantics for Propositional Attitudes' [43], Hintikka put forward an analysis of what it means to speak of one and the same individual across distinct scenarios. He never abandoned this analysis. On the contrary, he applied it throughout his subsequent work. However, as I will argue, Hintikka has not provided a coherent account of how his proposal should be construed. More precisely, his own understanding of the proposal has oscillated between two mutually incompatible interpretations: the transcendental and the epistemic interpretation. This has made it more difficult for everyone—Hintikka himself included—to clearly see what is being proposed and in what way the proposition is novel.

Hintikka's starting point was the analysis of propositional attitudes based on a distinction between scenarios compatible with an attitude and those incompatible with it [41]. He suggested that this analysis can be extended to perceptual experience [42].[22] The idea of compatibility can be phrased in terms of what are in our days standardly called accessibility relations (alternativeness relations): y is compatible with what the agent believes in x iff for all she believes in x, she is in y (*doxastic accessibility*); y is compatible with the agent's perceptual experience in x iff for all she can tell on the basis of her perceptual experience in x, she is in y (*perceptual accessibility*). Thus, in w_0, Alice believes that it is raining iff it is raining in all worlds compatible with Alice's overall belief state in w_0 iff it is raining in all worlds doxastically accessible from w_0. Likewise, in w_0, Bob perceptually experiences that Alice wears a red dress (it appears to Bob that Alice wears a red dress) iff Alice wears a red dress in all worlds compatible with Bob's perceptual experience in w_0 iff Alice wears a red dress in all worlds perceptually accessible from w_0. The reason why possible worlds or alternative scenarios are needed in the analysis of such notions is that one's perceptual experience or one's beliefs do not fix the world uniquely, but they leave open various alternatives as to how the world could be. Besides, even if the beliefs or the perceptual experience of an agent did fix a unique world, these states easily lead beyond the actual world w_0, which need not be compatible with what is believed or perceptually experienced.[23]

In 'Semantics for Propositional Attitudes', Hintikka first describes a framework that is sufficient for semantically dealing with such constructions as 'α believes p'. The formulation of the semantics of such expressions utilizes a set W of worlds and an alternativeness relation R defined on this set. For a given world w, the relation triggers a partition of W: the worlds in the set $\{v : R(w, v)\}$ are compatible with the totality of α's beliefs in w, whereas those in its complement with respect to W are not. Hintikka stresses that such a framework does not by itself allow speaking of objects in two worlds as identical. To that end, he says, we must be given ways of 'cross-identifying' individuals—ways of understanding whether 'an individual fig-

[22]He speaks of 'perception' instead of 'perceptual experience' but says explicitly that he uses the word without presupposing factiveness [44, p. 153].

[23]Here and henceforth, when speaking of the *actual world*—denoted by 'w_0'—I mean the scenario that happens to be the one in which the epistemic agent or language-user considered is situated. I do *not* wish to suggest that among all worlds, one specific world has, once and for all, been chosen as a distinguished world. In my usage, 'the actual world' is the current circumstance of evaluation or the situation in which an agent presently finds herself. It is simply convenient to agree that the symbol 'w_0' and the expression 'the actual world' stand for the relevant contextually determined scenario. Consequently, w_0 may but need not be the possible world or the spatiotemporally specific location in which the reader of these lines is situated. In linguistic settings, 'the actual world' is in my usage synonymous with 'the (initial) circumstance of evaluation', a phrase whose denotation evidently varies from case to case.

uring in one possible world is or is not identical with an individual figuring in another possible world' [44, p. 99]. Hintikka insists that such methods of cross-identification constitute a separate component in our semantic theory—something that must be added to a specification of a set of possible worlds and an alternativeness relation in order to make it generally possible to evaluate sentences in which expressions for propositional attitudes appear in the scope of quantifiers. Typically, sentences of this kind force us to consider an individual relative to several worlds. As Hintikka puts it in his intellectual autobiography, these 'identity criteria' do not reduce to descriptive requirements or to any other conditions that could serve to determine referential relations between linguistic expressions and elements of domains of worlds [49, pp. 27–8]. For the sake of exposition, Hintikka [44, p. 92] assimilates possible worlds to models used for evaluating formulas of first-order logic and says that the role of these methods of cross-identification is to relate to each other members of different domains of individuals [ibid. p. 100].[24]

Hintikka sketches a particular way in which cross-identification methods can be introduced into the formal semantics: by postulating 'a set of functions $\mathbf{F}$ each member f of which picks out at most one individual $f(\mu)$ from the domain of individuals $\mathbf{I}(\mu)$ of each given model μ' [ibid.].[25] He explicitly notes [p. 102] that the main role of the part of our semantic apparatus constituted by methods of cross-identification is to make sense of quantification into attitudinal contexts. Quantifiers range over the set $\mathbf{F}$. Hintikka remarks [p. 103] that a general formulation of the semantics requires that the relevant set of functions be allowed to vary with the agent and the world considered. He motivates his comment by saying that not everyone needs to be in all situations familiar with all the relevant methods of cross-identification. As will become clear when we proceed, Hintikka has a strong tendency to think of these cross-world links in *epistemic* terms—as a matter of an agent's capacity to reidentify one and the same individual over a number of situations.

[24]Assimilating worlds to first-order models does not mean ignoring the *relevant* distinction between worlds and models that Williamson [127, pp. 81, 83] hails as Kripke's decisive innovation [67]. Carnap [12] employed in his semantics state descriptions, which played simultaneously the role of worlds and first-order models. The totality of state descriptions represented all combinatorially possible ways of interpreting non-logical predicates of the language and constituted the one and only model of modal logic (modal structure). If elements of domains of modal structures are generically called 'contexts', all that matters is that we use modal structures in which each context is associated with a set of accessible contexts and that no specific way of selecting the associated sets is given a privileged status. We are free to think of contexts as first-order models if we so wish. This said, possible worlds must certainly not be strictly speaking identified with first-order models. As Hintikka [45] stresses, worlds must be seen as being structured by properties and relations, differently instantiated in different worlds, and interpretations of non-logical predicates must match these instantiations instead of being chosen arbitrarily; cf. footnote 5 in Sect. 3.2.

[25]For the notation $\mathbf{I}(\mu_i)$, cf. [44, p. 92].

1.5 Transcendental Interpretation of World Lines

There appears to be a trivial knockdown argument against Hintikka's claim that a set $\mathbf{F}$ of correlations between members of different domains must be separately given. Staying with Hintikka's notation, the objection would go as follows. Suppose $\mathbf{I}(\mu_1)$ and $\mathbf{I}(\mu_2)$ are sets of individuals existing, respectively, in the worlds μ_1 and μ_2. If $a \in \mathbf{I}(\mu_1)$, then for every $b \in \mathbf{I}(\mu_2)$, we either have $a = b$ or else $a \neq b$. For any pair of objects a and b, it is uniquely determined which of the two conditions holds. This fact in no way turns on any separately specified links between the domains $\mathbf{I}(\mu_1)$ and $\mathbf{I}(\mu_2)$. As I see it, this argument is indeed devastating to Hintikka's proposal, unless the argument rests on a presupposition that itself is unacceptable. Namely, if the argument is sound, there is built into the notion of 'method of cross-world identification' a criterion of correctness, and such methods of identification are at best methods for finding out matters of fact rather than conceptual building-blocks of the very notion of individual. If functions in the set $\mathbf{F}$ are meant to be codifications of methods of this kind, a cross-world identification method f is correct iff f is a constant function. If there is a function $f \in \mathbf{F}$ and worlds μ_1 and μ_2 such that $f(\mu_1) \neq f(\mu_2)$, the method of cross-world identification encoded in the function f is simply mistaken: it posits as identical objects that are not identical.

Such objectively mistaken correlations might be of some use when modeling epistemic agents' erroneous opinions as to how a given individual (perhaps the value of f in the actual world) appears in different circumstances. However, such functions hardly deserve to be considered as individuals—possible values of quantified variables. What is more, constant functions can scarcely be considered as *methods* of cross-identification: one would expect there to be several such methods of identification for any given individual, but taking correct methods to be constant functions does not admit sufficient variation. Finally, if objects $a \in \mathbf{I}(\mu_1)$ and $b \in \mathbf{I}(\mu_2)$ could be directly compared in terms of identity (i.e., if we always had either $a = b$ or $a \neq b$), it would seem strange and pointless to say, as Hintikka nevertheless does [44, p. 101], that the question of whether a is identical with b amounts to the question of whether there is a function $f \in \mathbf{F}$ such that $f(\mu_1) = a$ and $f(\mu_2) = b$.

The reasoning above can be blocked by denying its premise, according to which we may apply standard set-theoretical reasoning to the sets $\mathbf{I}(\mu_1)$ and $\mathbf{I}(\mu_2)$, so that for any $a \in \mathbf{I}(\mu_1)$ and $b \in \mathbf{I}(\mu_2)$, it is meaningful to ask whether $a = b$. As I pointed out in Sect. 1.3, such questions can be judged meaningless. Standard set-theoretic reasoning is applicable in extensional settings. According to the position developed in this book, distinct worlds should be thought of as being so fundamentally separated that insofar as the expressive power of our object language is concerned, we can only speak about *world-internal* applications of set-theoretic operations (e.g., intersection and difference). Asking whether $a = b$ for objects a and b in domains of distinct worlds is taken to be as devoid of sense as it would be to ask whether the Eiffel Tower is wise or whether a multiplication table is evil: the relevant categorial presupposition is not met.[26]

If indeed we cannot even meaningfully ask whether $a = b$ when a and b are objects in the domains of two worlds, then separately given correlations between elements of different domains are indispensable for speaking of cross-world identity. In that case, it really makes sense to say that $a \in \mathbf{I}(\mu_1)$ being identical to $b \in \mathbf{I}(\mu_2)$ amounts to the existence of a function $f \in \mathbf{F}$ such that $a = f(\mu_1)$ and $b = f(\mu_2)$. Namely, in that case, speaking of cross-world identity *means* speaking of world lines, and in semantics, we use functions to model world lines. As noted in Sect. 1.3, the only semantically relevant aspect of world lines is that they induce a relation of functional dependency between worlds and local objects, which is why in semantics we may take the induced partial functions, rather than the world lines themselves, to be values of variables. It becomes a precondition of cross-world talk that our semantics can make use of such functions. Construing the idea of cross-world links in this way leads to what I propose to call the transcendental interpretation of Hintikka's proposal. World lines are what renders cross-world talk of individuals possible.

The transcendental interpretation suggests itself when Hintikka speaks of his view as giving rise to semantic neo-Kantianism and maintains that '[e]ven such *prima facie* transparently simple notions as that of an individual turn out to depend on conceptual assumptions [concerning] different possible states of affairs' [44, p. 109]. It is this view he has in mind when explaining that he has looked for 'the tacit transcendental preconditions of. . .successful identification' [49, p. 556]. Similarly, it is in this way Hintikka construes his proposal when he writes in his essay 'The Intentions of Intentionality' from 1975:

> To the alleged primacy of individuals [with respect to possible worlds] we can contrast a view which admits that each possible world comes to us already analyzed categorially into individuals, their properties, their relations, etc. However, according to this view the identity of such entities, especially the identity of individuals from one world to another is not fixed by any absolute logical principles but is at least partly constituted by our comparisons between the two different possible worlds whose denizens the two respective individuals are [46, p. 209].

He proposes to use the suggestive terminology according to which the 'manifestations (roles, embodiments) of the same individual in different worlds are tied together by a line, the *world line* of this individual' and adds:

> the world lines…are not fixed by immutable laws of logic or God or some other equally transcendent power, but…they are as it were drawn by ourselves—of course not by each individual alone but by tacit collective decision embodied in the grammar and semantics of our language [ibid.].

[26] This view is easily obscured by mathematical models employed in semantic theorizing. Typically, the domains of these models are sets of mathematical objects, such as numbers. If A and B are sets of numbers, we are normally justified in forming the intersection $A \cap B$ and asking whether $A \cap B$ is empty. Likewise, if $a \in A$ and $b \in B$, we are normally justified in asking whether a equals b. That is, our means of formal representation are in this case, in this respect, misleading.

None of these comments makes sense if cross-world identity is taken to be an unproblematic, utterly simple notion and if it is considered to be meaningful to speak of one and the same individual as being literally a member of two domains.

As appears from the quotes above, the particular setting in which Hintikka defends the transcendental interpretation of world lines has a Kantian tone. With reference to the role of possible worlds and world lines in our conceptualizations, Hintikka says that '[a]s far as our thinking is concerned, reality cannot be in principle wholly disentangled from our concepts' and that anything we say concerning the reality is 'permeated throughout with concepts of our own making' [44, p. 109]. When construed in accordance with the transcendental interpretation, I take the core of Hintikka's view to be the thesis that thinking about individuals in modal settings is to think about world lines: it is a precondition for any modal cognition pertaining to individuals that we think of them as world lines. World lines are what must be given in order for us to be in a position to talk about individuals in modal settings. This understanding leads naturally to the conception of world lines as one of the determinants of our intentional states.

Hintikka's comments lead him further than Kant along Kantian lines. Starting with the view according to which we cannot step outside our conceptualizing activities and compare world lines to a reality devoid of such conceptualizations, he ends up saying that while world lines 'may be as solidly objective as houses or books', they are 'as certainly as these created by men. . .for the purpose of facilitating their transactions with the reality they have to face' [ibid. pp. 108–9]. Now, referring to arbitrary particulars as man-made without qualification is both an exaggeration and a deviation from the Kantian viewpoint. In a Kantian vein, it could indeed be claimed that it is a part of the *form* of our thinking about modal phenomena that our modal thoughts are structured in terms of worlds lines, in analogy with Kant's view, which takes appearances to have a form and to be structured according to the 'pure intuitions' of space and time—forms of our awareness of individuals. As Kant sees it, if the appearances were not structured, the subject could not have conscious experience with a specific content. On the other hand, the form of appearances does not arise out of sense experience. Therefore, we can with due caution say that the form of appearances is 'made-made', but this does not mean that in Kant's view the appearances themselves are man-made *tout court*. The appearances must have a matter and not only a form. The matter does not depend on the subject and is necessarily given a posteriori (see, e.g., Gardner [32, pp. 47–8]). Consequently, in a correctly formulated analogy, single world lines are not 'made-made'—if anything is, it is the 'formal' fact that we can only talk of individuals in many-world settings with reference to a structure of world lines.[27] The concept of cross-world identity

[27] That is, it is possible to formulate a well-founded analogy between Hintikka and Kant—although Kant's view has a somewhat more explicit epistemic stress than Hintikka's view has, according to its transcendental interpretation. For Kant, intuitions are awarenesses of individuals. For Hintikka, world lines are individuals. For Kant, intuitions occur in experience, which is always structured according to the forms of sensibility (spatiality, temporality). For Hintikka, world lines occur in many-world settings, which are always structured according to a system of cross-identification. The analogy can be further deepened: I show in Sect. 4.7 that intentional states (including perceptual

imposes by its nature a structural demand that must be fulfilled in one way or another so as to enable our modal cognition. The particular way in which it is fulfilled is not up to us. We do not choose *which* world lines there are.

1.6 Epistemic Interpretation of World Lines

I have already suggested that Hintikka's idea of individuals as world lines has, in his own thinking, mutually incompatible motivational sources. As illustrated in the previous section, at times, he considers there being world lines as a precondition of any modal talk of individuals—that is, he subscribes to a variant of the transcendental interpretation of world lines. On other occasions, Hintikka opts for what I call the epistemic interpretation: he views world lines as means of cross-identification, as some kinds of recipes that allow us to recognize a given object in different situations. Actually, whenever Hintikka hints at the former interpretation, he quickly switches to the latter. For example, immediately after having carefully distinguished world lines from elements of domains [44, pp. 99–100], Hintikka gives up the idea of world lines as correlations between elements of distinct domains, says that domains of distinct worlds may overlap—which presupposes that identity of elements of distinct domains is a notion understood independently of the idea of a world line—and reduces worlds lines to means of recognizing one and the same individual in different circumstances and in different courses of events [ibid. p. 101].

The two ways of construing world lines are incompatible. According to the transcendental interpretation, individuals *are* world lines. The proper formulation of the epistemic interpretation must start from conceding that we may unproblematically speak of individuals as appearing in different scenarios, the role of world lines being to allow us to recognize that an individual *here* is the same as an individual *there*. In any event, this way of formulating the epistemic interpretation is unavoidable if the formulation is to be compatible with Hintikka's overall philosophical views. This excludes an *anti-realist* reading, which would problematize the applicability of the very notion of cross-world identity, maintaining that we must possess certain epistemic capacities in order to be in a position to make use of this notion. I will comment on this interpretive option below. For now, let us note that if the transcendental interpretation is correct, world lines cannot be means of recognition, for according to this view, there is no individual to be recognized independently of a world line. This is simply because by hypothesis, individuals *are* world lines. Only when a world line is fixed is there an individual; afterwards, we may pose the epistemic question of how to recognize that such-and-such objects in different scenarios actually manifest this particular individual, but no matter what sorts of mechanisms there exist for

(Footnote 27 continued)
experience) can be analyzed as structures of worlds and intentionally individuated world lines. What is more, the latter can in suitable circumstances be 'awarenesses' (representations) of physically individuated world lines; see Sect. 4.8.

handling this task, they cannot be taken for world lines. Conversely, if the epistemic interpretation is accepted, the conceptual problem of cross-world identity is traded for the epistemic problem of recognition.

A more specific illustration of the fact that Hintikka tends to adopt the epistemic interpretation is provided by his discussion of the nature of our means of cross-world identification. Speaking of 'methods of individuation' associated with world lines, he affirms that these are based on 'such facts as bodily continuity [and] continuity of memory' [44, p. 170] and says that we can in principle try to find out by means of spatiotemporal continuity whether a physical object i_1 in a world w_1 and a physical object i_2 in another world w_2 are the same individual—we might be able to 'follow each of them in space and time in its respective world toward the common ground' that the two worlds supposedly share [50, p. 141]. Evidently, if world lines allow us to follow an individual in space and time, the thing that is being followed cannot be constituted by a world line, world lines being merely codifications of ways in which we can try to discover an individual.

As I hinted in passing, there is no intrinsic impossibility in interpreting world lines in the context of semantic anti-realism. Such an interpretation—which itself has an epistemic flavor—would consist of maintaining that we can only meaningfully say that an object of one scenario is the same as an object of another scenario if we are in a position to recognize, or have epistemic grounds for judging, that they are the same. This would be analogous to (and eventually constitutive of) the way in which an anti-realist like Dummett construes the notion of truth. By Dummett's standards, we can only ascribe truth to a statement if we possess means of recognizing it as true; truth-attributions are devoid of sense in the absence of such means (see, e.g., [23]). The anti-realist interpretation is not what Hintikka has in mind: he is positively hostile to the anti-realist theory of meaning. He takes the anti-realists to fallaciously infer from the fact that language-use is rule-governed to the conclusion that the applicability of semantic concepts depends on human agents [48, p. 18].

The transcendental interpretation leads to emphasizing the contribution of our cognitive faculties to the objects we talk and think about. When Hintikka switches to the epistemic interpretation, instead of maintaining that what is contributed is merely the way in which our modal thoughts must be structured (namely, in terms of world lines), he reasons as if our concrete means of knowledge-acquisition could play the role of such formal constraints. Taking the availability of world lines to be a transcendental precondition of our modal thoughts is naturally interpreted along the lines of transcendental idealism. If so, even this proposal has a certain epistemic flavor (since it concerns necessary conditions of our thinking), but these sorts of epistemic constraints must not be confused with the requirement of being able to apply knowledge-seeking mechanisms of scientific or everyday inquiry. It is this sort of confusion that appears as the best explanation of the two tendencies in Hintikka's comments on world lines. As a result of another type of confusion, necessary conditions of thought may come to be interpreted as requirements that are imposed on the epistemic capacities of language-users and without which the use of certain concepts is not warranted, leading to an anti-realist interpretation.

Chapter 2
The Nature of Modal Individuals

2.1 Introduction

In this chapter, I discuss, first, the nature of the proposal, according to which it is a 'transcendental precondition' of the way in which we speak and think about individuals in modal settings that they are categorized as world lines (Sect. 2.2). I then proceed to formulate a formal semantics of a quantified modal language in which quantifiers range over world lines (Sects. 2.3 and 2.4). I discern a general notion of content and show that both worlds and worlds lines can be seen as *modal unities* (Sects. 2.5 and 2.6). Contents are structures of interrelated modal unities. I close the chapter by clarifying how my world line framework is related to competing semantic and metaphysical views, notably those developed by Saul Kripke, David Lewis, and Kit Fine (Sect. 2.7).

2.2 Transcendental Preconditions

In Chap. 1, I spoke of a transcendental interpretation of world lines and suggested that construing individuals as world lines is a 'transcendental precondition' of our possibility to think and talk about them in modal settings. Saying so is not informative unless the relevant notion of precondition is clarified.

When discussing transcendental arguments, we are typically confronted with an inference that has two premises: a factual one, X, and a conditional one, 'necessarily, if X, then Y'. Here, X is typically a statement according to which we have certain experiences or thoughts or are capable of using language meaningfully in certain ways, and Y is often a proposition the skeptic doubts; it states either how things as a matter of fact are (strong version) or how we think they are (modest version). The reasoning is meant to establish that Y is a *transcendental precondition* of X. Here, Y is obtained from the two premises by modus ponens. I will refer to the conditional premise as a *transcendental claim*. In a transcendental claim 'necessarily,

T. Tulenheimo, *Objects and Modalities*, Logic, Epistemology,
and the Unity of Science 41, DOI 10.1007/978-3-319-53119-9_2

if X, then Y', the statement X is its *antecedent* and Y its *consequent*. As Barry Stroud [117, p. 156] stresses, the reasoning just described is in no way special. We should not take the *inference* to constitute a transcendental argument. Rather, a transcendental argument is an argument by means of which we could try to establish the transcendental claim (the conditional premise).

People often use phrases like 'Y is a necessary condition for the possibility of X' and 'In order for X to be possible, the condition Y must hold' when wishing to explain what a transcendental argument is supposed to establish. Superficially, it looks as though *possibility* qualifies the antecedent X and *necessity* qualifies the consequent Y in such formulations. In reality, it is the conditional 'if X, then Y' that is qualified by *necessity*. This is a case where grammar easily leads us astray. For example, when discussing Kant's philosophy, Sebastian Gardner [32, p. 30] writes, 'A transcendental proof has the peculiarity that it converts a possibility into a necessity: by saying under what conditions experience of objects is possible, transcendental proofs show those conditions to be necessary for us to the extent that we are to have experience of objects at all'. Here, the crucial expression is the hidden conditional of 'to the extent that'. No possibility is turned into anything, and nothing is turned into a necessity. It is just said that in any possible occasion in which we have experience of objects, a certain condition holds. This is what saying 'necessarily, if we have experience of objects, then such-and-such a condition holds' means.[1]

Following Quassim Cassam [13, pp. 83, 85], a distinction between *world-directed* and *self-directed* transcendental claims can be made. The antecedent of a transcendental claim of the former type states that we have certain experiences or thoughts. Its consequent states that the world in which these thoughts or experiences occur is a certain way. A world-directed transcendental claim itself states, then, that the world's being a certain way is a necessary condition for our having the thoughts or experiences mentioned in the antecedent. The antecedent of a self-directed transcendental claim, again, states that we have certain cognitive achievements, and its consequent states that our cognitive faculties are thus and so. Consequently, self-directed transcendental claims state that unless the thinking self has certain cognitive faculties, it lacks the cognitive achievements referred to in the antecedent. Self-directed transcendental claims can be seen as statements of *conceptual necessity*: they affirm that the employment of such-and-such concepts (those mentioned in the consequent) is necessary for our knowledge or experience or meaningful discourse.

Stroud argued in his famous 1968 paper that attempts to establish world-directed transcendental claims are deeply problematic: instead of proving that the *factual truth* of a proposition Y is necessary for our having such-and-such experiences, they merely appear to show that our *believing Y to be true* is necessary for those experiences [116]. This leaves still open the possibility of arguing for modest claims about connections between different ways of thinking [117, pp. 165–6], thereby in effect shifting attention toward self-directed transcendental arguments.

[1] This said, certainly nothing prevents X and Y from being modal statements—being, for example, of the form *possibly* Z or *necessarily* Z. I merely wish to stress that this is not a part of what saying 'Y is a necessary condition for the possibility of X' means.

I am interested in self-directed transcendental claims whose antecedent is a statement according to which we can speak meaningfully of objects exhibiting modal and temporal behavior and whose consequent states that we employ certain concepts to think about those objects. Cassam [13] takes up the question of whether self-directed transcendental claims are independent of the *subjective origin thesis* (SOT), according to which the cognitive faculties that a transcendental argument portrays as preconditions of our cognitive achievements are wholly subjective in nature. According to SOT, self-directed transcendental claims are committed to transcendental idealism. Cassam describes a way of viewing transcendental claims that—without rendering them superfluous—allows them to avoid a commitment to SOT, thereby making them compatible with a form of realism. Seen in this way, transcendental preconditions are taken to reflect the nature of mind-independent objects.

The realist position Cassam develops consists of seeing transcendental preconditions as *objectively necessary conditions*. These are world-dependent conditions, grounded at least partly in the nature of objects as they are in themselves. Cassam defines objectively necessary conditions as *conditionally* conceptually necessary conditions [13, pp. 103–4]. In this sense, Y is an objectively necessary condition of X iff Y is a conceptually necessary condition of X *given* certain assumptions about the objects as they are in themselves. If we write '[objective](X, Y)' for 'Y is objectively necessary for X' and '[conceptual](X, Y)' for 'Y is conceptually necessary for X', then affirming [objective](X, Y) means affirming a statement of the following form:

1. If Z, then [conceptual](X, Y),

where Z is a statement about the nature of those external objects to which our knowledge pertains or about which we can meaningfully speak according to the statement X. For example, Z could be the statement that the objects of our empirical knowledge are in themselves spatiotemporal, X and Y being, respectively, the statements that we have empirical knowledge of spatiotemporal objects and that we employ the concept of persisting space-occupying substance when thinking about these objects [ibid. p. 106]. Since [conceptual](X, Y) itself is a statement according to which Y is a necessary condition of X, the claim (1) amounts to (2):

2. If Z, then necessarily, if X, then Y.

Cassam refers to the realist position he describes as *conceptualist realism* [ibid. p. 104].

In the context of conceptualist realism, transcendental arguments are primarily arguments for conditional statements of the form (1). As Cassam sees it, for the conceptualist realist, such transcendental arguments have an *explanatory* role, not an anti-skeptical role [13, p. 109]. In a transcendental argument intended as establishing (1), the *explanandum* is the statement [conceptual](X, Y) and the *explanans* the whole conditional claim (1). According to the explanation in question, the conceptualizations mentioned in the statement Y are important for our cognitive achievements mentioned in the statement X, since the reality to which those cognitive achievements pertain is as described by Z. The claim (1) may of course hold even if Z is false. If Z

happens to be true, we can infer the corresponding claim of conceptual necessity—i.e., the claim [conceptual](X, Y)—with the help of (1).

We may now distinguish two versions of the 'transcendental interpretation' of world lines as sketched in Sect. 1.5: the *realist* and the *idealist* version. Consider the following statements P_1, P_2, Q, and R:

P_1: Some external (mind-independent) objects are temporally extended and have modal properties.
P_2: Some appearances (objects of experience conforming to our mode of cognition) are temporally extended and have modal properties.
Q: It is meaningful to speak of temporally extended objects and to talk about their counterfactual behavior.
R: Objects we speak of in modal settings are conceptualized as world lines.

In Q and R, by speaking of 'objects', I mean *physical objects*—as opposed to intentional objects. The conceptualist realist must provide a transcendental argument for the statement (3), whereas the transcendental idealist must establish (4):

3. If P_1, then [conceptual](Q, R)
4. [conceptual](Q, R).

Here, (4) is the statement that in order for us to meaningfully speak of temporally extended objects with modal properties, these objects *must be thought of as* world lines. This is precisely the thesis put forward by the transcendental interpretation of world lines. As explained in Sect. 1.3, thinking of modal individuals as world lines means holding that *individuals* and *worlds* are mutually independent but interacting 'modal unities'. The cross-world behavior of individuals does not reduce to, nor is supervenient on, world-internal local features. World lines are not determined by worlds, and worlds are not determined by world lines. An individual is realized in some but not necessarily all worlds. A world realizes some but not necessarily all individuals. Each version of the transcendental interpretation of world lines can be characterized as a conjunction of three statements that jointly entail R:

5. *Realist variant*: P_1 & Q & If P_1, then [conceptual](Q, R)
6. *Idealist variant*: P_2 & Q & [conceptual](Q, R).

In each case, the first statement describes how the relevant philosophical position views the nature of the objects spoken about; the second statement affirms the antecedent of the transcendental claim (4); and the third statement is the claim whose proof would constitute the relevant transcendental argument. In the realist case, such an argument is potentially easier to produce: it suffices to argue for (4) on the condition that P_1 holds, instead of establishing (4) categorically, as required in the idealist case. All three conjuncts of (5) are needed to derive R. By contrast, R is entailed already by the latter two conjuncts of (6).

The truth of the realist claim (3) can be seen as *explaining* our cognitive faculty of thinking of objects of meaningful discourse in a certain way (namely, as world lines).

For the conceptual realist, the condition R is *objectively* necessary, grounded in the truth of P_1, and the claim Q is understood as concerning our capacity to talk about external objects. The idealist variant differs from its realist cousin in that the claim (4) must be established unconditionally, and Q is understood as a claim about our capacity to speak of the temporal and modal behavior of *appearances* (as opposed to objects as they are in themselves). The idealist takes the relevant necessary condition R to originate in ‘the subjective constitution of our mind’ [cf. A23/B38], not in how the world is.[2] In order to show that self-directed transcendental claims are not committed to SOT, it suffices to argue that the idea of a conditionally conceptually necessary condition is coherent; cf. [13, pp. 104–5]. To this end, the realist need not be able to categorically rule out the idealist position P_2.

To defend either variant of the transcendental interpretation, the specific task is to produce the relevant transcendental argument.[3] For the realist, this means arguing for (3). The idealist needs to argue for (4). In both cases, we need an argument that is semantic by nature and consists of two steps. First, the meaning of the statement R must be clarified. We need a sufficiently comprehensive semantic theory that explicates what it means for quantifiers to range over world lines. This will show that it is at least intrinsically coherent to claim that meaningful discourse about individuals in modal settings is based on construing individuals as world lines. Second, grounds must be given for preferring world line semantics over alternative semantic accounts. I must back up my analysis by explicit comparisons between my view and views according to which cross-world identity is a simple notion, unproblematically transferrable from extensional to modal settings. If I succeed in showing that one must adopt the semantics of world lines to account for our actual meaningful discourse about temporally extended objects with modal properties, I have ipso facto provided a transcendental argument for the transcendental claim (4).

The choice between the realist and the idealist version of the transcendental interpretation must be based on general philosophical considerations. Cassam’s argumentation in his 1999 paper shows that the transcendental idealist faces considerable difficulties in maintaining that the realist cannot detach self-directed transcendental arguments from the subjective origin thesis. Within the confines of this book, I naturally cannot undertake an overarching defense of either transcendental idealism or (conceptualist) realism. What I say is compatible with either viewpoint. I am sympathetic to realism: I take it that we must postulate external objects. In addition to external physical objects, however, there are objects of thought. My goal is not to explain them away. Indeed, my overall semantic framework aims to defend a supplementary transcendental claim concerning intentional objects:

7. [conceptual](Q', R'),

where Q' and R' are the following statements:

Q': It is meaningful to speak of intentional objects.

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[2] All references of the form An/Bm or An or Bm are to Kant [60].

[3] The unspecific task would be to argue for P_1 or for P_2. However, it is beyond the scope of this book to undertake a global defense of realism or transcendental idealism.

R': Intentional objects are conceptualized as world lines.

Here, (7) affirms that in order for us to meaningfully speak of intentional objects, they *must be thought of as* world lines. I will argue that intentional objects must be viewed as being intrinsically modal—as world lines defined over the set of worlds compatible with an intentional state of an agent (e.g., the agent's perceptual experience or beliefs).[4] Chapters 3 and 4 provide a detailed discussion of intentional objects and their relation to physical objects. Until then, my analysis will remain schematic.

2.3 World Line Semantics

I proceed to describe schematically a semantics of a quantified modal language. Its quantifiers range over world lines. I refer to it as *world line semantics*. At this schematic level, I pay no attention to the nature of world lines. I merely wish to give a precise idea of how the semantics looks like. In Sect. 3.4, I enrich the framework to make it useful for discussing the contrast between intentional and physical objects.

Let *Var* be a set of variables and τ a *relational vocabulary*: a set of predicate symbols, each with an associated positive arity indicating how it syntactically combines with variables to form atomic formulas. For simplicity, sometimes I refer to predicate symbols as *predicates*. (In this sense, predicates are always linguistic entities.) The quantified modal language $L_0[\tau]$ of vocabulary τ is built according to the following syntax:

$$\phi ::= Q(x_1, \ldots, x_n) \mid x_1 = x_2 \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box\phi \mid \exists x\phi,$$

where n is a positive integer, the symbols $x, x_1, x_2, \ldots, x_n$ all belong to *Var*, and Q is an n-ary predicate belonging to τ.[5] Both *predications* $Q(x_1, \ldots, x_n)$ and *identities* $x_1 = x_2$ are *atomic* formulas. The operators $\vee$, $\rightarrow$, $\Diamond$, and $\forall$ are definable from the operators $\neg$, $\wedge$, $\Box$, and $\exists$ in the usual manner.

Models of vocabulary τ are structures $M = \langle W, R, \mathfrak{I}, Int\rangle$. Here, W is a non-empty set. Every member w of W has a specified non-empty domain $dom(w)$. Further, R is a binary relation on W, and *Int* is a function assigning to every n-ary

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[4] I take it that neither physical nor intentional objects should be considered as local objects, but both types of objects must be viewed as world lines. In the case of intentional objects, this is so even if we limit attention to objects that are not thought of as being temporally extended or having modal properties. This is because there is normally a great variety of scenarios compatible with an agent's intentional state; intentional objects are construed as world lines considered in relation to the totality of all such scenarios.

[5] The syntax is conveniently specified in Backus–Naur form. It should be understood as follows. If Q is a predicate and the x_i are variables, both $Q(x_1, \ldots, x_n)$ and $x_1 = x_2$ are formulas. The result of prefixing a formula by $\neg$, $\Box$, or $\exists x$ is likewise a formula. Finally, the result of combining a formula with a formula using $\wedge$ is a further formula. The symbol ϕ represents schematically any expression generated by the grammar, and distinct occurrences of ϕ need not represent the same expression.

predicate Q of τ and element w of W a subset $Int(Q, w)$ of $dom(w)^n$. Finally, $\mathfrak{I}$ is a collection of sets $\mathfrak{I}_w$ with $w \in W$. Each element of $\mathfrak{I}_w$ is a non-empty partial function on W, assigning an element of $dom(w')$ to every w' on which this partial function is defined.

The set W is the *domain* of M, in symbols $dom(M)$. Elements of W are referred to as *worlds*. Elements of the sets $dom(w)$, again, are termed *local objects*. If $w \neq w'$, in accordance with my assumptions about cross-world identity, I take it that elements of $dom(w)$ cannot be compared in terms of identity or numerical distinctness with elements of $dom(w')$.[6] The component R of the model is an *accessibility relation*. If $w \in W$, I write $R(w)$ for the set $\{w' : R(w, w')\}$. The component Int of the model is called an *interpretation*. Since each local object belongs to a unique world, the world in question is in principle recoverable from the object, and we could define the interpretation of an n-ary predicate Q directly as a subset of the set $\bigcup_{w\in W} dom(w)^n$ without relativizing the interpretation to a world. I stay with the definition given, however, as it makes the discussion of interpretations more transparent and in fact allows a smoother generalization when constant symbols will be allowed in the syntax (Sect. 3.4). The elements of the sets $\mathfrak{I}_w$ are *world lines over* W. Note that when calling these non-empty partial functions 'world lines', I am following the liberalized terminology agreed upon in Sect. 1.3. World lines themselves, in the sense in which I have spoken of them in connection with the transcendental interpretation, *are* not partial functions, but there is a one-to-one correspondence between world lines and suitable partial functions, which is why this abuse of terminology is harmless, and we may well utilize partial functions as surrogates of world lines in the formulation of the semantics. In order not to render the terminology needlessly heavy, I use the term 'world line' in both cases. There should be no serious risk of confusion. If $\mathbf{I} \in \mathfrak{I}_w$ and w' is a world on which $\mathbf{I}$ is defined, its value $\mathbf{I}(w')$ is the *realization* of $\mathbf{I}$ in w'. The domain of the partial function $\mathbf{I}$ is its *modal margin*, denoted $marg(\mathbf{I})$. The *domain of world lines* of M is the set $\bigcup_{w\in W} \mathfrak{I}_w$, denoted $\mathrm{WL}(M)$.

An *assignment* in M is a function of type $Var \to \mathrm{WL}(M)$. Thus, the values of variables are world lines, not local objects. If g is an assignment defined on x, then $g(x)$ is a world line. If this world line is realized in world w, the result $g(x)(w)$ of applying the function $g(x)$ to the world w is a local object that belongs to the domain of w. If g is an assignment and $\mathbf{I}$ is a world line, $g[x := \mathbf{I}]$ stands for the assignment that differs from g at most in that it assigns $\mathbf{I}$ to x. That is, if v is a variable, then $g[x := \mathbf{I}](v) = g(v)$ if v is by syntactic criteria distinct from x, whereas $g[x := \mathbf{I}](v) = \mathbf{I}$ if, syntactically, v equals x. The semantics of L_0 is defined by recursively specifying what it means for a formula ϕ to be *satisfied* in a model M

[6]There is a sense in which the question 'Is the set $dom(w) \cap dom(w')$ non-empty?' is ill formed: in any single world w, we only encounter its local objects, and remaining within w, we can never effect the relevant comparisons allowing us to meaningfully affirm that every $a \in dom(w)$ is distinct from every $a' \in dom(w')$. From a metatheoretic perspective, we can answer this question. Worlds partition the class of local objects into cells so that local objects a and a' can be compared in terms of numerical identity iff they belong to the same cell. The set $dom(w) \cap dom(w')$ is empty if w and w' correspond to distinct cells of this partition; otherwise, w equals w'. Cf. footnote 15 in Sect. 4.5.

at a world w under an assignment g, denoted $M, w, g \models \phi$. Here are the semantic clauses:

- $M, w, g \models Q(x_1, \ldots, x_n)$ iff for all $1 \le i \le n$, the world line $g(x_i)$ is realized in the world w, and the tuple $\langle g(x_1)(w), \ldots, g(x_n)(w)\rangle$ belongs to $Int(Q, w)$.
- $M, w, g \models x_1 = x_2$ iff the world lines $g(x_1)$ and $g(x_2)$ are both realized in the world w, and the local object $g(x_1)(w)$ is the same as the local object $g(x_2)(w)$.
- $M, w, g \models \neg\phi$ iff $M, w, g \not\models \phi$.
- $M, w, g \models (\phi \wedge \psi)$ iff $M, w, g \models \phi$ and $M, w, g \models \psi$.
- $M, w, g \models \Box\phi$ iff for all worlds w' with $R(w, w')$, we have: $M, w', g \models \phi$.
- $M, w, g \models \exists x\phi$ iff there is $\mathbf{I} \in \mathfrak{I}_w$ such that $M, w, g[x := \mathbf{I}] \models \phi$.

The most distinctive feature of this semantics is the way it treats quantifiers. In a world w, the quantifier $\exists$ ranges over the set $\mathfrak{I}_w$. Elements of $\mathfrak{I}_w$ are said to be *available* in w. Being available in w is not the same as being realized in w. It is not required that every world line available in w be realized in w, nor that every world line realized in w be available in w. The possibility of considering these features independently of each other will be important for analyzing quantification into modal contexts and for accommodating a sense of 'there is' without ontological commitments when discussing intentional objects. If the quantifier $\exists x$ is evaluated in world w, this results in assigning as the value of x an element $\mathbf{I}$ of $\mathfrak{I}_w$, a certain world line. When the evaluation reaches an atomic formula, say $P(x)$, it is checked whether the value $\mathbf{I}$ of x is realized in the world w' that is current *then*. If so, it is further checked whether the local object $\mathbf{I}(w')$ belongs to the set $Int(P, w')$. Note that the world w' can very well be distinct from w. This may happen if syntactically between the quantifier and the atomic formula there are occurrences of modal operators—for example, if the formula evaluated at w is $\exists x\Box P(x)$. Because availability does not entail realization, we can quantify into a modal context in w without getting thereby committed to the existence of the value of the relevant quantified variable in w—recalling that by 'existence' of a world line in a world, we mean its being realized in that world.[7]

The accessibility relation R may be relative to an agent. It could, for example, determine which alternatives the agent's perceptual experience leaves open in a given scenario (perceptual accessibility) or which scenarios are compatible with what an agent believes in a fixed scenario (doxastic accessbibility). The specific properties of the relation R depend on the modality it is taken to represent. In the case of a veridical perceptual experience, the relation must be reflexive, but not in connection with arbitrary perceptual experience and belief. I could introduce several types of modal operators into the syntax, each type having its associated accessibility relation as a component in the models. In fact, I will (Sect. 3.4). But, at this schematic phase, doing so would be a pointless complication.

[7] The distinction between availability and realization is considered in detail in Sect. 3.3. For a recent discussion of quantifying-in in general attitudinal contexts, see Jespersen [56].

As explained in Sect. 1.3, in my semantics, predicate symbols are suited for expressing properties that an individual has in a world depending on how it is in that world. According to the semantics above, predicate symbols are applied to local objects. If Q is an n-ary predicate symbol and a tuple $\langle a_1, \ldots, a_n \rangle$ belongs to the interpretation of Q in a world w, then the a_i are local objects (and belong to the domain of w). Predicate symbols are *not* applied to world lines. However, for every n-ary predicate Q of vocabulary τ, the semantics of $L_0[\tau]$ induces an $(n+1)$-ary relation R_Q as follows: $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n, w \rangle \in R_Q$ iff all world lines $\mathbf{I}_1, \ldots, \mathbf{I}_n$ are realized in w and $\langle \mathbf{I}_1(w), \ldots, \mathbf{I}_n(w) \rangle \in Int(Q, w)$. Consequently, we may view formulas $Q(x_1, \ldots, x_n)$ as n-ary 'intensional predicates'. An n-tuple $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ of world lines satisfies the intensional predicate $Q(x_1, \ldots, x_n)$ in a world w iff $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n, w \rangle \in R_Q$.

In the same way as the interpretation of an n-ary predicate symbol Q in a world w is the n-ary relation $Int(Q, w)$ on $dom(w)$, the fixed interpretation of the identity symbol in a world w is the relation of extensional identity on w—i.e., the binary identity relation $dom(w)$. Those philosophers who are perplexed by the very idea of a binary identity relation might prefer a formulation of the semantics of L_0 that does not expressly make use of such an identity relation.[8] It is, in fact, easy to reformulate the semantics of L_0 so as to meet this desideratum. Let K be the ternary relation of *co-realization* defined, relative to a given model M, as follows: whenever $\mathbf{I}, \mathbf{J} \in \bigcup_{v \in W} \mathcal{I}_v$ and $w \in W$, let $\langle \mathbf{I}, \mathbf{J}, w \rangle \in K$ iff both world lines $\mathbf{I}$ and $\mathbf{J}$ are realized in w and the local object $\mathbf{I}(w)$ is the same as the local object $\mathbf{J}(w)$. We may take the identity symbol to stand for the relation K in M, because the above satisfaction condition of identity formulas can be equivalently formulated in terms of K: we have $M, w, g \models x_1 = x_2$ iff $\langle g(x_1), g(x_2), w \rangle \in K$. Now, for having $\langle \mathbf{I}, \mathbf{J}, w \rangle \in K$, it is neither sufficient nor necessary that the world lines $\mathbf{I}$ and $\mathbf{J}$ be identical: $\langle \mathbf{I}, \mathbf{I}, w \rangle \notin K$ if $\mathbf{I}$ is not realized in w, and we may well have $\mathbf{I}(w) = \mathbf{J}(w)$ and therefore $\langle \mathbf{I}, \mathbf{J}, w \rangle \in K$ even if the world lines $\mathbf{I}$ and $\mathbf{J}$ are distinct—i.e., even if there is a world v in which the two world lines $\mathbf{I}$ and $\mathbf{J}$ do not have identical realizations (either because their realizations in v are distinct or because one but not the other fails to be realized in v). Thus, the relation K may seem less problematic as an interpretation of the identity symbol than a binary identity relation that every object bears to itself and to itself only. However, we do not get rid of binary identity relations (as certain relations-in-extension) just by reformulating our semantics so that an identity relation does not *explicitly* appear as the interpretation of the identity symbol in a world. Namely, the fact remains that

[8] Cf. the discussion in footnote 5 in Sect. 1.1.

world lines $\mathbf{I}$ and $\mathbf{J}$ being co-realized in a world *means* that they are realized in that world and that their realizations are extensionally identical.[9]

I assume the standard notion of free variable of a formula: an occurrence of a variable x is free in ϕ if it does not lie in ϕ in the scope of a quantifier carrying the variable x.[10] A formula containing no free variables is a *sentence*. I write $\phi(x_1, \ldots, x_n)$ to indicate that $x_1, \ldots, x_n$ are n distinct variables and that the free variables of ϕ are exactly the variables $x_1, \ldots, x_n$. If $g(x_i) = \mathbf{I}_i$ for all $1 \leq i \leq n$, we may express the condition $M, w, g \models \phi$ by writing $M, w, x_1 := \mathbf{I}_1, \ldots, x_n := \mathbf{I}_n \models \phi(x_1, \ldots, x_n)$, without thereby suppressing any relevant information.[11] If ϕ is a sentence, its satisfaction is entirely independent of the assignment considered. We may write $M, w \models \phi$ when ϕ is a sentence and $M, w, g \models \phi$ holds for at least one assignment g. When this condition holds, we say that ϕ is *true* in M at w. A formula ϕ of vocabulary τ is *valid* if for all models M of vocabulary τ, worlds $w \in dom(M)$, and assignments g in M, we have $M, w, g \models \phi$. A formula ϕ is *refutable* (respectively, *satisfiable*) if ϕ (respectively, $\neg\phi$) is not valid. Let us take some examples of how the semantics works.

Example 2.1 The formula $P(x)$ may fail to be satisfied in M at w under g for two reasons: either because the world line $g(x)$ is not realized in w in the first place or because it is but its realization $g(x)(w)$ lies outside the set $Int(P, w)$. Consequently, the negated formula $\neg P(x)$ can be satisfied in M at w under g for two reasons. The formula $x = x$ is *not* valid. In fact, $M, w, g \models x = x$ iff $g(x)$ is realized in w. The formulas $\forall x(P(x) \vee \neg P(x))$ and $\forall x\, x = x$ are not equivalent. The former *is* valid. Namely, suppose $\mathbf{I} \in \mathcal{I}_w$. If $\mathbf{I}$ is realized in w, the realization $\mathbf{I}(w)$ either does or does not belong to $Int(P, w)$, satisfying correspondingly either the left or the right disjunct of $P(x) \vee \neg P(x)$ at w. If, again, $\mathbf{I}$ is not realized in w, it satisfies the right

[9]Hintikka [40] showed that in first-order logic, the semantics of quantifiers can be interpreted 'inclusively' or 'exclusively' and that in the latter case, the expressive power of first-order logic is not diminished by disallowing the use of the identity symbol in the syntax (supposing we restrict attention to vocabularies in which neither constant nor function symbols occur). Unlike in the standard 'inclusive' interpretation, according to the 'exclusive' interpretation, a formula $\exists x\, \phi(x, x_1, \ldots, x_n)$ is satisfied in a model $\mathcal{M}$ under an assignment Γ iff there is in the domain of $\mathcal{M}$ an individual b *other than* any of the individuals $\Gamma(x_1), \ldots, \Gamma(x_n)$ such that $\Gamma[x/b]$ satisfies the formula $\phi(x, x_1, \ldots, x_n)$ in $\mathcal{M}$: the range of x *excludes* the values of all variables $x_1, \ldots, x_n$ that are free in the scope of the quantifier $\exists x$. Wehmeier [125] attempts to argue that we can dispense with the binary relation of identity, and as a partial motivation, he refers to Hintikka's result. However, as Hintikka [40, p. 228] himself stressed and contrary to what Wehmeier [125, Sect. 2] suggests, Hintikka's result merely shows that we do not need the *identity symbol* in the syntax of first-order logic; the result by no means suggests that we can dispense with the *notion of identity*. The exclusive interpretation of quantifiers merely provides an alternative way of dealing with the notion of extensional identity in first-order logic.

[10]Given the syntax of L_0, the only possible quantifier meeting this criterion could be $\exists x$. The obvious syntactic notion of subformula gives rise to the notion of scope in the usual way.

[11]As in first-order logic, also in L_0, the satisfaction of a formula ϕ under an assignment g evidently depends only on the values of g on *those* variables that are free in ϕ.

disjunct at w. By contrast, if $\mathbf{I}$ is not realized in w, it fails to satisfy $x = x$ at w.[12] □

Let Q be a fixed unary predicate. The formulas BF and CBF are, respectively, the *Barcan formula* and the *converse Barcan formula*:

BF $\quad \Diamond \exists x\, Q(x) \rightarrow \exists x \Diamond Q(x)$
CBF $\quad \exists x \Diamond Q(x) \rightarrow \Diamond \exists x\, Q(x)$.

For reasons to be explicated in Sect. 5.4, I will *not* say that all those formulas are Barcan formulas that are obtained from BF by replacing $Q(x)$ by a formula with x as its sole free variable. A similar remark applies to CBF. Accordingly, BF and CBF should not be seen as schemata in the usual sense.

Example 2.2 Neither BF nor CBF is valid. Let $w_1 \neq w_2$, and consider a model $M = \langle W, R, \mathcal{I}, Int \rangle$ defined as follows:

- $W = \{w_1, w_2\}$ and $R = \{\langle w_1, w_2 \rangle\}$
- $\mathcal{I} = \{\mathcal{I}_{w_1}, \mathcal{I}_{w_2}\}$, where $\mathcal{I}_{w_1} = \varnothing$ and $\mathcal{I}_{w_2} = \{\mathbf{I}\}$ with $marg(\mathbf{I}) = \{w_2\}$
- $Int(Q, w_1) = \varnothing$ and $Int(Q, w_2) = \{a\}$, where $a = \mathbf{I}(w_2)$.

We have $M, w_1 \models \Diamond \exists x\, Q(x)$, since w_2 is R-accessible from w_1 and $M, w_2 \models \exists x\, Q(x)$. The latter condition holds because $\mathbf{I} \in \mathcal{I}_{w_2}$ and $\mathbf{I}(w_2) = a \in Int(Q, w_2)$. Yet, $M, w_1 \not\models \exists x \Diamond Q(x)$. This follows from the fact that the set $\mathcal{I}_{w_1}$ is empty. We may conclude that BF is refutable. In order to see that also CBF is refutable, consider the model $M' = \langle W', R', \mathcal{I}', Int' \rangle$, where $W' = W$ and $R' = R$ and $Int' = Int$ and $\mathcal{I}' = \{\mathcal{I}'_{w_1}, \mathcal{I}'_{w_2}\}$, where $\mathcal{I}'_{w_1} = \{\mathbf{I}\}$ and $\mathcal{I}'_{w_2} = \varnothing$. Note that the very same world line that is available in w_2 in M is available in w_1 in M'. Now, $M', w_1 \models \exists x \Diamond Q(x)$, since $\mathbf{I} \in \mathcal{I}_{w_1}$ and $R(w_1, w_2)$ and $\mathbf{I}$ is realized in w_2, satisfying $\mathbf{I}(w_2) = a \in Int(Q, w_2)$. Still, $M', w_1 \not\models \Diamond \exists x\, Q(x)$, because the only world accessible from w_1 is w_2 and the set $\mathcal{I}_{w_2}$ is empty. Even though $\mathbf{I}$ is realized in w_2, neither it nor any other world line is available in w_2. Consequently, there is no world line $\mathbf{J} \in \mathcal{I}_{w_2}$ satisfying $M', w_2, x := \mathbf{J} \models Q(x)$. □

[12] No atomic formula $P(x)$ can be satisfied by a value $x := \mathbf{I}$ in a world w unless the world line $\mathbf{I}$ is realized in w. Thus, if 'haired' and 'bald' are construed as atomic predicates and $\mathbf{I}$ is not realized in w, the assignment $x := \mathbf{I}$ satisfies neither haired(x) nor bald(x) in w. As these predicates are used in English, qualifying anything as haired or bald in a context w would perhaps be taken to *presuppose* that the thing is present in w, rather than its presence in w being viewed as part of what is affirmed by such a qualification. In any event, natural-language semantics agrees that 'haired' and 'bald' cannot be satisfied in w by anything not realized in w. The same holds for such predicates as 'unfair'. If the value $\mathbf{I}$ of x is realized in w, we can safely say that $x := \mathbf{I}$ satisfies unfair(x) in w iff it satisfies $\neg$fair(x) in w, but generally, we could have $w, x := \mathbf{I} \models \neg\text{fair}(x)$ because $\mathbf{I}$ is not realized in w. In that case, we would not have $w, x := \mathbf{I} \models \text{unfair}(x)$, since this would require that $\mathbf{I}$ be realized in w. That is, unfair(x) is not simply the negation of fair(x). This said, unfair(x) can be defined in terms of fair(x): the formula unfair(x) is equivalent to $x = x \wedge \neg\text{fair}(x)$, since this latter is satisfied in w by $x := \mathbf{I}$ iff $\mathbf{I}$ is realized in w and $\mathbf{I}(w)$ fails to be fair. One could define a strong notion of negation ($\sim$) in L, by stipulating that $\sim P(x)$ means $x = x \wedge \neg P(x)$. Then, unfair(x) could indeed be defined as a negation of fair(x) in a certain sense, because unfair(x) is equivalent to $\sim$fair(x).

The above example shows that the semantics of L_0 validates neither BF nor CBF. It is important to note that this fact has nothing to do with local objects being world-bound. Values of variables are world lines, not local objects. The reason why BF is refutable is that world lines available to be quantified over in one world need not be available to be quantified over in another world. As for CBF, its counter-models must utilize a further property of the semantics of L_0, as well—namely, the fact that a world line may be realized in a world (and satisfy an atomic predicate therein) without being available in that world.

2.4 Types of Predicates

In world line semantics, an obvious distinction can be made between 'extensional' and 'intensional' predicates. The former apply to local objects (realizations of world lines), whereas the latter apply to world lines themselves. In models of L_0, elements of the non-logical vocabulary are treated as extensional predicates, which, in particular, satisfy the 'domain constraint': the interpretation of an n-ary predicate symbol in w is a subset of the n-th Cartesian power of $dom(w)$. However, formulas of L_0 give rise to intensional predicates. It has already been noted that an atomic formula $Q(x_1, \ldots, x_n)$ can be seen as an n-ary intensional predicate that is satisfied by a tuple $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ of world lines in a world w iff we have $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n, w \rangle \in R_Q$, where R_Q is an $(n+1)$-ary relation induced by the semantics of L_0 in the way explained in Sect. 2.3. In fact, any formula $\phi(x_1, \ldots, x_n)$ with n free variables can be considered as an intensional n-ary predicate that is atomic or complex, depending on whether the formula $\phi(x_1, \ldots, x_n)$ is atomic or complex. The predicate $\phi(x_1, \ldots, x_n)$ applies in a model M at a world w to those n-tuples of world lines that satisfy it in M at w. More generally, the semantics specified in Sect. 2.3 determines for all models M and L_0-formulas ϕ of n free variables a certain set of $(n+1)$-tuples—namely, the sequence of those 'parameters of evaluation' that satisfy ϕ in M.

Definition 2.1 (*Semantic value*) Let M be a model, and let $\phi(x_1, \ldots, x_n)$ be a formula of the language L_0. The *semantic value* $|\phi(x_1, \ldots, x_n)|^M$ of ϕ in M is the set of all $(n+1)$-tuples $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in dom(M) \times \mathrm{WL}(M)^n$ such that

$$M, w, x_1 := \mathbf{I}_1, \ldots, x_n := \mathbf{I}_n \models \phi(x_1, \ldots, x_n).$$

If ϕ is a sentence, then $|\phi|^M$ is a (possibly empty) subset of $dom(M)$—namely, the set of worlds w at which ϕ is true in M. □

Since the semantics of atomic formulas of L_0 is formulated in terms of extensional predicates, in fact all *intensional* predicates induced by L_0-formulas are analyzable in terms of extensional predicates. This holds trivially for *atomic* intensional predicates built from n-ary predicate symbols. A tuple $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ of world lines satisfies such an intensional predicate $Q(x_1, \ldots, x_n)$ at a world w iff all these world lines are realized in w and the tuple $\langle \mathbf{I}_1(w), \ldots, \mathbf{I}_n(w) \rangle$ satisfies the *extensional* predicate Q in w. Those dispositional predicates that can be represented by using complex L_0-formulas are likewise analyzable in terms of extensional predicates. Consider the intensional predicate $\Box(P(x) \rightarrow Q(x))$ that a world line satisfies in w if in all accessible scenarios in which it satisfies P, it satisfies Q, as well. The world line $\mathbf{I}$ satisfies this dispositional predicate in w iff in all those worlds w' accessible from w in which $\mathbf{I}$ is realized and in which its realization $\mathbf{I}(w)$ satisfies the extensional predicate P, this realization $\mathbf{I}(w)$ also satisfies the extensional predicate Q. As I said in Sect. 1.3, I do not wish to suggest that all philosophically interesting atomic predicates for some reason must ascribe local properties, and a fortiori I do not wish to claim that all intensional predicates must be analyzable in terms of local properties. However, some intensional predicates are so analyzable—cases in point being all *those* intensional predicates that are induced by L_0-formulas. In this book, I confine attention to intensional predicates of this kind.

If $\phi(x_1, \ldots, x_n)$ is an L_0-formula and $y_1, \ldots, y_n$ are variables, all of which are free for every variable $x_1, \ldots, x_n$ in ϕ, then $\phi[x_1 /\!\!/ y_1, \ldots, x_n /\!\!/ y_n]$ stands for the result of uniformly replacing all free occurrences of x_i in ϕ by y_i for all $1 \leq i \leq n$.[13] The following notions are useful when discussing varieties of predicates.

Definition 2.2 Let $\phi(x_1, \ldots, x_n)$ be a predicate in L_0. It is *existence-entailing* if the formula $\phi(x_1, \ldots, x_n) \rightarrow \bigwedge_{1 \leq i \leq n} x_i = x_i$ is valid. It is *pro mundo* if the formula $\big(\phi(x_1, \ldots, x_n) \wedge \bigwedge_{1 \leq i \leq n} x_i = y_i\big) \rightarrow \phi[x_1 /\!\!/ y_1, \ldots, x_n /\!\!/ y_n]$ is valid, given that each variable y_i is free for every variable x_j. It is *quasi-extensional* if it is both existence-entailing and *pro mundo*. □

Note that a predicate $\phi(x)$ is existence-entailing if a world line $\mathbf{I}$ cannot satisfy $\phi(x)$ in a world w unless $\mathbf{I}$ is realized in w. It is *pro mundo* if its satisfaction in w by a world line depends only on the realization (if any) of the world line in w: if $\mathbf{I}(w) = \mathbf{J}(w)$ and $\mathbf{I}$ satisfies $\phi(x)$ in w, then also $\mathbf{J}$ satisfies $\phi(x)$ in w.

Fact 2.1 *(a) Being existence-entailing and being* pro mundo *are mutually independent properties of predicates. (b) The set of existence-entailing predicates is not closed under applications of the operators* $\neg$ *or* $\Box$*. (c) The set of* pro mundo *predicates is not closed under applications of the operator* $\Box$*.*

Proof Let us begin with item (a). If P is unary and atomic, the predicate $x = x \wedge \Diamond P(x)$ is trivially existence-entailing, but it is not *pro mundo*: world lines $\mathbf{I}$ and $\mathbf{J}$ can coincide in w while only one of them satisfies $\Diamond P(x)$ in w. In order to see that, conversely, being *pro mundo* does not guarantee existence-entailment, note first that the predicate $\neg P(x)$ is *pro mundo*: if $\mathbf{I}(w) = \mathbf{J}(w)$, then $\mathbf{I}(w) \notin Int(P, w)$ iff $\mathbf{J}(w) \notin Int(P, w)$. Yet, it fails to be existence-entailing: $\neg P(x)$ is satisfied in w by any world line not realized in w. For items (b) and (c), observe that $P(x)$ is both

[13] We say that y is *free for* x in ϕ iff x does not occur free in the scope of the quantifier $\exists y$ in ϕ. If y is not free for x in ϕ, substituting y for a certain free occurrence of x in ϕ results in a formula in which that occurrence of y is bound.

existence-entailing and *pro mundo*. Yet, $\neg P(x)$ is not existence-entailing. Neither is $\Box P(x)$. For having $w, x := \mathbf{I} \models \Box P(x)$, it suffices that $\mathbf{I}$ is realized and satisfies P in all worlds v accessible from w; $\mathbf{I}$ need not be realized in w unless the accessibility relation corresponding to $\Box$ happens to be reflexive. Finally, $\Box P(x)$ also fails to be *pro mundo*: having $\mathbf{I}(w) = \mathbf{J}(w)$ and $w, x := \mathbf{I} \models \Box P(x)$ does not entail $w, x := \mathbf{J} \models \Box P(x)$. The fact that $\mathbf{I}$ and $\mathbf{J}$ coincide locally, in w, does not generally guarantee that they behave similarly in all worlds accessible from w. $\square$

A predicate ϕ is *non-modal* if it contains no occurrences of $\Box$. It is *positive* if all its atomic subformulas occur in the scope of an even number of negation-signs. All non-modal predicates are *pro mundo*. Not all of them are quasi-extensional. For example $\neg P(x)$ is not, as it fails to be existence-entailing. By contrast, all positive non-modal predicates are quasi-extensional. A case in point is $Q(x) \wedge \neg(\neg R(x, y) \wedge \neg P(y))$. Also some modal predicates are quasi-extensional. An example is $P(x) \wedge \theta$ if θ is any modal sentence, no matter how many nested boxes it contains. It is characteristic of quasi-extensional predicates that their semantic values can be encoded by interpretations of extensional predicates in the following sense. Suppose $\phi(x_1, \ldots, x_n)$ is a quasi-extensional predicate of vocabulary τ. Let Q_ϕ be an n-ary predicate symbol with $Q_\phi \notin \tau$. For every model M of vocabulary τ, expand its interpretation function *Int* by setting $Int(Q_\phi, w) := \{\langle a_1, \ldots, a_n\rangle \in dom(w)^n$: there are $\mathbf{I}_1, \ldots, \mathbf{I}_n \in \mathrm{WL}(M)$ such that $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle \in |\phi(x_1, \ldots, x_n)|^M$ and $a_1 = \mathbf{I}_1(w)$ and...and $a_n = \mathbf{I}_n(w)\}$. Now, the set $Int(Q_\phi, w)$ provides enough information for us to tell whether a given tuple $\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n\rangle$ belongs to $|\phi|^M$. Namely, for all $\mathbf{J}_1, \ldots, \mathbf{J}_n \in \mathrm{WL}(M)$, we have:

$$\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n\rangle \in |\phi|^M \text{ iff } \langle \mathbf{J}_1(w), \ldots, \mathbf{J}_n(w)\rangle \in Int(Q_\phi, w).$$

For the direction from left to right, note that if $\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n\rangle \in |\phi|^M$, then each $\mathbf{J}_i$ is, indeed, realized in w. This is because ϕ is existence-entailing. It follows by the definition of $Int(Q_\phi, w)$ that $\langle \mathbf{J}_1(w), \ldots, \mathbf{J}_n(w)\rangle \in Int(Q_\phi, w)$. Conversely, if $\mathbf{J}_1, \ldots, \mathbf{J}_n$ are world lines such that $\langle \mathbf{J}_1(w), \ldots, \mathbf{J}_n(w)\rangle \in Int(Q_\phi, w)$, there are, by the definition of the set $Int(Q_\phi, w)$, world lines $\mathbf{I}_1, \ldots, \mathbf{I}_n$ such that $\mathbf{J}_j(w) = \mathbf{I}_j(w)$ for all $1 \leq j \leq n$ and $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle \in |\phi|^M$. Because ϕ is *pro mundo*, it follows that $\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n\rangle \in |\phi|^M$.

In metaphysical literature, especially in connection with four-dimensionalism, a distinction between sortal and non-sortal predicates is made.[14] Among predicates of the former variety, there are '— is a soccer ball', '— is an apple', and '— is a dog', while '— is spherical', '— is red', and '— barks' are predicates of the latter kind. Four-dimensionalism is a metaphysical view according to which individuals are not wholly present at any moment at which they exist. They have temporal parts, and those parts are what we encounter at specific instants. These temporal parts are not objects of a terribly extraordinary variety. As Wasserman [123] puts it, if you want to know what a temporal part looks like, just look in the mirror: there, one sees one's

[14] For sortals, see, e.g., Grandy [35], Lowe [81], Wiggins [126].

current temporal part. World lines give rise to a view of individuals generalizing four-dimensionalism: the four-dimensionalist's individuals are world lines defined over a set of instants within one and the same structured world. There are different varieties of four-dimensionalism—perdurantism, stage theory. These views differ in how they construe the semantics of sortal predicates. Perdurantists hold that sortal predicates are applicable to material things themselves, not to their temporal parts. Using the terminology above, this would mean that sortal predicates are not *pro mundo*: they are irreducibly intensional predicates. Stage theorists, again, are happy to let sortal predicates apply to *stages* of material things (their brief temporal parts). A stage theorist would not object to ascribing sortal predicates to realizations of world lines or treating them as quasi-extensional predicates, provided that the contexts over which world lines are defined are of a suitable kind—they should be instantaneous, or in any event, they should not themselves involve change.[15]

My discussion is primarily driven by logical considerations. I wish to remain as neutral as I can in metaphysical matters. Also, I do not aim at an all-englobing logical analysis of modal phenomena. I wish to keep my formalism relatively simple. It will always be possible to extend a well-understood formalism, while a sketchy account of a messy formalism serves no purpose. I deliberately confine attention to extensional predicates at the atomic level. From the perdurantist viewpoint, this decision blocks the possibility of representing ascriptions of sortal predicates. Then again, if '— is a lion' does not apply to a local object, '— is a realization of a lion' will. Alternatively, we can save the spirit of the perdurantist view on sortal predicates by distinguishing two types of extensional predicates: those that apply to local objects unconditionally and those that apply to a local object b in a structured world w at an instant t only on condition that there is an individual $\mathbf{I}$ with $b = \mathbf{I}(w, t)$ and a more or less large temporal interval X with $t \in X$ such that the predicate applies to $\mathbf{I}(w, t')$ for all $t' \in X$. The former predicates correspond to non-sortal predicates, the latter to sortal predicates. Viewed in this way, a sortal predicate is an extensional predicate whose applicability in one context presupposes its applicability to the realizations of a fixed world line over a whole set of contexts.

2.5 Contents

I define a general concept of *content*; such contents may but need not be propositional. Further, using the notion of semantic value defined in the previous section, I specify what it means for a content to (locally or uniformly) *support* a formula.

Definition 2.3 (*Content, situated content, internal modal margin*) Let M be a model. Let $V \subseteq dom(M)$ and $\mathbf{I}_1, \ldots, \mathbf{I}_n \in \mathrm{WL}(M)$. The structure $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ is an *n-ary content* over M. The set V is its *propositional component*, and the $\mathbf{I}_j$ are its *world line components*. A content is *propositional* if $n = 0$, otherwise it is said

[15] For variants of four-dimensionalism, see, e.g., Lewis [76], Hawley [38, 39], Sider [112].

to have a propositional and a non-propositional aspect. If R is a binary relation on $dom(M)$, $w^* \in dom(M)$, and $V = R(w^*)$, the structure $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n, w^* \rangle$ is an *R-situated n-ary content*. The set $V \cap marg(\mathbf{I}_j)$ is the *internal modal margin* of $\mathbf{I}_j$. □

In the definition above, the world w^* may but need not belong to the set V. Further, it is allowed that $V \nsubseteq marg(\mathbf{I}_j)$, and it is likewise allowed that $marg(\mathbf{I}_j) \nsubseteq V$. Note that contents as defined above are indeed *structures* and not sets: distinct orders of the world line components $\mathbf{I}_1, \ldots, \mathbf{I}_n$ give rise to distinct contents. I define the notion of content in this way, because I wish to be able to utilize contents when talking about the evaluation of formulas. If in formulas we use variables indexed by positive integers, it will be understood that the variable with the index i takes as its value the i-th world line in the list $\mathbf{I}_1, \ldots, \mathbf{I}_n$.

Let $Cont_n[M]$ be the set of all n-ary contents over M. The semantic value of a formula $\phi(x_1, \ldots, x_n)$ in M gives rise to a subset $Cont(\phi, M)$ of $Cont_n[M]$ as follows.

Definition 2.4 (*Contents generated by a formula*) Let M be a model. The set $Cont(\phi, M)$ of *contents generated by $\phi(x_1, \ldots, x_n)$ in M* is the smallest subset of $Cont_n[M]$ satisfying the following condition: if V is non-empty and $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in |\phi|^M$ for all $w \in V$, then $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in Cont(\phi, M)$. □

If $\mathbf{J}_1, \ldots, \mathbf{J}_n$ are world lines over M, write $W^{\phi}_{\mathbf{J}_1 \ldots \mathbf{J}_n}$ for the set of worlds w such that $\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n \rangle \in |\phi|^M$. We may note that $Cont(\phi, M)$ is the set of all tuples $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ such that $\mathbf{I}_1, \ldots, \mathbf{I}_n \in \mathrm{WL}(M)$ and V is a non-empty subset of $W^{\phi}_{\mathbf{I}_1 \ldots \mathbf{I}_n}$. In particular, $\langle \{w\}, \mathbf{J}_1, \ldots, \mathbf{J}_n \rangle \in Cont(\phi, M)$ whenever $\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n \rangle \in |\phi|^M$. Further, $\langle W^{\phi}_{\mathbf{J}_1 \ldots \mathbf{J}_n}, \mathbf{J}_1, \ldots, \mathbf{J}_n \rangle$ belongs to $Cont(\phi, M)$ if $W^{\phi}_{\mathbf{J}_1 \ldots \mathbf{J}_n} \neq \varnothing$. The sets $Cont(\phi, M)$ and $|\phi|^M$ provide two ways of encoding the same information—the former way being more complex than the latter. The set $Cont(\phi, M)$ is empty iff ϕ is contradictory. Whenever $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in Cont(\phi, M)$, the set V is non-empty.

I will use situated contents to model agents' intentional states. The following notions facilitate discussing how formulas may be used for describing such states.

Definition 2.5 (*Formulas supported by a content*) Let $C = \langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n, w^* \rangle$ be a situated n-ary content over M, and let $\phi(x_1, \ldots, x_n)$ be an L_0-formula. *C locally supports ϕ* (in symbols $C \Vdash_{loc} \phi$) if $\langle w^*, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in |\phi|^M$. It *uniformly supports* ϕ (in symbols $C \Vdash_{uni} \phi$) if $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in Cont(\phi, M)$. □

The following fact is a straightforward consequence of Definition 2.5.

Fact 2.2 *If $\phi(x_1, \ldots, x_n)$ is an L_0-formula and $C = \langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n, w^* \rangle$ is an R-situated n-ary content over M, the following four conditions are pairwise equivalent:*

(a) $C \Vdash_{uni} \phi$
(b) $C \Vdash_{loc} \Box\phi$
(c) $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n, v \rangle \Vdash_{loc} \phi$ *for all* $v \in V$

(d) $V \subseteq W^{\phi}_{\mathbf{I}_1 \ldots \mathbf{I}_n}$.

Proof Note that $V = R(w^*)$. Let $g(x_j) = \mathbf{I}_j$ for all $1 \leq j \leq n$. Now, (a) holds iff $\big(M, v, g \models \phi$ for all $v \in V\big)$ iff $\big(M, v, g \models \phi$ for all $v \in R(w^*)\big)$ iff $M, w^*, g \models \Box\phi$ iff (b) holds. Further, (c) is a roundabout way of expressing the condition (b), and (d) means that $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle \in \mathit{Cont}(\phi, M)$—that is, it means that (a) holds. □

2.6 Systems of Modal Unities

In the semantics of the language L_0, values of variables are partial functions mapping worlds to local objects. This is a way of representing world lines for semantic purposes. Individuals are a paradigmatic case of values of variables: I take them to be world lines semantically represented by such partial functions. Whether we adopt a realist or idealist viewpoint on individuals, we will certainly not wish to say that they *are* functions. They are world lines. Unlike functions, world lines are not abstract entities. Yet, for every world line, there is a unique partial function that serves to model the world line.[16] That non-abstract objects can be correlated with abstract ones is hardly controversial. The form of my desk can be represented as a subset of $\mathbb{R}^3$, even if my desk is definitely not a set of triples of reals. The whereabouts of a person over a timespan can be represented by a function from instants to points in space, even if the subject of change is not a function but a person. In fact, as soon as a feature of one type depends on how a feature of another type is, this dependence relation induces a function in the mathematical sense. Not every relation of functional dependence among features corresponds to a non-abstract entity, but some do. In the case of world lines, the relevant dependence relation is between worlds and local objects, and the induced function indeed corresponds to a non-abstract entity—namely, a world line whose realizations those various local objects are.

The specific character of the position I am putting forward consists of claiming that in modal settings, individuals and worlds are related in a certain way: individuals are world lines. Somewhat surprisingly, perhaps, my framework can be reformulated in a way that allows viewing worlds and individuals as objects of the *same* general type—as interrelated objects that live in distinct dimensions, so to say. As hinted at in Sect. 1.3, they can both be modeled as *sets of local objects*; cf. Tulenheimo [120]. Individuals are extended in the 'dimension of worlds': one and the same set corresponding to an individual can intersect with several sets, each of which corresponds to a world. Worlds are extended in the 'dimension of individuals': one and the same set corresponding to a world can intersect with several sets, each of which corresponds to an individual. Both worlds and individuals are *modal unities*. Contents in the sense of Definition 2.3 emerge as *systems of modal unities*. Let us formulate these ideas more precisely.

[16]As already noted in footnote 15 in Sect. 1.3, this means that I take world lines to be related to partial functions in the same way as variable embodiments are related to principles of variable embodiment in Fine's metaphysics (see Sect. 2.7.3).

Let **A** be a non-empty set. Its elements are referred to as local objects. These are things for which the notion of local identity is, by definition, utterly simple and unproblematic and for which the question of cross-world identity cannot be posed. Elements of **A** are purely 'extensional objects'. There are different ways in which they can be grouped together. Any non-empty subset of **A** is a *modal unity*. Worlds and individuals are naturally modeled as covers of the set **A**.[17] Indeed, let **B** and **C** be covers of **A**. We declare that elements of **B** are *worlds* and that those of **C** are *individuals*. Consequently, both **B** and **C** are sets of modal unities—i.e., sets of sets of elements of **A**. The very idea that the elements of **A** are *local* objects entails that **B** must be a partition (and not an arbitrary cover) of **A**: distinct elements of **B** cannot have common elements. Unless we wish to enter into a discussion on the material constitution of physical objects, we may assume that an element of **A** cannot be shared by several individuals, either—i.e., we may also take **C** to be a partition of **A**. Regarding the interrelations of **B** and **C**, the following is required:[18]

$$\text{if } \mathbf{b} \in \mathbf{B} \text{ and } \mathbf{c} \in \mathbf{C}, \text{ then } |\mathbf{b} \cap \mathbf{c}| \leq 1.$$

In other words, either a world and an individual do not intersect at all or their intersection consists of a single element. If $\mathbf{b} \cap \mathbf{c} = \{\mathbf{a}\}$, then **a** is said to be the *realization* of **c** in **b**. If, again, the set $\mathbf{b} \cap \mathbf{c}$ is empty, we say that **c** is *not realized* in **b**.[19] The two partitions **B** and **C** give rise to two 'dimensions': a fixed individual (element of **C**) can be realized in several worlds (elements of **B**), and a fixed world (element of **B**) can serve to realize several individuals (elements of **C**). In order to obtain an alternative description of the notion of a model of vocabulary τ as defined in Sect. 2.3, the structural information provided by sets **A**, **B**, and **C** must be complemented by a binary relation R on set **B** and an interpretation function Int assigning to every n, every n-ary predicate Q, and every $\mathbf{b} \in \mathbf{B}$ a subset $Int(Q, \mathbf{b})$ of the set $\mathbf{b}^n$. Elements of $Int(Q, \mathbf{b})$ are n-tuples of elements of **A**—more specifically, n-tuples of elements of **b**. The quintuple $\langle \mathbf{A}, \mathbf{B}, \mathbf{C}, R, Int \rangle$ is a *system of modal unities*. Systems of modal unities give us a symmetric notion of content: if $\mathcal{S} = \langle \mathbf{A}, \mathbf{B}, \mathbf{C}, R, Int \rangle$ is a system of modal unities, $\mathbf{B}_0 \subseteq \mathbf{B}$, and $\mathbf{C}_0 \subseteq \mathbf{C}$, then the pair $\langle \mathbf{B}_0, \mathbf{C}_0 \rangle$ is a *content* over $\mathcal{S}$.

Systems of modal unities defined as above correspond to models $\langle W, R, \mathcal{I}, Int \rangle$ in the special case that all world lines are available in all worlds ($\mathcal{I}_w = \mathcal{I}_{w'}$ for all $w, w' \in W$) and no two world lines overlap (for all $w, w', u \in W$ and all $\mathbf{I} \in \mathcal{I}_w$, $\mathbf{I}' \in \mathcal{I}_{w'}$, if $\mathbf{I}(v) = \mathbf{I}'(v)$ for some $v \in W$, then *either* neither **I** nor **I**′ is realized in u *or else* $\mathbf{I}(u) = \mathbf{I}'(u)$). Systems of modal unities can, however, be defined more generally as structures $\langle \mathbf{A}, \mathbf{B}, \mathfrak{C}, R, Int \rangle$, where $\mathfrak{C} = \{\mathbf{C}_{\mathbf{b}} : \mathbf{b} \in \mathbf{B}\}$ is a collection of subcovers of **A** indexed by elements of **B**, satisfying $|\mathbf{b} \cap \mathbf{c}| \leq 1$ for all $\mathbf{b}, \mathbf{b}' \in \mathbf{B}$

[17] Let X be a set, κ a cardinal number, and $\mathcal{C} = \{C_i : i < \kappa\}$ a collection of non-empty, not necessarily pairwise disjoint subsets of X. If $X = \bigcup_{i<\kappa} C_i$ and $X \subseteq Y$, then $\mathcal{C}$ is a *cover* of X and a *subcover* of Y. If $\mathcal{C}$ is a cover of X and the elements of the collection $\mathcal{C}$ are pairwise disjoint, then $\mathcal{C}$ is a *partition* of X and a *subpartition* of Y.

[18] If S is a set, I write $|S|$ for its cardinality.

[19] We could opt for a symmetric concept of **b** and **c** being 'co-realized'. However, I prefer to view **B** as providing the contexts in which elements of **C** may or may not be realized.

and $\mathbf{c} \in \mathbf{C}_{\mathbf{b}'}$.[20] In the general setting, contents must be relativized to elements of $\mathbf{B}$, a content over $\langle \mathbf{A}, \mathbf{B}, \mathfrak{C}, R, Int \rangle$ relative to $\mathbf{b}$ being a pair $\langle \mathbf{B}_0, \mathbf{C}_0 \rangle$, where $\mathbf{B}_0 \subseteq \mathbf{B}$ and $\mathbf{C}_0 \subseteq \mathbf{C}_{\mathbf{b}}$.

The transcendental interpretation of world lines poses the availability of individuals in the sense of modal unities as a necessary condition of our talking and thinking about individuals. The modal unities we call 'worlds' provide a medium relative to which we may consider the modal unities we call 'individuals'. One way of viewing the proposal the transcendental interpretation puts forward is along the lines of Kant's transcendental idealism. It was noted in Sect. 1.5 that taking our modal thoughts to be structured in terms of world lines can be compared with Kant's view, according to which appearances are structured in terms of pure intuitions of space and time. However, in order to motivate an idealist construal of the transcendental interpretation, we should find—over and above such a formal analogy—a role for world lines in relation to the cognitive operations that can be considered as rendering the experience of objects possible. In Kant's framework, there is actually a candidate that fits the bill in connection with sense experience: synthetic unities resulting from the combination of a manifold of intuition. According to Kant, sense experience gives us a plurality of representations. By themselves, such manifolds do not give rise to an experience of objects; Kant maintains that the concept of object is not derived from experience. At the same time, the concept of object cannot have the status of a category (a pure concept of the understanding): categories are only applicable to an experience structured in terms of objects. It is the cognitive operation of synthesis that puts different representations together and comprehends their manifoldness in one cognition [B 103]. What results from a synthesis is a 'synthetic unity', and it is such a synthetic unity that enables us to say that we have an experience of an object [A105, B130]. As examples of synthetic unities, Kant mentions a house viewed from different angles—the process of viewing takes time but gives rise to a spatial unity [A190/B235]—and a line in space that is cognized by drawing it, thereby synthetically bringing about a determinate combination of a given manifold [B138].

If anyone wanted to formally develop what Kant calls transcendental logic—proceeding from the manifold of sensibility and moving toward the applicability of the pure concepts of the understanding [A77/B102]—then the way in which *objects* should be represented in such a logic would be precisely in terms of world lines defined over the relevant manifolds. They constitute unities of the appropriate type

[20]We could go much further in generalizing the notion of system of modal unities, but the type of language to be used for talking about such systems imposes limits to what are reasonable generalizations. Modal operators do not carry syntactic variables, and they are evaluated in terms of binary relations among worlds, which is why we cannot end up evaluating a formula relative to n worlds for $n \geq 2$. Neither is the set of worlds accessible at a given world dependent on values of first-order variables. Thus, it would be pointless to replace $\mathbf{B}$ by a collection of subcovers of $\mathbf{A}$ indexed by elements of $\mathbf{C}$ or to replace the relation R (or, more generally, a collection of binary relations) by a collection of relations on $\mathbf{B}$ with arbitrary arities. When predicate symbols are interpreted extensionally, it suffices to define the function *Int* as above. Otherwise, its values should be defined as sets of tuples of elements of the set $\bigcup_{\mathbf{b} \in \mathbf{B}} \mathbf{C}_{\mathbf{b}}$.

in as straightforward a sense as one can hope for. This does not mean that Kant equates objects with unities of representations; it only means that this is how he takes objects to emerge in our experience (cf. Gardner [32, p. 101]). According to him, the manifold of an intuition is united in the concept of an object [B137], and it is only in terms of the concept of an object that experience of objects is possible.

2.7 Relation to Other Views

As indicated in Sect. 2.2, in order to defend the position according to which individuals are world lines, we must compare it with widely discussed ways of understanding individuals in modal settings. I will briefly comment on Kripke's view on the one hand and Lewis's view on the other hand. Further, I relate the transcendental interpretation of world lines to Fine's metaphysical theory of variable embodiments. Finally, I discuss different notions taken up in the literature that superficially resemble the concept of world line but should not be confounded with it.

2.7.1 Kripke's Stipulative Account

According to Kripke [69, 71], for any actual individual, we can envisage different scenarios in which this very individual appears—by *stipulating* that we are speaking of what might have happened to it. Kripke takes it that once we fix attention to an individual in this world, nothing prevents us from asking how *it* would behave in counterfactual situations. This is how he puts it [71, pp. 52–3]:

> I have the table in my hands, I can point to it, and when I ask whether *it* might have been in another room, I am talking, by definition, about *it*. . . . If I am talking about it, I am talking about *it*, in the same way as when I say that our hands might have been painted green, I have stipulated that I am talking about greenness.

Kripke contrasts his view with the idea that we may only speak of individuals as inhabitants of several worlds in terms of some sort of criteria of transworld identity—means of recognizing the same individual in different circumstances. He takes such an opposing view to suggest that counterfactual scenarios can only be considered purely qualitatively, so that speaking of an actual individual in a counterfactual situation w would require possessing means to locate it among the inhabitants of w on the basis of its properties. In Kripke's view, we talk about the table *directly*. When we reflect on the possibility for the table to be in another room, properties of the table need not be used for identifying it in counterfactual situations, nor need they be used to identify it in the actual world.

If information about the local features of a world is systematically insufficient for establishing those cross-world links that allow us to speak of an individual relative to a number of worlds (indeed, if world lines are independent of worlds), then the seemingly innocent idea of fixing attention to a locally manifested individual in a world and considering *it* elsewhere conceals the fact that such a stipulation has a conceptual precondition that is not met merely by having fixed attention to a local inhabitant of a world: a suitable world line must be given. Assuming the perspective on identity in modal settings formulated in this book, Kripke's account, with its recourse to the counterfactual behavior of stipulatively fixed individuals, presupposes what it is meant to clarify. As Hintikka stresses on many occasions, the conceptual confusion is deepened by phrasing the idea in linguistic terms, with reference to rigid designation (cf. [49, pp. 27–8]).

The very idea of rigid designation—the idea that such expressions as proper names refer to the same object in all worlds in which the object exists, unmediated by a sense representing properties of the object—requires that the referent be a sort of entity that can be found in several worlds.[21] As Hintikka points out, exponents of the idea of rigid designation appear to mix questions of reference with questions of identity (cf. [49, pp. 24–34], [51, 52]). The problem with the idea of rigid designation is not the suggestion that proper names refer directly without ascribing properties to their bearers. The problem is that the very notion of referring to the *same object* in distinct worlds presupposes the notion of cross-world identity. Yet, Kripke wishes to use the notion of rigid designation to clarify the meaning of statements involving individuals that appear in several possible worlds. That is, his strategy is question-begging: he attempts to employ a notion that presupposes the possibility of speaking of cross-world sameness to clarify the meaning of statements about cross-world sameness. The notion of reference is linguistic and world-relative—it is about interpreting non-logical symbols world by world. The notion of cross-world identity, again, does not presuppose recourse to language, and it is about world lines determining which objects in which domains realize the same individual. Correlating objects of distinct worlds is not a matter of reference—not a linguistic matter to begin with.

[21] In any event, this is the most natural way of understanding what Kripke says. Admittedly, if the referents of rigid designators were world-bound objects, there would by hypothesis be no issue of cross-world identity concerning them. Since Kripke rejects the domain constraint, he could speak of ascribing predicates to a world-bound object of world w relative to a distinct world w'. However, in fact, Kripke does not assume that referents of rigid designators are world-bound but allows them to be objects that exist in several worlds. Independently of this interpretive issue, we may note, as Kaplan [61, pp. 492–3] does, that Kripke characterizes his notion of rigid designator in mutually incoherent ways. At times, he says that a rigid designator refers to the same object in *all* worlds [71, p. 48]. At other times, he takes a rigid designator to refer to the same object in all *those* worlds in which the object exists ([68, p. 146], [71, p. 49]). The two formulations are equally problematic from the viewpoint of cross-world identity adopted in this book. Kripke's notion of rigid designator was anticipated in the work of Marcus [82]. She speaks of proper names as 'identifying tags' whose descriptive meaning is lost or ignored.

The notion of cross-world identity must, then, be secured before we can even attempt to define the notion of rigid designation.[22] This fact has repercussions on the possibility of the substitutional interpretation of quantifiers in modal logic (cf. Kripke [70]). One cannot explicate the semantics of a formula like $\exists x \Box P(x)$ by saying that its truth in w amounts to there being a rigid designator n such that $P(n)$ is true in all worlds accessible from w. No singular term can *be* a rigid designator unless it makes sense to speak of its referent in several worlds. The semantics of $\exists x \Box P(x)$ must clarify what renders it meaningful to speak of cross-world identity; this cannot be accomplished by resorting to conceptualizations that simply presuppose the meaningfulness of such a discourse. Therefore, quantification into modal contexts cannot be accounted for in terms of rigid designators.[23] Further, it would not help to turn attention to local objects. If quantification is understood objectually, the truth of $\exists x \Box P(x)$ in w does not amount to there being an object b of the domain of w such that this same object b, when considered in any world v accessible from w, is P in v. According to the analysis I am propagating, such transportation of objects to other worlds is impossible, and quantification into modal contexts must rely on world lines. In sum, the question of whether the 'horizontal' requirement of cross-world identity is satisfied cannot be approached in terms of the 'vertical' question of world-relative reference, nor by simply fixing attention to a local object.

The criticism levelled against Kripke's view on the above grounds is appropriate in the context of the transcendental interpretation of world lines. By contrast, the epistemic interpretation is indeed vulnerable to Kripke's critique against the qualitative view of individuals in modal settings. I have argued in Sect. 1.6 that Hintikka confuses the conceptual issue of individuation with the epistemic issue of reidentification and ends up speaking as if the epistemic capacity of recognition itself were a transcendental precondition of modal talk. This leaves the problem of cross-world identity unanswered or amounts to declaring that there is no such problem. Due to the ambiguity of Hintikka's motivations, it is understandable that his case against Kripke's position has not been perceived as being particularly strong.

Formally, Kripke's semantics of quantified modal logic can be seen as the result of denying the distinction between local objects and world lines and giving up the domain constraint adopted in world line semantics. Here is how Kripke's semantic framework can be obtained by modifying systems of modal unities as defined in Sect. 2.6. Given a set **A** of objects, let **B** be a cover of **A**. Elements of **A** will be individuals in Kripke's sense, while elements of **B** will be worlds. It will precisely

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[22] As was explained above, already the language-independent idea of considering how *this* object behaves in counterfactual circumstances presupposes the notion of cross-world identity. A fortiori, then, this same presupposition is involved in the idea of taking the actual referent of a linguistic expression as one's starting point and considering how this referent behaves in counterfactual circumstances. The linguistic detour cannot remove the heart of the problem, though it can serve to hide it. In particular, the identity of a proper name does not translate into the identity of its actual referent: it may be unproblematic to say that two occurrences of 'Hesperus' are two occurrences of the same name, but this linguistic fact has no bearing on the issue of whether it makes sense to say that 'Hesperus' refers to one and the same non-linguistic entity on the two occasions.

[23] For a critique of the substitutional interpretation applied in modal logic, see [49, p. 28], [51].

not be assumed that $\mathbf{B}$ is a *partition* of $\mathbf{A}$: distinct worlds may have elements of $\mathbf{A}$ in common. (Consequently, elements of $\mathbf{A}$ are not substantially speaking *local* objects.) Further, if $\mathbf{b} \in \mathbf{B}$, let $\mathbf{C_b}$ consist of singletons of elements of $\mathbf{b}$. That is, $\mathbf{C_b} = \{\mathbf{c} :$ there is $\mathbf{a} \in \mathbf{b}$ such that $\mathbf{c} = \{\mathbf{a}\}\}$. Consequently, each $\mathbf{C_b}$ is a very particular sort of subpartition of $\mathbf{A}$—namely, a subpartition whose cells are the singleton sets of elements of $\mathbf{b}$. We may consider elements $\{\mathbf{a}\}$ of $\mathbf{C_b}$ and elements $\mathbf{a}$ of $\mathbf{b}$ as two equivalent ways of representing Kripkean individuals existing in the world $\mathbf{b}$. Like in world line semantics, also here every individual has at most one realization in a given world: if $\mathbf{b}, \mathbf{b}' \in \mathbf{B}$ and $\mathbf{c} \in \mathbf{C_{b'}}$, then $|\mathbf{b} \cap \mathbf{c}| \leq 1$. Indeed, $|\mathbf{b} \cap \mathbf{c}| = 1$ iff $\mathbf{a} \in \mathbf{b}$, where $\mathbf{a}$ is the unique element of the set $\mathbf{c}$. In particular, every individual belonging to the domain $\mathbf{C_b}$ of the world $\mathbf{b}$ is realized in the world $\mathbf{b}$: if $\mathbf{c} \in \mathbf{C_b}$, then $\mathbf{c} = \{\mathbf{a}\}$ for some $\mathbf{a} \in \mathbf{b}$, and therefore, $|\mathbf{b} \cap \mathbf{c}| = 1$. On the other hand, in Kripke's framework, the same object $\mathbf{a} \in \mathbf{A}$ can be the realization of an individual in several worlds: if $\mathbf{a} \in \mathbf{b} \cap \mathbf{b}'$, then $\{\mathbf{a}\} \in \mathbf{C_b} \cap \mathbf{C_{b'}}$, and indeed, $\mathbf{a}$ is the realization $\{\mathbf{a}\}$ in both worlds $\mathbf{b}$ and $\mathbf{b}'$. Predicate symbols are interpreted relative to worlds, but it is not required that the interpretation $Int(Q, \mathbf{b})$ of an n-ary predicate Q in $\mathbf{b}$ be a subset of $\mathbf{b}^n$. It is merely required that $Int(Q, \mathbf{b})$ be a subset of $\mathbf{A}^n$. Thereby, the domain constraint is given up. Kripke's semantics can be obtained as a variant of world line semantics based on systems of modal unities $\langle \mathbf{A}, \mathbf{B}, \mathfrak{C}, R, Int \rangle$ whose components are specified as above, with $\mathfrak{C} = \{\mathbf{C_b} : \mathbf{b} \in \mathbf{B}\}$. By contrast, world line semantics cannot be obtained as a variant of Kripke's semantics, since the latter does not make the requisite distinction between extensional identity and cross-world identity and correspondingly fails to distinguish between local objects and world lines. In particular, Kripke's framework cannot simulate the distinction between availability and realization, essential for my analysis of intentional objects understood as intentionally individuated world lines that can lie in the range of intentional quantifiers in a world without being realized in that world.[24]

2.7.2 Lewis on Counterparts and Humean Supervenience

In this book, I do not explore ways of construing the notion of world line epistemically: my focus is neither on the epistemic nor on the anti-realist interpretation of world lines. Still, on the face of it, at least, thinking of individuals as world lines *need* not be motivated by the transcendental interpretation. It was already hinted at in Sect. 2.4 that nothing prevents us from considering world lines *metaphysically*. Thus understood, they give rise to a view of individuals generalizing four-dimensionalism.

Lewis [76] is a four-dimensionalist who takes the basic entities to be temporally extended objects with temporal parts. In his analysis, the quantifiers 'something' and 'everything' range over world-bound cross-temporal hybrids whose identity over time is taken to be utterly simple and unproblematic. Concerning the similarities between Lewis's view and the approach I adopt, it was already noted in Sect. 1.3

[24] Cf. the discussion in Sect. 1.3. For details, see Chap. 3.

that *local objects* in my sense are world-bound, like Lewis's *individuals*. If attention is confined to a single temporally extended world—so that the 'contexts' to be considered are pairs of structured worlds and times—*individuals* in my sense are four-dimensional, like *individuals* in Lewis's sense, and *local objects* are bound not only to a structured world but also to a time, like *temporal parts* of individuals are according to Lewis.

Lewis's account of statements pertaining to temporally extended individuals within one and the same world is categorically different from his analysis of statements that seem to be about one and the same individual in distinct possible worlds. He has one thing to say about cross-temporal identity and another story to tell about seeming cases of cross-world identity. He takes the former relation to be utterly simple and unproblematic but denies the reality of the latter. To compensate for the denial, Lewis develops his modal counterpart theory, which is supposed to provide an analysis of representation *de re*—an account of how the modal properties of a temporally extended individual of one world can be represented in terms of other worlds [75, 76]. Lewis's individuals remain within one world and may merely have *counterparts* in other worlds. According to my view, individuals have a modal margin, comprising not only various spatiotemporal locations within a single structured world but also locations in distinct structured worlds. I take cross-context identity to be a notion in need of analysis, and the analysis I give is uniform: modal behavior is generally inbuilt within the individual itself, not just its temporal behavior. Cross-temporal identity within a fixed structured world is no less problematic than identity across structured worlds. The preservation of identity over any sorts of contexts is uniformly analyzed in terms of world lines.[25]

Kripke [71, p. 45] criticizes Lewis's counterpart theory, according to which the sentence 'Humphrey might have won the election' is true in the actual world if in a counterfactual world a certain numerically distinct individual—namely, a counterpart of Humphrey—indeed wins the election. Kripke finds this a doubtful analysis, as according to him, Humphrey presumably could not care less that another person would have been victorious in another possible world. Kripke takes it that the qualitative similarity between Humphrey and his counterpart would not render the success of this counterpart any more interesting from Humphrey's viewpoint. This type of critique is not available against world line semantics. In my analysis, the sentence 'Humphrey might have won the election' is about a certain (physically individuated) world line—Humphrey the physical individual. Writing $\mathbf{J}$ for Humphrey, the sentence means that $\mathbf{J}$ is realized not only in the actual world w_0 but in another world w, as well, and the realization $\mathbf{J}(w)$ of $\mathbf{J}$ satisfies the predicate '— wins the election'. Winning the election is a contingent property of Humphrey himself, a property *he* has in some but not all worlds. Humphrey (the world line $\mathbf{J}$) must absolutely not be

[25] Hawley [39] calls attention to the fact that in Lewis's analysis, the analog between time and modality is not complete (ordinary objects are world-bound but not time-bound) and discusses the possibility of achieving uniformity by explicating not only sameness across worlds but also sameness over time in counterpart-theoretic terms. In Sider's stage theory [111], statements about identity over time are indeed accounted for in terms of temporal counterpart relations.

confused with this or that realization of the world line **J**—in particular, not with the realization $\mathbf{J}(w_0)$.[26]

Lewis [77, 78] defends the thesis of Humean supervenience. One way of attempting to concisely formulate this thesis would be as follows: all facts, other than facts about spatiotemporal distance, supervene upon local matters of particular fact.[27] This thesis leads to what Lewis calls anti-haecceitism [76, p. 221]: counterpart relations between denizens of distinct worlds are supervenient on features internal to worlds.[28] Indeed, any world-bound individuals that qualitatively resemble one another are thereby counterparts of one another. *All facts* include modal facts—in particular, facts about the modal behavior of actual individuals. Now, these modal facts rely on representations *de re* articulated in terms of counterpart relations. If these facts also supervene on local matters of fact (i.e., matters of fact concerning each world taken by itself), it follows that counterpart relations among worlds are determined as soon as a set of worlds and the geometrical arrangement of their space-time points are fixed: counterpart relations supervene on local features of the worlds. Were this not so, modal facts could vary while the distribution of qualitative properties and relations within worlds remains the same, which would contradict anti-haecceitism. Lewis says that he intends the thesis of Humean supervenience to be *contingent* [78, p. 474]. (For a discussion, see Hall [37].) Indeed, if in some worlds, though perhaps not in the actual one, we must take into account irreducible *non-spatiotemporal relations* as constitutive of the structure of the world, then Humean supervenience, as formulated above, can at best be true of worlds in which, by chance, such relations are not instantiated.

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[26]For contingently satisfied predicates, see Sect. 4.2. The fact that Kripke's Humphrey-objection is not applicable against my analysis does not depend on how we choose to deal with proper names in our semantics. In this book, I opt for construing proper names as standing for local objects relative to a world: for every world w in the modal margin of **J**, the interpretation of 'Humphrey' in w equals $\mathbf{J}(w)$. Another option would be to construe proper names intensionally, letting 'Humphrey' to stand for the world line **J**. For a discussion, see Sect. 3.4 and footnote 29 in Sect. 6.6.

[27]See Bricker [7, p. 287]. Lewis considered different versions of the thesis. Arguably, his final formulation [78] does not even count as a supervenience thesis (cf. Weatherson [124]).

[28]Lewis defines *haecceitism* as the doctrine that there are at least some cases of 'haecceitistic difference' between worlds: there are worlds that do not differ qualitatively in any way but differ in what they 'represent *de re*' concerning some individual. According to *anti-haecceitism*, there are no cases of haecceitistic difference. Haecceitism is, for instance, compatible with there being the worlds w_0, w_1, and w_2 satisfying the following conditions: (1) each world w_i has exactly two inhabitants (a_i and b_i); (2) individual a_1 is P but b_1 is not P, while a_2 is not P though b_2 indeed is P; and (3) both individuals a_1 and a_2 are counterparts of a_0. The worlds w_1 and w_2 are qualitatively exactly alike, but w_1 represents *de re* concerning a_0 that it is P, while w_2 represents *de re* concerning a_0 that it is not P. Lewis [76, p. 225] maintains that a haecceitist need not accept that an individual is distinguished from all other individuals by a *haecceity*—an unanalyzable non-qualitative property that this individual has and all other individuals lack. In the example, haecceitism without haecceities would mean that not only is the fact that a_1 and a_2 are counterparts of a_0 not triggered by qualitative properties of the three worlds w_0, w_1, and w_2, but this is a primitive fact not determined by any property at all that would belong to the individuals a_0, a_1, and a_2. It must be noted that Lewis's characterization of haecceitism is not neutral but depends on the idea of *de re* representation conceptualized in terms of counterparts of world-bound individuals.

The position I defend is utterly antithetical to Humean supervenience—and, more generally, to any view entailing that all facts about a structured world supervene on properties of world-bound objects or on relations among world-bound objects within a fixed world, no matter what the specific nature of those relations is. There are two conceptually independent factors to consider: worlds and world lines. Precisely because these factors are mutually independent, it will not be generally possible to provide an account of what unifies world lines by making use of world-internal features—just like it will not be generally possible to analyze in terms of features of world lines what unifies worlds.[29] My view does not exclude the possibility that world lines indeed supervene on local properties: this is what would happen if the remarkable metaphysical coincidence referred to in Sect. 1.3 was actualized and, as a matter of contingent fact, world lines were not independent from worlds. Assuming Humean supervenience as a *fait accompli* would in any event be theoretically harmful. It would lead attention away from the fact that worlds and world lines are conceptually independent of each other.

Lewis's counterpart theory can be seen as the result of supposing that world lines are in fact generated by world-internal characteristics—by qualitative resemblance of world-bound objects. Formally, here is how Lewis's semantic framework can be simulated in the context of world line semantics.[30] Consider a system of modal unities $\langle \mathbf{A}, \mathbf{B}, \{\mathbf{C_b} : \mathbf{b} \in \mathbf{B}\}, R, Int\rangle$, where $\mathbf{B}$ is a partition of $\mathbf{A}$. While this assumption has merely interpretative but not formal consequences, let us suppose that it so happens that the sets $\mathbf{c} \in \mathbf{C_b}$ are supervenient on elements of $\mathbf{B}$. We suppose that distinct local objects $\mathbf{a}, \mathbf{a}' \in \mathbf{c}$ resemble each other qualitatively; world-internal qualitative features of local objects determine the elements of the sets $\mathbf{C_b}$ with $\mathbf{b} \in \mathbf{B}$. It will be further assumed that every local object $\mathbf{a}$ of world $\mathbf{b}$ is the realization of some world line $\mathbf{c} \in \mathbf{C_b}$ available in $\mathbf{b}$, so that $\mathbf{c} \cap \mathbf{b} = \{\mathbf{a}\}$. Let us use the sets $\mathbf{C_b}$ with $\mathbf{b} \in \mathbf{B}$ for defining a relation CP on the set $\mathbf{A}$ as follows: if $\mathbf{a} \in \mathbf{b}$ and $\mathbf{a}' \in \mathbf{b}'$, where $\mathbf{b}$ and $\mathbf{b}'$ may but need not be distinct worlds, let $CP(\mathbf{a}, \mathbf{a}')$ iff there is $\mathbf{c} \in \mathbf{C_b}$ such that

[29] World lines should not be viewed as generated by any sorts of properties—in particular, not by anything qualifiable as 'haecceities'; cf. the comments on essences in Sect. 2.7.4. My position is certainly closer in spirit to haecceitism without haecceities than to anti-haecceitism—although literally, my position is anti-haecceitist by Lewis's criteria, since I maintain that there are no cases of haecceitistic difference between worlds, for the simple reason that I maintain that there are no Lewisian *de re* representations in the first place. World lines are not supervenient on world-internal facts, so there could be a world line $\mathbf{I}$, worlds w_1 and w_2, and local objects $a_1 \in dom(w_1)$ and $a_2 \in dom(w_2)$ such that as to their internal qualitative features, w_1 and w_2 are exactly alike, the local object a_1 is P while a_2 is not P, and yet a_1 and a_2 could both be realizations of the world line $\mathbf{I}$. (For 'exact likeness', cf. the notion of internal indistinguishability discussed in Sect. 4.5.) Even though my view resembles haecceitism, I do not subscribe to haecceitism in *Lewis's sense*: the world w_2 does not involve a representation *de re* concerning the local object a_1 in virtue of the fact that $a_2 = \mathbf{I}(w_2)$ and $a_1 = \mathbf{I}(w_1)$. Local objects or the worlds in which they are located do not in any sense represent local objects to be found in other worlds. What may but need not happen is that local objects of two worlds are *realizations* of one and the same world line.

[30] In Sect. 5.7, I explain how my modal language L_0 (and its extension L, to be introduced in Sect. 3.4) can be translated into first-order logic. This translation makes it particularly easy to observe the formal similarities between world line semantics and Lewis's counterpart theory—the latter being standardly presented in terms of first-order logic, cf. [75].

$\mathbf{a} \in \mathbf{c}$ and $\mathbf{a}' \in \mathbf{c}$. (Note that if $\mathbf{b} = \mathbf{b}'$, we can have $\mathbf{a} \in \mathbf{c}$ and $\mathbf{a}' \in \mathbf{c}$ only if $\mathbf{a} = \mathbf{a}'$.) The terms of the relation CP are, then, local objects belonging to one and the same world line available in $\mathbf{b}$—the same element $\mathbf{c}$ of $\mathbf{C_b}$. The relation CP is reflexive: if $\mathbf{a} \in \mathbf{b}$, there is $\mathbf{c} \in \mathbf{C_b}$ such that $\mathbf{c} \cap \mathbf{b} = \{\mathbf{a}\}$, whence we have $CP(\mathbf{a}, \mathbf{a})$. The relation CP is not symmetric: having $CP(\mathbf{a}, \mathbf{a}')$ for local objects $\mathbf{a} \in \mathbf{b}$ and $\mathbf{a}' \in \mathbf{b}'$ merely requires that there be $\mathbf{c} \in \mathbf{C_b}$ such that $\mathbf{a} \in \mathbf{c}$ and $\mathbf{a}' \in \mathbf{c}$, but it need not happen that $\mathbf{c}$ is available in $\mathbf{b}'$, as well: we may have $\mathbf{c} \notin \mathbf{C_{b'}}$. The relation CP is not transitive, either—again, because distinct sets of world lines may be available in distinct worlds.[31] These observations are enough to show that the world line framework allows accommodating counterpart-theoretic ideas.[32] By contrast, world line semantics cannot be obtained as a variant of Lewis's counterpart semantics, since the latter assumes that counterpart relations are generated by qualitative resemblance among world-bound individuals, while in world line semantics, worlds and world lines are taken to be, generally, mutually independent and cross-world identity is taken to be a notion that defies all attempts of analysis in world-internal terms. In particular, counterpart theory cannot simulate the distinction between availability and realization. In Lewis's account, individual-valued quantifiers evaluated relative to a world w always take as their values world-bound individuals of the world w. Counterparts enter the picture only insofar as modal claims concerning those inhabitants of the world w are being made. By contrast, in my analysis, intentionally individuated world lines can lie in the range of intentional quantifiers in a world w without being realized in the world w.

[31] Lewis resorts to non-transitive counterpart relations in his reply to Chisholm's identity paradox (see [14], [76, pp. 243–8]). The paradox can be presented as follows. Take two individuals existing in w_0—say, Adam and Noah. Suppose Adam has properties $P_1, \ldots, P_n$ and Noah the properties $Q_1, \ldots, Q_n$ in w_0. (For simplicity, let us suppose that the properties $P_1, \ldots, P_n, Q_1, \ldots, Q_n$ are pairwise independent.) Presumably, at least some of Adam's and Noah's properties could get exchanged. If so, by repeated exchanges, we arrive at a sequence of worlds $w_0, \ldots, w_n$ such that in w_i, Adam has the properties $Q_1, \ldots, Q_i, P_{i+1}, \ldots, P_n$ and Noah the properties $P_1, \ldots, P_i, Q_{i+1}, \ldots, Q_n$ (for all $1 \leq i \leq n$). In w_n, Adam's properties are those of Noah in w_0, and vice versa. Supposing there are no other individuals in w_0 and w_n, these worlds are qualitatively exactly alike, so individuals appear to have a 'bare identity' entirely unrelated to their properties. This suggests that for any property and any individual, there is a world in which the individual has this property. Lewis blocks this reasoning by appealing to the idea of world-bound individuals. Adam and Noah are located in w_0. Even if there were individuals a_1 of w_1 and a_2 of w_2 such that a_1 is a counterpart of Adam and a_2 is a counterpart of a_1—with a_1 having the properties of Adam except for P_1, and a_2 having the properties of Adam save for P_1 and P_2—we cannot infer that a_2 is a counterpart of Adam, since the counterpart relation need not be transitive. From my viewpoint, Adam is a world line. He is realized in w_0. He has a fixed modal margin, which may or may not contain the worlds $w_1, \ldots, w_n$. World lines are not supervenient on qualitative world-internal features. Whatever (extensional) predicates a local object of a world w_i satisfies, it need not be the realization of Adam in w_i. This is why Chisholm's reasoning cannot be carried out when world line semantics is assumed.

[32] Note that Lewis does not preclude the possibility of an individual having several counterparts in a given world. Unless it is required that every $\mathbf{C_b}$ with $\mathbf{b} \in \mathbf{B}$ be a partition of $\mathbf{A}$, we can indeed have $CP(\mathbf{a_0}, \mathbf{a_1})$ and $CP(\mathbf{a_0}, \mathbf{a_2})$ with $\mathbf{a_1} \neq \mathbf{a_2}$, where $\mathbf{a_1}, \mathbf{a_2}$ belong to the same element $\mathbf{b}'$ of $\mathbf{B}$. Namely, there can be $\mathbf{b} \in \mathbf{B}$ with $\mathbf{b} \neq \mathbf{b}'$, and distinct $\mathbf{c_1}$ and $\mathbf{c_2}$ in $\mathbf{C_b}$, such that $\mathbf{a_0} \in \mathbf{c}_1 \cap \mathbf{c}_2 \cap \mathbf{b}$, while $\mathbf{a_1} \in \mathbf{c}_1 \cap \mathbf{b}'$ and $\mathbf{a_2} \in \mathbf{c}_2 \cap \mathbf{b}'$.

2.7.3 Fine's Notion of Variable Embodiment

In Lewis's metaphysics, it is the postulation of Humean supervenience that allows him to avoid recognizing world lines as an independent component of modal reality. Surveying recent metaphysical literature, there is one proposal that is formally analogous to the idea of construing individuals as world lines: Kit Fine's theory of *variable embodiments* [25]. Fine formulates his view in a temporal setting relative to a single world, but he definitely rejects Humean supervenience.

Fine's goal is to sketch a theory of the general nature of material things. He proposes a novel way of thinking about mereology. According to him, there are two operations by means of which wholes can be formed from parts: one operation produces *rigid embodiments* and the other *variable embodiments*. The former operation is supposed to explain the mereological structure of an object at a time. The latter is meant to account for the variation of an object over time. A rigid embodiment is a special sort of hylomorphic entity consisting of a number of objects (its matter) interrelated according to a fixed relation (its form, a principle of rigid embodiment). The matter is given independently of the form. An example is a ham sandwich: a piece of ham and two slices of bread spatiotemporally arranged in a certain way. *Principles of variable embodiment* are of special interest here. Such a principle F is a function mapping times to things, and it determines an object $/F/$, referred to as a *variable embodiment*. Values of F are *manifestations* of $/F/$. Typically, such manifestations are rigid embodiments.[33] The variable embodiment $/F/$ does *not* supervene on its manifestations. In fact, the matter of a variable embodiment (the plurality of its manifestations) is specified by its form (the principle of variable embodiment) instead of being available independently. An example is a car with a varying constitution: at each time, it consists of an engine, chassis, and body related in a certain way, but neither the matter nor the form of one manifestation need be transferred to another manifestation: at different times, the constituents of the car may be differently related, and besides, the constituents may vary. For example, the carburetor of the car at t_1 may be replaced by a new one at t_2.

Fine develops his position as a response to problems he detects in traditional mereology. He maintains that standard mereology is incapable of accounting for the notions of timeless part and temporary part. First, suppose a ham sandwich has two slices of bread (s_1 and s_2) and a piece of ham (h) as its *timeless parts*. Each of the three components is itself a temporally extended object. According to standard mereology, the sum $s_1 + s_2 + h$ exists, at all instants at which at least one of its components exists. It would follow, absurdly, that the sandwich exists as soon as the relevant piece of ham does, even if the slices of bread are yet to come into existence. Second, supposing that objects have time-slices (instantaneous timeless parts), it could be proposed that x is a *temporary part* of y at t if the time-slice x_t of x at t is a timeless part of the time-slice y_t of y at t. It does not help the mereologist to restrict attention to instants

[33] Fine [25, p. 73] allows even variable embodiments as values of F (cf. Koslicki [63, p. 78]).

at which all three parts of the ham sandwich exist simultaneously. Namely, consider the sum $h + c$ of the piece of ham and Cleopatra (c). The sum exists according to the standard mereology, which subscribes to unrestricted mereological composition. Not only is h a temporary part of the sandwich at t—even $h + c$ is one, because $(h + c)_t = h_t$ is a timeless part of the time-slice of the sandwich at t. The conclusion appears absurd: among the temporary parts that the ham sandwich has at t, there is a certain object with Cleopatra as one of its parts. Similar absurdities result if, instead of a temporally relatively stable object, like a ham sandwich, we consider an object with a variable constitution, such as a car with one carburetor at one time and another one at another time.

Formally, variable embodiments are related to their manifestations exactly as world lines are related to their realizations. Principles of variable embodiment are related to variable embodiments as partial functions representing world lines are related to world lines themselves. Like variable embodiments, world lines also have both a formal and a material aspect. The matter of a world line is what gets realized in various worlds, its form being the fact that the relevant local objects jointly constitute a single world line. Unlike a principle F, the corresponding variable embodiment $/F/$ is not an abstract object. It can have properties of the sort concrete objects have. Fine stipulates that the *pro tem* properties of $/F/$ at a time t are those of its manifestation $F(t)$ at t. This sort of 'transfer principle' is likewise built into the semantic clause for atomic formulas of the language L_0: saying that a world line satisfies a predicate P in w means that its realization in w satisfies P in w. Formally, the main difference between variable embodiments and world lines is that the manifestation $F(t)$ of a variable embodiment $/F/$ at t need not be local in any way: typically, it is a rigid embodiment composed of suitably arranged temporally extended objects (and it may even itself be a variable embodiment), whereas realizations of a world line at t are temporally limited to the instant t. I do not enter into systematic mereological discussions in this book. However, my framework could be generalized by allowing local objects to have a mereological structure.[34] Parts of local objects would themselves be local.

Fine's notions of composition are supposed to avoid absurdities of the type described above. The ham sandwich as a rigid embodiment consists of the two slices of bread and a piece of ham arranged in a certain way, and this structured object exists only when s_1, s_2, and h are thus related; it does not suffice that one or more of these objects exist. In his theory of variable embodiment, Fine formulates a somewhat elaborate definition of temporary part (in terms of manifestations and a relation of simultaneity among parts), which allows him to disqualify the combination of Cleopatra and the piece of ham as a temporary part of the ham sandwich, retaining, however, the piece of ham as its temporary part. In my setting, the ham sandwich is a world line $\mathbf{I}$ over an interval of time, and there are three further world lines involved: slices of bread $\mathbf{J}_1$ and $\mathbf{J}_2$ and a piece of ham $\mathbf{J}_3$. If my framework was generalized so as to allow speaking of timeless parts of local objects, it would become possible to

[34] For further remarks in this direction, see Sect. 4.6, esp. footnote 17.

analyze the interrelations of the four world lines: each of the $\mathbf{J}_i$ is realized at every instant t at which $\mathbf{I}$ is realized, the local object $\mathbf{J}_i(t)$ being a timeless part of $\mathbf{I}(t)$. As Fine's analysis, this view does not suggest that the world lines $\mathbf{J}_1$, $\mathbf{J}_2$, and $\mathbf{J}_3$ being realized at t is a sufficient condition for the realization of $\mathbf{I}$ at t. Concerning the case of monster objects like Cleopatra combined with a carburetor, I think we had better deny their existence altogether. Indeed, if there exists a variable embodiment manifested initially as Cleopatra and later on as a carburetor of a car—this much Fine grants without hesitation—it should not be particularly shocking that the car has the hybrid of Cleopatra and the carburetor as its temporary part during the life span of the carburetor. Fine takes the absurdity of the latter claim as a reason to reject traditional mereology. However, it is already absurd to countenance the existence of some one thing that is partially Cleopatra and partially a carburetor.

Kathrin Koslicki [63] criticizes Fine's view for proliferation of *sui generis* relations of composition—those producing rigid embodiments and variable embodiments, plus arbitrary hierarchical combinations of compositions of these two types. She also criticizes his theory for its commitment to a superabundance of objects—objects generated from already available objects by ever more involved compositions.[35] I think Koslicki's criticism is justified. At any point in space, there could be an object, but that does not mean there is one at every point. A function over points in space could describe the spatial form of an object in space, but that does not mean there is an object thus described. I do *not* assume that for every selection of local objects, one object per world, there is a corresponding world line (unrestricted cross-world composition of local objects), *nor* that for every selection of local objects within a fixed world, there is a further local object having those objects as parts (unrestricted world-internal composition of local objects), *nor* that every selection of world lines with pairwise disjoint modal margins can be conjoined into a single world line (unrestricted composition of world lines). The question of which individuals are available to be talked about in which world cannot be settled on a priori grounds—not even conditionally to a prior specification of the set of all relevant worlds. This is why each model of L_0 has a domain of world lines as a separate component. Additional world lines over the same set of worlds are perfectly conceivable, but this does not mean that all conceivable world lines should be present in every model.

Rejecting unrestricted composition of world lines is compatible with world lines being divisible into component world lines. In particular, one world line could be divisible in many ways. Whenever there are such divisions, there is of course the corresponding composition. If so, some world lines can be obtained by composition from other world lines. Further, rejecting the above-mentioned two forms of unrestricted composition of local objects is compatible with local objects being divisible.

[35] Priest [95, pp. 46–7] takes 'objects' to be functions from worlds to 'identities' (see Sect. 3.8 below) and notes that in his framework, there is a risk of a similar superabundance problem. If d is an object and $d(w)$ is its identity in w, can $d(w)$ be viewed as a *part* of d at w? Priest discusses this idea but dismisses it because he sees it as giving metaphysical priority to identities, and he takes it that this would lead to an uncontrolled proliferation to objects: any function from worlds to identities would count as one.

Koslicki proposes to restrict allowed compositions and divisions by postulating an ontology of kinds. This is a reasonable approach if one interprets world lines metaphysically, as Fine does. I prefer to understand world lines in accordance with the realist version of the transcendental interpretation. According to this position, if the external objects we can meaningfully talk about are temporally extended and have modal properties, we must think of them as world lines. This view does not lead us to postulate any more world lines than those we employ in thinking about the reality. Since our doing so need not involve arbitrary compounds of world lines, nor arbitrary compounds of local objects, this position gives no support to any form of unrestricted composition.

2.7.4 What World Lines Are Not

I have explained how the transcendental interpretation of world lines formulated in Sects. 1.5 and 2.2 differs from alternative ways of construing this idea: epistemic (Sect. 1.6), anti-realist (Sect. 1.6), and metaphysical (Sects. 2.7.2 and 2.7.3) interpretation. There are certain notions that are much discussed in the literature and bear some resemblance to the notion of world line but should not be confounded with it. Since the risk of confusion is real, it is worthwhile to spend some words on what world lines are *not*.

It was noted in Sect. 1.3 that in semantics, world lines (individuals) can be represented by partial functions whose arguments are worlds and whose values are world-bound local objects. Note that the values of these functions are not individuals but local objects—possible realizations of individuals. Now, it is neither necessary nor sufficient for cross-world identity that cross-world identical local objects satisfy a given descriptive condition. Consequently, it would be a mistake to suppose that a partial function induced by a world line must be defined by some descriptive condition that all values of the function satisfy. Conversely, if we take an arbitrary partial function from worlds to corresponding local objects, there is no reason to think that this function is induced by some world line. In particular, even if all values of such a function satisfy some descriptive condition, this does not guarantee that the values (which are local objects) are realizations of one and the same world line.[36]

Let us proceed to consider concrete examples of much-discussed notions that one might misconstrue as being capable of taking up the conceptual role of world lines.

[36]For a blatant example, the condition expressed by the definite description ‘the president of the US’ picks out, in every world in which it is applicable at all, the realization of a unique individual, but these different realizations do not belong to any one individual: no world line is first manifested as (a realization of) Bill Clinton and later on as (a realization of) George Bush Jr. This definite description defines a certain partial function from worlds to local objects, but there is no reason to assume that this function could be a value of a quantified variable—i.e., that it corresponds to a world line. Cf. footnote 38 in Sect. 6.7.

Following Carnap [12], the expression 'individual concept' is often used in the philosophy of language for a function that selects for every context (out of some relevant class of contexts) an individual as the referent of a given singular term, such as a proper name or a definite description. In the special case of proper names, such functions are usually taken to be constant functions—that is, they are taken to select the same individual as the referent of the name in each relevant context. One might be tempted to think that world lines in my sense are such individual concepts. This would be erroneous for several reasons. First, unlike the notion of individual concept (individual concept *of* a singular term), the notion of world line is not language-relative. Intensions of singular terms are of interest for semantic reasons, but the mere fact that someone introduces a novel singular term into our language does not mean that thereby *there are* novel things to talk about—i.e., does not create new values for (physical or intentional) quantifiers to range over. Second, in order for an individual concept of a proper name to be able to assign the *same* individual as the referent of the name in two contexts, the domains of these contexts must have elements in common—this being a nonsensical idea, according to my analysis. Third, while it is indeed formally correct that a world line, like an individual concept, is a function assigning to any context on which it is defined an element of the domain of this context, it is just as much correct formally that values of individual concepts are always individuals, whereas values of world lines (that is, their realizations) are never individuals. For construing world lines as individual concepts, see, e.g., Kraut [65].[37]

World lines are not essences of any kind.[38] If $\phi(x)$ is a unary intensional predicate, let us say that a world line $\mathbf{I}$ satisfies $\phi(x)$ *necessarily*, if for every world w in which $\mathbf{I}$ is realized, $\mathbf{I}$ satisfies $\phi(x)$ in w.[39] There are at least two ways in which we may use the expression 'essence' in my framework if we so wish, but essences in neither sense have any role in clarifying the notion of cross-world identity. First, we might take the essence of $\mathbf{I}$ to be the totality of those predicates that $\mathbf{I}$ satisfies necessarily. A more refined idea would be to identify the essence of a world line $\mathbf{I}$ with the function that assigns to every world w in $marg(\mathbf{I})$ the set of predicates that $\mathbf{I}$ satisfies in w.[40] If for all worlds w in which a physical object is realized, we have available full information about the predicates it satisfies in w, then we can read off its essence in either sense. It must be noted that essences of either sort can by no means be substitutes for

[37] Also, Kracht and Kutz [66] assimilate world lines to what they call individual concepts, but in the sense in which they take world lines to be individual concepts (world lines being extracted from counterpart relations), these individual concepts could not be constant functions.

[38] A haecceity can be viewed as a *trivial* individual essence, as opposed to an informative or *non-trivial* individual essence—a set of qualitative properties whose possession by the individual would be a necessary and sufficient condition for its being the individual it is. It was remarked in footnote 29 of this chapter that world lines are not haecceities. See also footnote 28 in this chapter.

[39] In the special case that $\phi(x)$ is atomic, this amounts to a condition concerning the *realizations* of $\mathbf{I}$. The world line $\mathbf{I}$ is necessarily $P(x)$, if for all worlds w in the modal margin of $\mathbf{I}$, the realization $\mathbf{I}(w)$ of $\mathbf{I}$ belongs to the interpretation of P in w.

[40] This is basically how Plantinga [94, pp. 76–7] defines the notion of essence.

world lines. They cannot in any way generate or determine world lines. There is no guarantee that a given set of (world-indexed) predicates is the essence of a world line, although conversely, a given world line automatically has an essence in both senses. Kripke [71, p. 114] understands the essence of a physical object in terms of material constitution and causal continuity, in particular with reference to the way in which the material constitution of the object has evolved over time from a specific origin. It might well turn out that essences of physical objects (in one or both of the above senses of 'essence') can be profitably discussed in terms of the notions of material constitution and spatiotemporal continuity. However, while these notions might indeed play an important role in the characterization of the *epistemic* task of recognizing that two local objects are realizations of the same physically individuated world line, they cannot be of the slightest use in explicating the notions of world line and cross-world identity. Namely, in order to make sense of the very ideas of the origin of a physical object and its evolution over time, the notion of cross-temporal sameness must already be presupposed. Generally, nothing we can say about local objects and their possible mereological structure at this or that time, will be enough to deliver the requisite notion of cross-contextual sameness. Piecewise information about worlds cannot, generally, generate information about world lines. World lines are independent of worlds.

It may be useful to stress separately that world lines must not be equated with Fregean senses of singular terms. A Frege-inspired philosopher may speak of an agent as thinking of the planet Venus under the sense (mode of presentation) associated with the description 'the morning star'. Understood as an individual concept, this sense picks out the unique morning star from each context in which one is available. There is no valid analogy to the case of world lines: there is no reason whatsoever to think that one and the same description would uniformly apply to each and every realization of a world line.

World lines are not criteria of identity in the epistemic sense: their role is not to provide us means to recognize an individual in different circumstances. In searching for an answer to the question 'What is a criterion of identity?', E. J. Lowe [80, pp. 12–3] similarly stresses that what he calls criteria of identity are not epistemic or heuristic principles for settling questions of identity concerning individuals. His positive view is that criteria of identity are *semantic* principles governing the meaning of certain sorts of *general* terms.[41] By contrast, world lines are not semantic in character; they are language-independent. Besides, the linguistic items whose semantics can be explicated with reference to world lines are not predicates but quantifiers.

[41]Lowe makes much of Frege's discussion [29, Sects. 62–9] of identity criteria (*Kennzeichen*) in connection with the mathematical practice of defining abstract entities (like directions) as equivalence classes of somewhat less abstract entities (like lines). This leads Lowe to postulate that various sortal terms Φ have an associated 'criterial relation' R so that whenever x and y satisfy Φ, we have $x = y$ iff $R(x, y)$. This is a dubious generalization, since here, identity is applied to entities of the same logical type as those to which the criterial relation is applied, while Frege applies identity to *sets* of lines (directions) and the criterial relation—parallelism—to lines themselves.

Given that world lines cannot be construed as Carnapian individual concepts or as descriptive modes of presentation, one might insist that actually the most natural notion of individual concept is the notion of a *de re* sense formulated by Evans [24] and McDowell [84]. Such *de re* senses are *non-descriptive* modes of presentation, but they count as Fregean senses by being constituents of thoughts and by determining an object as their denotation. However, world lines cannot be analyzed as *de re* senses either, because the mere fact that objects o_1 and o_2 are determined as denotations of one and the same *de re* sense on two occasions does not guarantee that they are one and the same object. Rather, the very idea of determining the same object in different circumstances presupposes the notion of cross-world identity. It is a part of the notion of *de re* sense that it determines the same individual on a variety of occasions—in this respect, *de re* senses behave just like Carnapian individual concepts associated with proper names. It is not up to a mode of presentation—descriptive or not—to constitute the notion of cross-world identity. Modes of presentation and senses are means of having access to entities that are already there. They cannot provide an analysis of what it means to be the same individual. They can only provide means of accessing individuals whose cross-world identity is independently secured.

Given that I am not putting forward a metaphysical understanding of world lines, it is useful to make clear in what sense this viewpoint is and in what sense it is not ontologically committing. As will be seen in Chap. 3, we must distinguish between two ways of individuating world lines: the physical and the intentional. Talking about physically individuated world lines (physical objects) is ontologically committing, whereas speaking of intentionally individuated world lines (intentional objects) is not. What is more, speaking of certain objects as physical ascribes to them a certain sort of objectivity, whereas no such objectivity is assumed in connection with intentional objects. Physically individuated world lines are constrained by objective regularities articulated in physical theories. As for intentionally individuated world lines, they are severely conditioned by an agent, and yet, the agent cannot choose them at will. As remarked in Sect. 1.5, the fact that I take it to be a part of the *form* of our modal cognition that our modal thoughts are structured in terms of world lines does not mean that it is up to us to create the specific objects we are thinking of. Analogously, it is not Kant's view that the pure intuitions and the pure concepts of understanding alone render it possible for us to have experience of objects. The appearances must have a *matter* and not only form.

Chapter 3
Two Modes of Individuation

3.1 Introduction

I will enrich the framework of Chap. 2 by discerning two modes of individuation: the physical and the intentional (Sect. 3.2). In modal settings, I take physical objects to be physically individuated world lines. Intentional objects are viewed as intentionally individuated world lines, defined on worlds compatible with an agent's intentional state. The difference between the two types of world lines can be characterized by using the notions 'availability' and 'realization' (Sect. 3.3, cf. Sect. 2.3). Distinguishing the two modes of individuation leads to a generalization L of the modal language L_0. There are two types of quantifiers in L differing in the types of world lines they range over: physical and intentional quantifiers (Sect. 3.4). I spell out specific hypotheses about the interaction of the two quantifier types. I take up examples illustrating the semantics: negative general existential statements, statements combining quantifiers and extensional identity, and statements about intentional identity (Sect. 3.5).

I relate my analysis to Hintikka's epistemically motivated distinction between the public and perspectival mode of identification (Sect. 3.6) and to Williamson's necessitism (Sect. 3.7). I point out that necessitism involves two mutually independent claims. According to the first, quantifiers range in all worlds over the same set of objects (the quantification pool). According to the second, the identity formula $x = x$ is satisfied in all worlds by the same objects (the identity pool). My framework gives no support to necessitism about intentional objects. I do not claim to provide a knockdown argument against necessitism about physical objects, but I suggest that we can make sense of modal and temporal claims concerning physical objects without postulating unrestricted quantification. The necessitist's 'static' view of reality may well be the result of confusing a useful mathematical model of the reality with the reality itself. I close the chapter by commenting on Meinongianism and comparing my analysis of intentionally individuated world lines with Graham Priest's Meinongian account of objects of thought (Sect. 3.8).

T. Tulenheimo, *Objects and Modalities*, Logic, Epistemology, and the Unity of Science 41, DOI 10.1007/978-3-319-53119-9_3

In order to understand many of the things that epistemic agents and language-users think and say, we must distinguish between objects of two types: physical and intentional objects. As was explained in Sect. 1.3, the notions of local object and world line are semantic notions that admit different interpretations. The same holds true of the notions of physically and intentionally individuated world line. The 'intended' way of understanding these semantic notions is to construe physically individuated world lines as physical objects and, similarly, to interpret intentionally individuated world lines as objects of thought. This leads us to view both physical and intentional objects as world lines, and it leads to a certain understanding of the differences and similarities between these two types of objects.[1] Intentional objects are construed as world lines considered over the set of all worlds compatible with an agent's intentional state. This way of understanding intentional objects is not refuted by the fact that epistemic agents would presumably not describe their phenomenological contents of experience in terms of possible worlds and world lines. As I suggested in Sect. 1.3, world lines (also intentionally individuated world lines) may well be *epistemically* prior with respect to local objects. If so, our intentional states do not involve being aware of world lines as correlations between local objects of distinct worlds, even if world lines can be so analyzed.

In order to avoid false expectations, it may be useful to stress that when formulating my semantic analysis, I do not assume impossible worlds or impossible objects, and I do not advocate any sort of dialetheism. This is not to say that I would deny the fact that people sometimes appear to have contradictory beliefs and to think of things satisfying contradictory predicates.[2] However, whichever modal-logical analysis one might wish to provide for dealing with such cases, one must *first* get clear about the notion of cross-world identity, and one must find an insightful analysis of the difference between physical and intentional objects. This book is about cross-world identity. My approach cannot be validly criticized on the basis that it does not, in addition, discuss this or that further problem. By contrast, one could validly criticize any approach that ignores the problem of cross-world identity and proceeds right away to formulate proposals on how to modal-logically analyze contradictory beliefs or contradictory attributions of predicates to objects of thought.

Given the standard modal-logical analysis that I follow when discussing propositional beliefs, an agent's believing a contradictory proposition in a world w means that there are no worlds at all compatible with what the agent believes at w: a contradictory proposition can be true in *all* doxastically accessible worlds only if there are no such worlds. Consequently, my analysis is not able to make distinctions between different contradictory beliefs—say, between the belief *that Socrates is sitting and not sitting* and the belief *that the continuum hypothesis can be proven from the axioms*

[1]The details of the resulting understanding of physical and intentional objects depend on how one interprets the notion of world line. Above, I have given preference to the transcendental interpretation of world lines and discerned a number of alternative construals: epistemic, anti-realist, and metaphysical interpretation.

[2]For using world line semantics to analyze cases in which an agent has contradictory beliefs about a physical object, see Sect. 6.3.

of set theory. More generally, I do not attempt to provide an analysis of *fine-grained* or *hyperintensional* modal contexts—that is, contexts in which one must be able to distinguish between conditions that are not only extensionally but even intensionally equivalent. A case in point would be provided by a sentence such as ‘Alice sees a half empty, but not half full, glass of water’. I do not deny that one’s overall theory of modal expressions must be able to cope with fine-grained modalities, but I do not find it negligence on my part that I do not discuss such cases in this book, whose focus is on presenting a novel detailed theory of cross-world identity. It would, on the other hand, be questionable for anyone to try the reverse—that is, to move on to pronounce on hyperintensionality in connection with quantified modal languages without having paid due attention to the notion of cross-world identity.[3]

3.2 Intentional and Physical Mode of Individuation

Arthur Prior distinguished two types of objects of thought: *what we think* and *what we think of* [99, pp. 3, 111]. The former are usually called *propositions*. The latter can be called *intentional objects*. Tim Crane discusses objects of thought in the latter sense. He takes intentional objects to be what mental states are directed on. These may be things in the world but, equally well, things we merely represent to be in the world [21, p. 4]. As Crane puts it, intentional objects may but need not ‘exist’, although even in the latter case, we can correctly say that ‘there are’ such objects or that ‘some’ objects are intentional [ibid. Chap. 2]. Crane does not wish to invoke a special category of *entities* termed ‘intentional objects’ or ‘non-existent objects’. Saying that there are non-existent intentional objects just means that we can think about things that do not exist and that we think about them in the same way we think about existing things. As Crane sees it, the notion of a non-existent object is needed for a theory of intentionality [ibid. p. 5].

As I will show, my framework allows accommodating intentional objects without attaching ontological importance to them. It also enables explicating the idea that when a thought is directed on an existing thing, the corresponding intentional object simply is an ordinary physical object. It is this way of understanding the relation between intentional and physical objects that underlies the intentional theory of perceptual experience.[4] The view extends naturally to objects of thought in general. My goal is to provide a logical analysis compatible with this view on intentional objects. Both physical and intentional objects are construed as world lines.

Let us consider objects of belief. It was recalled in Sect. 1.4 that belief can be regarded as a modality. If R is the doxastic accessibility relation for a fixed agent α, the set $R(w_0)$ consists of scenarios compatible with what α believes in w_0. Discussing

[3] For an example of this latter type of approach, see, e.g., Jespersen [56].

[4] For an overview of this theory, see, e.g., Crane [20].

intentionality, Hintikka [46] put forward the thesis of *Intentionality as Intensionality*, according to which intensionality is a criterion for the intentionality of a concept. A concept is intentional if its semantics must be phrased by taking simultaneously into account different alternative scenarios. Instead of intensionality, we may speak of the modal character of such notions. By this criterion, the concept of belief is intentional: what α believes in w_0 is to be understood with reference to the set $R(w_0)$. According to Hintikka [ibid. pp. 201–3], intentional states are by character *informational* and *conceptual*. First, the set $R(w_0)$ yields a measure of the information that α has regarding the world w_0. The smaller the set, the more specific information it provides, and the closer it is qualitatively to w_0, the more accurate the information is. It is certainly not obvious how to define a quantitative measure of information on the basis of such sets. Yet, the idea that $R(w_0)$ correlates in some way with the information α has at her disposal in w_0 is easy to appreciate and suffices for my purposes. Second, it is a part of the nature of worlds that they are organized in terms of properties and relations attributed to elements of their domains. The fact that worlds are thus organized is often represented linguistically, by world-relative interpretations of predicate symbols. I follow this practice in this book.[5] Worlds relevant for the analysis of intentional states should be thought of as being 'luminous' in the sense that the characteristics in terms of which they are organized are characteristics α uses to conceptualize her environment. If X is an n-ary relation and α does not understand what it is to exemplify the characteristic X, then the membership of a world w in $R(w_0)$ does not depend on whether a tuple $\langle a_1, \ldots, a_n \rangle \in dom(w)^n$ exemplifies X.

While I share with Hintikka the idea that beliefs must be analyzed modally, I do not agree that their being thus analyzed accounts for the fact that belief is an intentional state. Hintikka takes it to be a consequence of his thesis of Intentionality as Intensionality that possible worlds are primary with respect to individuals, in the sense that individuals—understood as world lines—are constructed from the raw material provided by possible words. I disagree, not because I hold there to be an independent stock of individuals ready to take up the role of denizens of possible worlds but because I hold that worlds and world lines are two intertwined but mutually independent types of modal unities. Neither type is primary with respect to the other, and elements of either type can be considered over the medium provided by elements of the other type: an individual realized over a number of worlds, a world realizing a number of individuals. Accordingly, I take the intentionality of an agent's beliefs in w_0 to be the joint result of there being a set $R(w_0)$ of worlds compatible with what the agent believes at w_0 *and* a set $\mathfrak{I}$ of world lines available to this agent in w_0.

[5]The notion of a possible world thus represented is *not* meant to be the same as that of a first-order model: the totality of possible worlds does *not* allow arbitrary reinterpretations of predicates (cf. footnote 24 in Sect. 1.4). As Hintikka [45, p. 378] puts it, '[in each possible world] those and only those things are to be called red that *are* red'. Predicates are merely linguistic surrogates for sets of instantiations of characteristics. In possible world semantics, a predicate standing for a set of instantiations of a characteristic in one world stands for the set of *its* instantiations in all worlds. As long as we need not quantify over properties and relations, we can content ourselves with speaking of characteristics via their linguistic surrogates. (Should we wish to quantify over characteristics, even properties and relations should be construed as world lines of a suitable type.)

The world lines in the set $\mathcal{J}$ must be thought of as being *intentionally individuated*. The agent's thoughts are to be analyzed in terms of the content $\langle R(w_0), \mathcal{J}\rangle$. Hintikka [46, pp. 194–5] takes the thesis of Intentionality as Intensionality to be in contrast with the idea of intentionality as object-directness. By distinguishing the two components of beliefs understood as intentional states—worlds and world lines—it becomes possible to maintain that intentional states have a modal character *and* are directed toward objects. The set $R(w_0)$ taken by itself encodes propositional information. The world lines $\mathbf{J} \in \mathcal{J}$ impose an additional structure on the set $R(w_0)$ and serve to model the intentional objects of the agent's thoughts.

Speaking of physical objects is to speak of world lines defined over a number of contexts. Ordinary physical objects are temporally extended and have dispositional properties, which leads us to consider their realizations not only at several instants in one and the same structured world but also in distinct structured worlds. There is *no difference in kind* between intentional and physical objects: both are world lines with a modal margin, both are correlations between local objects, and both can be modeled as functions associating relatively simple worlds with local objects. In particular, speaking of physical and intentional objects does not mean distinguishing local objects of two types. It serves clarity to speak of two *modes of individuation*: the intentional and the physical. The distinction does not concern two separate categories of being but two sources for modal unities, both of which must be recognized so that we can describe what there is in the world, what we believe and experience, and what we speak about. Physical objects are viewed as physically individuated world lines and intentional objects as intentionally individuated world lines.

The contrast between physically and intentionally individuated world lines exemplifies certain important distinctions—the same distinctions that Crane wants the contrast between existing things and intentional objects to exemplify. I explain in the following section that world lines of the latter variety need not 'exist' and speaking of them has no ontological repercussions. At the same time, some intentional objects 'are' physical objects. The affinity of my semantic approach to Crane's account of objects of thought is not a result of a shared overall framework. My focus is semantic—I wish to develop a formal semantic analysis of certain sentences ascribing intentional states to agents—and this leads me to explicitly reason in terms of alternative scenarios or possible worlds. Crane, in his turn, operates with the basic notion of representation and has no wish to analyze it in terms of possible worlds. He admits that resorting to propositions or possible worlds may be theoretically useful in connection with semantics, but he finds it mistaken to accord these notions a fundamental role in a psychological account of mental states [21, pp. 128–9].

More specifically, Crane distinguishes two conceptions of the content of experience: the *semantic* and the *phenomenological* [22, Sects. 5–6]. Phenomenological contents are representations in an intrinsic sense. Being a representation in this sense does not amount to having an associated correctness condition or bearing a relation to a proposition. Phenomenological contents are spatiotemporally determinate, concrete, particular, unrepeatable, and unshareable occurrences. They are specific to an agent. According to Crane, what is phenomenologically given or conveyed to a subject in her experience is a content in this sense. By contrast, semantic contents are propositional, they have a correctness condition, and they are abstract both in the sense of not having a spatiotemporal location and in the sense of abstracting from the concrete reality of an experiential episode. They are not tied to any particular moment of thinking or experiencing, and they can be shared by a number of agents: they can be contents of many acts of thinking. Crane proposes to view them as *modeling* mental states—being their idealized, theoretical, external ‘descriptions’.

I am basically happy with such a division of labor: I do not wish to claim that phenomenological contents of experience should or could be equated with contents in my semantic sense (Definition 2.3). This said, my semantic analysis goes deeper than Crane [21, p. 128] expects a semantic analysis to go: contents in my sense need not be propositional. I agree with Crane that there are object-directed non-relational non-propositional states not reducible to intentional states with a propositional content [ibid. Sects. 4.4–4.5]. However, insofar as Crane thinks that contents of those non-propositional states must be in his sense phenomenological rather than semantic, I disagree. Semantic contents are not limited to propositional contents. Contents in my sense are shareable, repeatable, abstract, and not bound to an agent. They count as semantic by Crane’s standards. Yet a content in my sense is, generally, object-directed. Its object-directedness has, furthermore, an intrinsic character: its being a representation does not mean that it stands in a relation to a physical object. In my analysis, object-directedness is not reserved for phenomenological contents but is a possible feature of semantic contents, as well. Crane notes that we cannot credibly claim that a set of worlds is phenomenologically conveyed to us in experience [22, p. 238]; a fortiori, it would be hard to maintain that a structure of worlds and worlds lines is given to a subject in experience. That is, we cannot dispense with the distinction between phenomenological and semantic contents.

Now, intentional objects involved in phenomenological contents are certainly not experienced as world lines correlating local objects that belong to distinct possible worlds. They are experienced in a more direct and less analyzed manner. Intentionally individuated world lines are epistemically prior to local objects. However, we need not for this reason adopt an instrumentalist understanding of world lines, suggesting that analyzing intentional contents in terms of worlds and world lines is *merely* a matter of modeling. I maintain that due to their very nature, intentional states are compatible with a number of alternative scenarios, and intentional objects involved in such states are not reducible to these worlds. Instead, by being world lines, they constitute an independent component of the content. Phenomenological contents admit an analysis in terms of worlds and world lines. The result of such an analysis is a content in the semantic sense. In order to understand what someone is thinking, we need information of the sort encoded in a content in the semantic sense: which scenarios are compatible with the agent’s beliefs and how the agent’s objects of thought behave in relation to those scenarios. Phenomenological contents in Crane’s sense give rise to semantic contents. Genetically, they have priority over semantic contents, but one’s experience cannot have a phenomenological content without thereby having a semantic content, as well. In Sect. 6.4, I explain what it means to say that a content in my sense has an accuracy condition.

3.3 Availability and Realization

The two modes of individuation can be profitably compared with reference to the notions of availability and realization. When we talk about physical or intentional objects, we talk about world lines, and when we talk, we are located in a world. In a given world, the set of world lines of which we may speak is limited, or, at least on the face of it, there is no reason to suppose otherwise.[6] That is, for every world w there is a set $\mathcal{P}_w$ of world lines *physically available* at w: those physically individuated world lines we can speak about in w. For each world w and for every agent α, there is a set $\mathcal{I}_w^\alpha$ of world lines *intentionally available* to α at w: these are the objects of α's thought in w.

Instead of a single collection of world lines as in models discussed in Sect. 2.3, two such collections will be considered: $\mathcal{P} = \{\mathcal{P}_w : w \in W\}$ and $\mathcal{I} = \{\mathcal{I}_w^\alpha : w \in W, \alpha \in A\}$, where A is the set of agents we wish to take into account. In our language, we will have two types of quantifiers: $\exists$ ranges over physically individuated world lines, while E_a ranges over world lines intentionally individuated by the agent referred to by the term 'a'. Locutions such as 'there is' and 'some' are interpreted in terms of such quantifiers and therefore in terms of *availability*. I take these locutions to be ambiguous between a reading in terms of $\exists$ and a reading employing E_a. The sense of 'existence' needed for explicating the notion of ontological commitment is clarified with reference to *realizations*. Saying that the value of a variable (a certain world line) exists in a world w means that this world line is realized in w.

To describe features of physical and intentional objects understood as modal unities, the notions of availability and realization are both needed. Especially, they are needed to explicate typical differences between objects of these two varieties. Speaking of physical or intentional objects leads always or, at least, normally to considering a whole set of worlds. The world in which we think or talk about a world line need not be among the worlds in which the world line is realized.

I take it that in connection with *physically* individuated world lines, the notions of availability and realization coincide:

1. $\mathbf{I} \in \mathcal{P}_w$ iff $\mathbf{I}$ is realized in w, for all $w \in W$ and $\mathbf{I} \in \bigcup_{v \in W} \mathcal{P}_v$.

Assuming that availability entails realization means taking quantification over physical objects to be ontologically committing. If a physical object $\mathbf{I}$ is available to be talked about in w, it is realized in w. I take the objective character of physical individuals to require also the converse entailment. If $\mathbf{I}$ is realized in w, then $\mathbf{I}$ is available to be talked about in w—it lies in the range of values of the quantifier $\exists$ in w. With physical objects, the whole package comes with a single aspect, not in the sense that a physical object would be 'wholly present' in every world in which it is realized but in the sense that already a single realization guarantees the possibility to speak

[6] In any event, proceeding in this way leaves all options open. If it turns out that there is a well-defined totality of all individuals that we can talk about independently of the world in which we find ourselves (unrestricted quantification), this assumption is readily accommodated by stipulating that the set of available individuals is the same for all worlds.

of the whole physical object.[7] By contrast, for intentional objects, availability and realization are mutually independent properties:

2. *Intentional availability does not entail realization:* there are agents α, worlds $w \in W$, and world lines $\mathbf{J} \in \bigcup_{v \in W} \mathcal{I}_v^\alpha$ such that $\mathbf{J} \in \mathcal{I}_w^\alpha$ but $\mathbf{J}$ is not realized in w.
3. *Realization does not entail intentional availability:* there are agents α, worlds $w \in W$, and world lines $\mathbf{J} \in \bigcup_{v \in W} \mathcal{I}_v^\alpha$ such that $\mathbf{J}$ is realized in w but $\mathbf{J} \notin \mathcal{I}_w^\alpha$.

Clause (2) is an expression of the fact that intentional objects need not exist—a feature Crane stresses as crucial for any theory of intentionality. Quantification over intentional objects is not ontologically committing: the fact that an intentionally individuated world line is available as a value of an intentional quantifier in a world does not entail that this world line 'exists' in that world—i.e., that it is realized therein. This is exactly as it should be.[8] The fact that α has an object of thought does not entail that this object of thought exists. Features of an intentionally individuated world line $\mathbf{J}$ available in w_0 may be compared with features of the set $R(w_0)$ of worlds compatible with what α believes. Just as the world w_0 may lie outside $marg(\mathbf{J})$, the world w_0 may lie outside $R(w_0)$. The latter is what happens when α believes a false proposition, the former when an object of α's beliefs is non-existent. In both cases, the actual world fails to belong to a set. Incidentally, the two phenomena are independent: α may have a false belief about an existent object and a true belief about a non-existent object.[9] Non-realized but available world lines are possible values of such natural-language quantifiers as 'there is' or 'some', when these are understood as ranging over intentional objects. In my analysis—unlike in Crane's—there is an important uniformity between ontologically committing and ontologically non-committing quantification: both types of quantifiers range over world lines. The difference resides in the mode of individuation of the respective values of quantified variables. Crane's analysis uses the basic notion of representation to account for

[7] What is important for the purposes of the present book is that the distinction between typical cases of physical objects (stones, planets, tables, elephants, and persons) and typical cases of intentional objects (objects of perceptual experience, fictional objects) is articulated in a way that can be utilized in semantic theorizing and that is compatible with the idea that an existing intentional object *is* a physical object. It is not important to fix precise metaphysical interpretations of the semantic notions of physically individuated world line and intentionally individuated world line. For example, I leave open the question of whether numbers or events can be considered as physically individuated world lines. For the metaphysical question of how physical objects are to be distinguished from other types of objects, see van Inwagen [122] and Markosian [83].

[8] Recall that it was indicated in Sect. 1.3 that by the 'existence' of a world line in a world, I mean that it is realized in that world. The fact that a world line does not in this sense exist in a world precisely does not preclude that it is available in it—i.e., that it is a possible value of a suitable quantifier in that world. It is indeed typical of intentional objects that they may be available without existing—without being realized.

[9] A simple example of a true belief about a non-existent object is the belief that Pegasus does not exist. Another example is α's belief that Pegasus is a winged horse, supposing that α does not seriously believe that Pegasus exists. If α believed that Pegasus exists, Pegasus would be realized throughout $R(w_0)$. This would rule out the possibility that $w_0 \in R(w_0)$ and indeed the truth of the belief, given that Pegasus does not exist in w_0.

non-committing quantification. Since quantification over physical objects is non-representational, the formal similarity between the two types of cases is not captured in his framework.

Clause (3) expresses the following important feature of intentional objects: their existing in a world does not entail that we can talk about them therein. This phenomenon would be difficult to conceptualize without the distinction between availability and realization. Crane certainly does not recognize it. To see that this phenomenon can indeed occur, consider a temporally extended world. Suppose instant t is earlier than instant s, and suppose that α has at t an object of thought $\mathbf{J}$. That is, $\mathbf{J} \in \mathcal{I}_t^\alpha$. It might happen that according to what α believes at t, the object of thought $\mathbf{J}$ will be Q at s. If so, $\mathbf{J}$ is realized at s. (The predicate Q is atomic and therefore existence-entailing.) When time passes and the instant s becomes present, α may have forgotten what she was thinking about at t. If so, $\mathbf{J} \notin \mathcal{I}_s^\alpha$. That is, $\mathbf{J}$ is realized but not available at s. The intentional object $\mathbf{J}$ *exists* in s, but α cannot talk about it at s, though α was able to talk about it at t. Other examples of this type—existence without availability—are found by considering multiple agents or by comparing intentional objects with physical ones. Suppose, for example, that an intentional object $\mathbf{J}$ of agent α is both available and realized in w. This by no means guarantees that $\mathbf{J}$ is available in w to another agent β, as well—which does not remove that fact that $\mathbf{J}$ is realized in w. Further, the fact that a physical object is realized (and therefore available) in w is perfectly compatible with its not being intentionally available to an agent in w. Actually, the indeterminate and temporally limited nature of intentional objects in practice blocks the possibility that an intentional object is literally identical to a physical object, rather than merely coinciding with one over a subset of its modal margin.[10]

It is a significant feature of my analysis that intentionally and physically individuated world lines may indeed coincide over a set of worlds. There may be worlds w and v, agents α, world lines $\mathbf{I} \in \mathcal{P}_w$ and $\mathbf{J} \in \mathcal{I}_v^\alpha$, and non-empty sets U of worlds such that $\mathbf{I}(u) = \mathbf{J}(u)$ for all $u \in U$. Similarly, world lines intentionally individuated by two agents may share their realizations over a set of worlds: there can be distinct agents α and β and world lines $\mathbf{I} \in \mathcal{I}_w^\alpha$ and $\mathbf{J} \in \mathcal{I}_v^\beta$ such that $\mathbf{I}(u) = \mathbf{J}(u)$ for all $u \in U$. The former possibility is needed to account for perceptual experience as a form of intentionality and to explicate how objects of beliefs can be related to external objects. The latter possibility is called for to make sense of intentional identity. In principle, one and the same world line could be both physically *and* intentionally individuated—or intentionally individuated by two agents. As noted above, the former possibility never occurs in practice. The possibility for intentional objects of two agents to be literally identical depends on the way these objects are characterized. Insofar as the characteristics that the agents ascribe to their

[10]In Sect. 4.8, it will be seen that the idea of α's intentional state representing a specific physical object $\mathbf{J}$ in w_0 cannot be taken to mean that $\mathbf{J}$ itself is intentionally available to α in w_0. Rather, there must be an intentional object $\mathbf{I} \in \mathcal{I}_{w_0}^\alpha$ and a large enough subset U of $R(w_0)$ such that $\mathbf{J}(w) = \mathbf{I}(w)$ for all $w \in U$. It is necessary but not sufficient that $w_0 \in U$.

respective intentional objects are mutually incompatible, it is questionable whether their intentional objects can even share a realization, let alone be identical.

3.4 World Line Semantics Generalized

I sketched in Sect. 2.3 a formal semantics to be used when speaking of individuals in modal settings. This semantic framework must be generalized to incorporate both physical and intentional objects. As a further modification, I allow using constant symbols in the syntax. For all $n \geq 0$, let τ_n be a set of n-ary predicate letters. Constant symbols are elements of τ_0 (nullary predicate letters). The set $\tau := \bigcup_{i \in \mathbb{N}} \tau_i$ is a *vocabulary*. Variables and constant symbols are collectively referred to as *terms*. I write *Term* for the set $Var \cup \tau_0$. Syntactically, all terms behave similarly. However, while semantic values of variables are world lines, semantic values of constant symbols are local objects. Let $\mathbb{A}$ and $\mathbb{I}$ be finite non-empty sets of *agent markers* and *indices*, respectively. The semantics specifies for each index its own accessibility relation. Since my modal-logical analysis will be applied to sentences ascribing intentional states to agents, such accessibility relations should be thought of as being relative to a specific intentional state and a specific agent. If $\mathsf{a} \in \mathbb{A}$, I write α for the agent denoted by 'a' and A for the set of agents denoted by markers in the set $\mathbb{A}$.

Frames are structures $\langle W, \mathcal{R}, \mathcal{P}, \mathcal{I}\rangle$, where $\mathcal{R} = \{R_{\mathsf{i}} : \mathsf{i} \in \mathbb{I}\}$ is a family of accessibility relations, and $\mathcal{P} = \{\mathcal{P}_w : w \in W\}$ and $\mathcal{I} = \{\mathcal{I}^{\alpha}_w : w \in W, \alpha \in A\}$ are families of world lines over W. *Models* of vocabulary τ are structures $M = \langle F, Int\rangle$, where F is a frame and *Int* is an interpretation function defined otherwise as in connection with L_0, except that it associates every constant symbol c and world w with an element of the set $dom(w) \cup \{*\}$, where $* \notin \bigcup_{v \in W} dom(v)$. The value $Int(c, w) = *$ is taken to indicate that c has no referent in w. Note that if $Q \in \tau_n$ and $n \geq 1$, then $* \notin Int(Q, w)$.

I write $\mathrm{WL_P}(M)$ for the set $\bigcup_{w \in W} \mathcal{P}_w$ and $\mathrm{WL_I}(M)$ for $\bigcup_{w \in W, \alpha \in A} \mathcal{I}^{\alpha}_w$. Further, let $\mathrm{WL}(M) := \mathrm{WL_P}(M) \cup \mathrm{WL_I}(M)$. The set $\mathrm{WL}(M)$ depends only on the frame F; when discussing frames, I write $\mathrm{WL}(F)$ to denote $\mathrm{WL}(M)$. As in Sect. 2.3, also here I refer to the partial functions in $\mathrm{WL}(M)$ as *world lines*, although literally they are mathematical representations of world lines. I refer to the elements of $\mathrm{WL_P}(M)$ as *physical objects* (or physically individuated world lines) and to those of $\mathrm{WL_I}(M)$ as *intentional objects* (or intentionally individuated world lines), even though, strictly speaking, they are of course mathematical representations of physically and intentionally individuated world lines—representations of physical and intentional objects. As explained in Sect. 3.3, an element of $\mathrm{WL_P}(M)$ is assumed to be available in a world iff it is realized therein, whereas for elements of $\mathrm{WL_I}(M)$, availability and realization are mutually independent properties. The following further hypotheses H1 through H4 are made concerning the sets $\mathrm{WL_P}(M)$ and $\mathrm{WL_I}(M)$:

H1. *No two physically individuated world lines overlap:* If $\mathbf{I}, \mathbf{J} \in \mathrm{WL_P}(M)$ and there is w such that $w \in \mathit{marg}(\mathbf{I}) \cap \mathit{marg}(\mathbf{J})$ and $\mathbf{I}(w) = \mathbf{J}(w)$, then for all $v \in W$ we have: [either $v \notin \mathit{marg}(\mathbf{I}) \cup \mathit{marg}(\mathbf{J})$, or $v \in \mathit{marg}(\mathbf{I}) \cap \mathit{marg}(\mathbf{J})$ and $\mathbf{I}(v) = \mathbf{J}(v)$].
H2. *Realizations of physically individuated world lines are local objects:* If $w \in W$ and $\mathbf{I} \in \mathcal{P}_w$, then $\mathbf{I}(w) \in \mathit{dom}(w)$.
H3. *Realizations of intentionally individuated world lines are local objects:* If $\alpha \in A$ and $w, v \in W$ and $\mathbf{I} \in \mathcal{I}_w^\alpha$ and $\mathbf{I}$ is realized in v, then $\mathbf{I}(v) \in \mathit{dom}(v)$.
H4. *Every local object is the realization of some physical object:* If $w \in W$ and $b \in \mathit{dom}(w)$, then there is $\mathbf{I} \in \mathcal{P}_w$ such that $b = \mathbf{I}(w)$.

The connection between availability and realization that was assumed to hold for physical objects does not by itself entail H1: without this hypothesis, a given realization could correspond to several physical objects. H1 should be viewed as an idealization. It would be too restrictive in connection with a general analysis of material constitution: the possibility of coincident objects cannot be excluded (say, a statue and a lump of clay). Hypotheses H2 and H3 hold automatically when world lines are defined as in Sect. 2.3. However, as will be pointed out in Sect. 4.6, in connection with certain phenomena, it may be preferable to give up these assumptions. Hypothesis H4 means that the simplest building blocks of models (local objects) have a 'physical basis', but this hypothesis in no way limits possible correlations between local objects in terms of intentionally individuated world lines. If $\mathbf{I}$ is an intentional object, then for every $w \in \mathit{marg}(\mathbf{I})$ there is a physical object $\mathbf{J}_w$ such that $\mathbf{I}(w) = \mathbf{J}_w(w)$, but it does not follow that there is a single physical object $\mathbf{J}$ such that for all $w \in \mathit{marg}(\mathbf{I})$ we have $\mathbf{I}(w) = \mathbf{J}(w)$. It ensues from H4, along with the fact that the domain of every world is non-empty, that all sets $\mathcal{P}_w$ are non-empty, as well.[11]

Since there are two types of *terms*, it is convenient to fix a general definition of a *value* $\mathsf{t}^{M,w,g}$ of term t in model M at world w under assignment $g : \mathit{Var} \to \mathrm{WL}(M)$:

$$\mathsf{t}^{M,w,g} = \begin{cases} \mathit{Int}(\mathsf{t}, w) & \text{if } \mathsf{t} \in \tau_0 \text{ and } \mathit{Int}(\mathsf{t}, w) \neq * \\ g(\mathsf{t})(w) & \text{if } \mathsf{t} \in \mathit{Var} \text{ and } g(\mathsf{t}) \text{ is realized in } w. \end{cases}$$

By this definition, the value $\mathsf{t}^{M,w,g}$ is a local object—an element of $\mathit{dom}(w)$—whenever the value is defined in the first place. The value is not defined if $\mathsf{t} \in \tau_0$ and $\mathit{Int}(\mathsf{t}, w) = *$, nor if $\mathsf{t} \in \mathit{Var}$ and $g(\mathsf{t})$ fails to be realized in w.

I consider an extended language L that is recursively defined like L_0, except that it is closed under applications of modal operators $\Box_\mathsf{i}$ with $\mathsf{i} \in \mathbb{I}$ and applications of

[11] I am not suggesting that it would be somehow contradictory to consider worlds with no local objects or worlds with a local object that is not the realization of any physically individuated world line. However, from the interpretative viewpoint, it is a reasonable (and modest) assumption that in every world, we can talk about at least one physical object—a physical object realized in that world.

quantifiers E_{a} with $\mathsf{a} \in \mathbb{A}$. Moreover, atomic formulas can employ arbitrary terms. That is, the language $L[\tau]$ of vocabulary τ is built according to the following syntax:

$$\phi ::= Q(\mathsf{t}_1, \ldots, \mathsf{t}_n) \mid \mathsf{t}_1 = \mathsf{t}_2 \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_{\mathsf{i}}\phi \mid \exists x\phi \mid \mathsf{E}_{\mathsf{a}}x\phi,$$

where $n \geq 1$ and $Q \in \tau_n$ and $\mathsf{t}_1, \mathsf{t}_2, \ldots, \mathsf{t}_n \in \mathit{Term}$ and $x \in \mathit{Var}$ and $\mathsf{i} \in \mathbb{I}$ and $\mathsf{a} \in \mathbb{A}$. I refer to $\exists$ as a *physical quantifier* and to the E_{a} as *intentional quantifiers*.

The semantic clauses are otherwise as in L_0, except that in a world w, the physical quantifier $\exists$ ranges over the set $\mathcal{P}_w$ and the intentional quantifier E_{a} over the set $\mathcal{I}^{\alpha}_{w}$. Further, the accessibility relation associated with the modal operator $\Box_{\mathsf{i}}$ depends on the index i. Finally, the clauses for atomic formulas must be modified, since atomic formulas may contain constant symbols. Here are the clauses that need modifications:

- $M, w, g \models Q(\mathsf{t}_1, \ldots, \mathsf{t}_n)$ iff for all $1 \leq i \leq n$, the value $\mathsf{t}_i^{M,w,g}$ of the term t_i in M at w under g is defined, and the tuple $\langle \mathsf{t}_1^{M,w,g}, \ldots, \mathsf{t}_n^{M,w,g} \rangle$ belongs to $\mathit{Int}(Q, w)$.
- $M, w, g \models \mathsf{t}_1 = \mathsf{t}_2$ iff for all $i \in \{1, 2\}$, the value $\mathsf{t}_i^{M,w,g}$ of the term t_i in M at w under g is defined, and $\mathsf{t}_1^{M,w,g}$ equals $\mathsf{t}_2^{M,w,g}$.
- $M, w, g \models \exists x\phi$ iff there is $\mathbf{I} \in \mathcal{P}_w$ such that $M, w, g[x := \mathbf{I}] \models \phi$.
- $M, w, g \models \mathsf{E}_{\mathsf{a}}x\phi$ iff there is $\mathbf{I} \in \mathcal{I}^{\alpha}_{w}$ such that $M, w, g[x := \mathbf{I}] \models \phi$.
- $M, w, g \models \Box_{\mathsf{i}}\phi$ iff for all worlds w' with $R_{\mathsf{i}}(w, w')$ we have: $M, w', g \models \phi$.

The operators $\vee$, $\rightarrow$, and $\Diamond_{\mathsf{i}}$ can be defined from $\neg$, $\wedge$, and $\Box_{\mathsf{i}}$ in the usual way. Further, I introduce a quantifier E by stipulating that $\mathsf{E}x\phi$ means the same as $\bigvee_{\mathsf{a}\in\mathbb{A}} \mathsf{E}_{\mathsf{a}}x\phi$. (This disjunctive formula is well formed in L, because the set $\mathbb{A}$ is finite.) The universal quantifiers $\forall$, A_{a}, and A are definable as the duals of $\exists$, E_{a}, and E, respectively. Consequently, $\mathsf{A}x\phi$ is equivalent to $\bigwedge_{\mathsf{a}\in\mathbb{A}} \mathsf{A}_{\mathsf{a}}x\phi$.

Some clarificatory comments on my use of the expression 'constant symbol' are in order. In the present modal-logical setting, the way in which I employ this term may appear strange for the following reason. Whenever the semantic value of a constant symbol in a world w is defined, this value is a local object of the domain of w. Therefore, one and the same constant symbol c cannot stand for the same object in two worlds: if $w_1 \neq w_2$ and $a_1 = \mathit{Int}(c, w_1) \neq * \neq \mathit{Int}(c, w_2) = a_2$, then a_1 is a local object of w_1 and a_2 is a local object of w_2, whence a_1 and a_2 cannot possibly be extensionally identical. One must understand that the expression 'constant symbol' is used here as a technical term: it is not presupposed that it is a term whose referent remains the same across worlds. My terminology is justified by analogy with first-order logic. In first-order logic, a constant symbol is an expression whose fixed semantic value is an element of the domain. Further, semantic values of constant symbols are entities of the same type as those entities to which unary predicates can be applied. These two features are retained in my framework. Here, a constant symbol is an expression whose fixed semantic value in a *world* is an element of the domain of that world (unless the constant symbol has no referent at all in that world). Further, just like the semantic value of a unary predicate in a world is a set of local objects of the world, the value of a constant symbol in a world is, when defined, a local object of that world. It is possible to consider imposing additional rules that would regulate the use of constant symbols. It could be required that referents of a constant symbol in distinct worlds be realizations of one and the same physical object. This would mean that for every constant symbol c, there is a physically individuated world line $\mathbf{I}_c$ such that if $a_1 = Int(c, w_1) \neq * \neq Int(c, w_2) = a_2$, then $a_1 = \mathbf{I}_c(w_1)$ and $a_2 = \mathbf{I}_c(w_2)$. Alternatively, notably when considering the use of constant symbols in connection with attitudes or experiences of an agent α, the restriction could be imposed that referents of a constant symbol in distinct worlds be realizations of one and the same intentional object of the agent α. This would entail that for every such constant symbol c, there is a world line $\mathbf{J}_c$ intentionally individuated by α such that if $a_1 = Int(c, w_1) \neq * \neq Int(c, w_2) = a_2$, then $a_1 = \mathbf{J}_c(w_1)$ and $a_2 = \mathbf{J}_c(w_2)$. In this book, I stay with the general semantics in which no cross-world restrictions are imposed on world-relative interpretations of constant symbols.

3.5 Examples

The logical behavior of constant symbols will be discussed in Sect. 5.3. Already here we may note the following consequence of not having required that all constant symbols be referring.

Example 3.1 The constant symbol c is referring in w iff $c = c$ is true in w. Therefore, $c = c$ is refutable. By contrast, $P(c) \vee \neg P(c)$ is valid. In particular, it is true at any world in which c fails to refer: if $Int(c, w) = *$, then $Int(c, w) \notin Int(P, w)$. □

Let us proceed to observe some features of the semantics of L that are in keeping with the expected behavior of intentional and physical objects. Recall that the existence of a world line in a world w means that it is realized in w. Thus, the world line $\mathbf{I}$ exists in w iff the formula $x = x$ is satisfied in w by the assignment $x := \mathbf{I}$. The formula $x = x$ is *not* satisfied by all assignments. However, the only condition this formula imposes on the value of x in w is that its value is realized in w.

Example 3.2 (*Negative general existentials: intentional objects*) Not all intentional objects exist. Some do not. There are non-existent intentional objects. These are claims we wish to make about intentional objects. The logical renderings of these statements should be satisfiable. Now, these claims are of the form $\mathsf{E}x\neg x = x$, and this formula is indeed satisfiable.[12] Let $A = \{\alpha\}$. Define a model $M = \langle W, \mathcal{R}, \mathcal{P}, \mathcal{I}, Int\rangle$ by selecting $\mathcal{R}$, $\mathcal{P}$, and Int in an arbitrary fashion and letting W and $\mathcal{I}$ be as follows:

- $W = \{w_1, w_2\}$ with $w_1 \neq w_2$
- $\mathcal{I} = \{\mathcal{I}^\alpha_{w_1}, \mathcal{I}^\alpha_{w_2}\}$, where $\mathcal{I}^\alpha_{w_1} = \{\mathbf{I}\}$ and $\mathcal{I}^\alpha_{w_2} = \varnothing$ with $marg(\mathbf{I}) = \{w_2\}$.

[12] In Sect. 5.6, we will see that the notion of logical form is quite intricate in the logic L, but as a matter of fact, this notion is unproblematic as long as we stay with formulas not using any non-logical predicates—that is, formulas whose only atomic subformulas are identities.

Unless **I** were realized in w_2, **I** would be empty—and it has been stipulated in Sect. 2.3 that the empty function is not a world line. We have $M, w_1 \models \mathsf{E}_{\mathsf{a}} x \neg x = x$. Namely, **I** belongs to $\mathcal{I}^{\alpha}_{w_1}$, and yet, $M, w_1, x := \mathbf{I} \not\models x = x$, because **I** is not realized in w_1. A fortiori, then, the formula $\mathsf{E} x \neg x = x$ is true in M at w_1. □

For comparison, according to my semantics, we cannot consistently claim that a physical object fails to exist. Claims to that effect cannot be taken at face value.

Example 3.3 (*Negative general existentials: physical objects*) The formula $\exists x \neg x = x$ is not satisfiable. Quantification over physically available world lines is ontologically committing. If **I** is the value of x in w and $\mathbf{I} \in \mathcal{P}_w$, then **I** is realized in w, whence the assignment $x := \mathbf{I}$ satisfies the identity formula $x = x$ in w. □

Crane argues that the problem of negative existentials is merely a special case of problems of non-existence, the real problem being how to understand *representation-dependent* characteristics of non-existent intentional objects. He distinguishes between *substantial* and *pleonastic* properties and relations. The former are existence-entailing and 'characterize the nature of real existing things' [21, pp. 64–9]. In his view, true sentences about non-existent intentional objects ascribe pleonastic properties, and this fact has no metaphysical consequences. Crane holds that all properties of non-existent intentional objects are pleonastic, and almost all of them are representation-dependent. For him, the property of non-existence is an exception and so would be the logical property of identity if it was applicable to non-existent objects—though Crane argues that it is not [ibid. pp. 69, 120, 166–7]. I will discuss predicates ascribed to intentional objects in Sect. 4.2. Already here it is useful to observe, by way of example, what such predications involve in my framework.

Example 3.4 (*Predicate ascriptions*) I take it that when an agent ascribes a predicate P to an intentional object **I**, this amounts to affirming that **I** is P in all worlds in the internal modal margin of **I**—i.e., all worlds in which **I** is realized *and* that are compatible with the agent's intentional state. (Recall the notion of internal modal margin from Definition 2.3.) Consider the set $R(w_0)$ of worlds compatible with Alice's total body of beliefs in w_0. Suppose Alice has a non-existent object of thought **I** to which she ascribes the predicate '— is a winged horse'. Here, **I** is a world line intentionally individuated by Alice. Given the proposed analysis of intentional predication, it follows that **I** satisfies this predicate in every world w belonging to the intersection $marg(\mathbf{I}) \cap R(w_0)$.[13] If '— is a winged horse' is taken to be an atomic predicate, this means that the *realization* $\mathbf{I}(w)$ of **I** in w is a winged horse whenever $w \in marg(\mathbf{I}) \cap R(w_0)$. By contrast, if **J** is a physical horse (as opposed to an object of thought characterizable as a horse), ascribing a predicate like '— is asleep' to **J** in

[13]If Alice thinks of her intentional object **I** as being non-existent, $R(w_0) \not\subseteq marg(\mathbf{I})$. Her belief about **I**'s non-existence is correct if $w_0 \in R(w_0)$, even though $w_0 \notin marg(\mathbf{I})$. For a systematic discussion of why the inclusions $marg(\mathbf{I}) \subseteq R(w_0) \subseteq marg(\mathbf{I})$ may fail, see Sects. 4.7 and 6.2.

w_0 amounts to ascribing it to the realization $\mathbf{J}(w_0)$ of $\mathbf{J}$ in w_0. Intentional predication pertains to a whole set of worlds, physical predication to a single world. □

Indeterminacy of an intentional object amounts to there being predicates that are not uniformly distributed over its internal modal margin. More specifically, an intentional object **I** is indeterminate with respect to predicate Q if, in its internal modal margin, some but not all its realizations satisfy Q. For example, if Alice's object of thought is a winged horse, this object of thought is presumably indeterminate with respect to weight. Quite possibly, it can be characterized neither as weighing 520 kg nor as not weighing 520 kg: some of its realizations weigh 520 kg, others do not. In order for an intentional object to be indeterminate, it is, of course, not required that it be indeterminate with respect to *all* predicates. If Alice's object of thought **I** indeed is a winged horse, then the predicate '— is a winged horse' precisely is uniformly distributed over the internal modal margin of **I**: all realizations of **I** in worlds belonging to its internal modal margin satisfy this predicate. What is more, all intentional objects are trivially determinate with respect to predicates such as '— is or is not a pig' and '— is such that $2 + 3 = 5$'. The phenomenon of indeterminacy will be discussed in Sect. 4.5.

Example 3.5 (*Identity and quantification*) The formula $\mathsf{A}x\ \ x = x$ is refutable. To see this, consider a model M in which a set $\mathcal{I}^{\alpha}_{w}$ contains a world line $\mathbf{J}$ not realized in w. Thus, the assignment $x := \mathbf{J}$ fails to satisfy the formula $x = x$ in w. It follows that $M, w \not\models \mathsf{A}_{\mathsf{a}}x\ \ x = x$ and therefore $M, w \not\models \mathsf{A}x\ \ x = x$. Even the formula $\mathsf{E}x\ \ x = x$ is refutable: it suffices to consider a world v such that for all agents α and all world lines $\mathbf{J} \in \mathcal{I}^{\alpha}_{v}$, we have $v \notin marg(\mathbf{J})$. The formula $\forall x\ \ x = x$, again, is trivially valid. So is, in fact, $\exists x\ \ x = x$: since we have assumed that the domain of every model contains at least one world and that the domain of every world contains at least one local object, it follows by hypothesis H4 that in every world w, there is available at least one physically individuated world line, trivially realized in w. By contrast, the formula $\forall x \Box x = x$ is refutable: there can be a physical object $\mathbf{I} \in \mathcal{P}_w$ realized in w but not in all worlds w' accessible from w. Finally, the necessitist formula $\Box\forall x\Box\exists y\ x = y$ is refutable.[14] Consider a model N with $R(w, w')$ and $R(w', w'')$ such that $w' \neq w''$ and $\mathcal{P}_{w'} = \{\mathbf{I}\}$ and $\mathcal{P}_{w''} = \{\mathbf{J}\}$, and the only world in which $\mathbf{I}$ is realized is w' and the only world in which $\mathbf{J}$ is realized is w''. Now, $N, w'', x := \mathbf{I}, y := \mathbf{J} \not\models\ x = y$, because $w'' \notin marg(\mathbf{I})$. Since $\mathcal{P}_{w''} = \{\mathbf{J}\}$, it follows that $N, w'', x := \mathbf{I} \not\models \exists y\ x = y$, and because $R(w', w'')$, it further ensues that $N, w', x := \mathbf{I} \not\models \Box\exists y\ x = y$. As $\mathbf{I} \in \mathcal{P}_{w'}$, we have $N, w' \not\models \forall x\Box\exists y\ x = y$, and we may conclude that $N, w \not\models \Box\forall x\Box\exists y\ x = y$. □

Two agents may share an object of thought. They may be thinking of the same intentional object, even if the intentional object does not exist. Or at least so it seems. How should such claims be understood? It should be possible to discern the logical form of statements, according to which, to put it as Geach [34, p. 627] does, 'a number of people, or one person on different occasions, have attitudes with a

[14] For necessitism, see Sect. 3.7.

common focus, whether or not there actually is something at that focus'.[15] In Geach's famous example, two farmers are said to have beliefs pertaining to a specific witch, whence the common object of their thoughts is a non-existent intentional object.

Example 3.6 (*Intentional identity*) Let us construe $\Box_{\mathsf{a}}$ and $\Box_{\mathsf{b}}$ as belief-operators. One way of reading the statement

1. α believes that a witch is P and β believes that the same witch is Q

is to take it to have the form

2. $\mathsf{E}_{\mathsf{a}} x\ \mathsf{E}_{\mathsf{b}} y\ \big(\Box_{\mathsf{a}}[W(x) \wedge P(x)] \quad \wedge \quad \Box_{\mathsf{b}}[W(y) \wedge Q(y)] \quad \wedge \quad \Box_{\mathsf{b}}\, x = y\big).$

This proposal does not suffer from the scope problems to which Geach called attention [34, pp. 628–30], simply because intentional quantifiers are not ontologically committing. Formula (2) says in w_0 that α has an object of thought to which α intentionally ascribes the predicate '— is a witch who is P', β has an object of thought to which β intentionally ascribes the predicate '— is a witch who is Q', and moreover, the two intentional objects coincide when attention is restricted to what β believes. That is, there is a world line $\mathbf{I}$ intentionally individuated by α and a world line $\mathbf{J}$ intentionally individuated by β such that $R_{\mathsf{a}}(w_0) \subseteq \mathit{marg}(\mathbf{I})$; $R_{\mathsf{b}}(w_0) \subseteq \mathit{marg}(\mathbf{J})$; for all $w_1 \in R_{\mathsf{a}}(w_0)$, $\mathbf{I}(w_1)$ satisfies the predicates W and P; and for all $w_2 \in R_{\mathsf{b}}(w_0)$, $\mathbf{J}(w_2)$ satisfies the predicates W and Q, and moreover, we have $\mathbf{I}(w_2) = \mathbf{J}(w_2)$.

The truth of (2) requires that we can speak of α's intentional object $\mathbf{I}$ relative to the set of worlds compatible with β's intentional state. The modal margin of $\mathbf{I}$ must comprise *both* sets $R_{\mathsf{a}}(w_0)$ and $R_{\mathsf{b}}(w_0)$, but $R_{\mathsf{b}}(w_0)$ need not be included in $R_{\mathsf{a}}(w_0)$. Actually, $R_{\mathsf{a}}(w_0)$ and $R_{\mathsf{b}}(w_0)$ must be disjoint if P and Q are logically incompatible. Generally, the truth of (2) requires that there be *collective* objects of thought. An intentional object $\mathbf{J}_1 \in \mathcal{I}^{\gamma}_{w_0}$ is in this sense collective if there is an agent δ distinct from γ and an intentional object $\mathbf{J}_2 \in \mathcal{I}^{\delta}_{w_0}$ such that (i) $\mathbf{J}_1$ is realized in at least one world belonging to $R_{\mathsf{d}}(w_0)$ but not to $R_{\mathsf{c}}(w_0)$, and (ii) $\mathbf{J}_1$ locally captures $\mathbf{J}_2$ in the sense that $\mathbf{J}_1(w) = \mathbf{J}_2(w)$ for all $w \in \mathit{marg}(\mathbf{J}_2) \cap R_{\mathsf{d}}(w_0)$. In a precise sense, such intentional objects 'transcend' the beliefs of γ. The main semantic feature of (2) that can cast doubt on its being a good formal rendering of (1) is that it is asymmetric with respect to the agents. Its truth does not require that the set $R_{\mathsf{a}}(w_0)$ be included in the set $R_{\mathsf{b}}(w_0)$, which is why it does not entail (3):

3. $\mathsf{E}_{\mathsf{a}} x\, \mathsf{E}_{\mathsf{b}} y\big(\Box_{\mathsf{a}}[W(x) \wedge P(x)] \quad \wedge \quad \Box_{\mathsf{b}}[W(y) \wedge Q(y)] \quad \wedge \quad \Box_{\mathsf{a}}\, x = y\big).$

It is not obvious that a pretheoretical understanding of claim (1) motivates assuming any such asymmetry. □

As noted earlier in this section, Crane holds that it makes no sense to speak of identity in connection with non-existent intentional objects. He takes intentional

[15] We must be able to pinpoint the logical form of such statements if we take it to make sense in the first place to say anything of the sort. For *this* issue, it is irrelevant how unlikely it might be that a given statement of such a form is in fact true.

identity not to be identity of any kind but similarity of representation [21, p. 164]. If so, a witch α thinks of cannot be the same as a witch β thinks of. All that can happen is that the two representations are qualitatively more or less alike. Twisting a Quinean phrase, Crane expresses this by saying that there is no identity without an entity [ibid. p. 162]: he maintains that the notion of identity does not apply to non-existent intentional objects. I disagree: insofar as we may quantify over intentional objects, it must be meaningful to say that two agents are thinking of the same object at a fixed time or that a fixed agent is thinking of the same object at two times. I do not mean to say that cases of intentional identity proper occur often in practice, but I insist that if we are entitled to speak of intentional objects in the first place, it cannot be devoid of sense to apply the notion of identity to them. To twist Quine's words once more: no value of a quantified variable without identity. I agree that intentional identity is problematic in a way that identity of physical objects is not. The problem is not, however, that we could not speak of identity or numerical distinctness of intentional objects. In terms of the language L, we can do so indirectly, by speaking directly of the local identity or distinctness of their realizations. The problem is that there can be many interpretational options concerning the set of worlds over which intentional objects correlated with two agents are to be compared. In the example above, should we consider $R_{\mathsf{a}}(w_0)$ or $R_{\mathsf{b}}(w_0)$ or rather $R_{\mathsf{a}}(w_0) \cup R_{\mathsf{b}}(w_0)$? The very idea of a collective intentional object is to some extent puzzling. However, not all cases of intentional identity involve collective intentional objects in the sense specified in Example 3.6. One way of understanding the statement

4. α and β share an object of thought

is to take it to be of the form

5. $\mathsf{E}_{\mathsf{a}} x \; \mathsf{E}_{\mathsf{b}} y \left(\Box_{\mathsf{a}} \, x = y \quad \wedge \quad \Box_{\mathsf{b}} \, x = y\right).$

In order for (5) to be true, it suffices that $R_{\mathsf{a}}(w_0) = \mathit{marg}(\mathbf{I}) = \mathit{marg}(\mathbf{J}) = R_{\mathsf{b}}(w_0)$ and $\mathbf{I}(w) = \mathbf{J}(w)$ for all worlds w in the common internal modal margin of these world lines. Thus, $\mathbf{I}$ and $\mathbf{J}$ are not collective intentional objects in the potentially problematic sense.

3.6 Hintikka on Two Modes of Identification

Given that the idea of using world lines in the semantics of quantified modal logic stems from Hintikka and given especially that he has wished to use in his modal language two types of quantifiers with their associated 'modes of identification', it is appropriate to briefly clarify how his ideas are related to mine.

Starting from his earliest writings on the semantics of quantified modal logic, Hintikka discerned two modes of cross-identification: the *perspectival* and the *public* mode. He lumps together a number of related but different ideas by speaking of the distinction, at least in all of the following ways: perceptual vs. physical method of individuation [42]; cross-identification by acquaintance vs. descriptive cross-identification, demonstrative (or perspectival or contextual) vs. physical (or public) method of cross-identification (or individuation) [46]; perspectival vs. public mode of identification [47, 49, 52]; and subject-centered vs. object-centered mode of identification [51, 52]. Given Hintikka's inclination toward viewing world lines epistemically, it is not surprising that he understands the two modes in squarely epistemic terms. Consequently, just as his general interpretation of world lines cannot serve as a foundation for quantified modal logic, the way he construes the two modes of identification cannot be used for semantic purposes. (Saying this is not to deny the epistemological interest of Hintikka's discussion.) The problem is, as explained in Sect. 1.6, that world lines, when epistemically understood, presuppose the relevant notion of cross-context identity instead of giving rise to it.

Hintikka takes the distinction between the two modes to have clearest applications in the case of visual perception. As already noted, he views perception as a modality. The totality of α's visual stimuli in a situation w_0 leaves open a number of scenarios—namely, the set $S(w_0)$ of scenarios v such that for all α can tell on the basis of her visual perception in w_0, she could be in v. This set can be said to consist of α's 'visual alternatives' at w_0. As likewise observed already, in Hintikka's view, a set such as $S(w_0)$ leaves open the questions of identity and nonidentity between members of domains of these scenarios. To settle these questions, cross-identifications between elements of distinct domains are needed. Hintikka [46, pp. 48, 52] suggests that in connection with perception, memory, and knowledge, such cross-identifications can be based on methods of two essentially different kinds: the perspectival and the public. He attempts to clarify the relevant notion of identification by saying that identifying an object means placing it in some framework or map [52, p. 96]. The different types of identification are supposed to be distinguished by the sorts of frameworks in which objects are placed. In perspectival identification, α's visual space is taken to provide the relevant coordinate system. Here, Hintikka assumes that we can unproblematically speak of the *same position* in α's visual space relative to α's distinct visual alternatives. Perspectival identification is agent-relative but not for this reason subjective or private in character. Concerning public identification applied to persons, Hintikka says that the requisite framework is 'something like the social organization of the people in question', based on their roles in public life; see [49, p. 31], [52, p. 96]. While perspectival identification is supposed to employ positions in one's visual space, public identification of persons is meant to utilize such features as people's names and social security numbers. Other features are bodily continuity, continuity of memory, and physical laws regulating movements of material bodies [44, p. 170]. Hintikka says that in the presence of the two modes of identification, it is 'tempting to speak of visual objects and public objects' when one means, respectively, visually and publicly identified objects. Yet, he stresses that in any scenario, there is just one set of objects [49, p. 31]: speaking of visually and publicly identified objects is to speak of one stock of ordinary objects relative to two frameworks.

Described as above, one could expect that for Hintikka, the two modes are mutually independent and constitute two ways in which cross-world identifications may emerge in connection with perception. Yet, he affirms that '[i]n a typical successful public identification, the perceiver finds (on the basis of [the available] momentary visual information) a slot for some given visual object. . .among his or her public objects'. In an example he gives, *that woman over there* is identified as Lady Bird Johnson. Further, he says that 'in a typical visual identification, the perceiver finds a slot for some known public entity among his or her visual objets'. Here, Hintikka's example is Dr. Livingstone identified as *that gentleman over there*. (For these examples, see [49, p. 31], cf. [47, p. 221].) Hintikka is saying that public identification means placing a perspectivally identified object on a public map, whereas perspectival identification means placing a publicly identified object on a perspectival map.

The phenomena illustrated by the examples above are of unmistakeable epistemological interest, but resorting to them in trying to make sense of the two modes of identification is rather desperate. If public identification is an operation applicable only to results of perspectival identification and perspectival identification is an operation applicable only to results of public identification, we never get to apply either operation in the intended way. In Hintikka's examples, it is clearly intended that the former First Lady of the US is publicly identified without any intervening perspectival identifications on the part of the agent, and similarly, *that gentleman over there* is meant to be perspectivally identified independently of any previous public identifications. While Hintikka fails to notice this, in effect he uses the terms 'public identification' and 'perspectival identification' in two senses, one of which must be considered, by his own standards, as more fundamental than the other. First, there are cross-identifications and, for that matter, two modes of carrying them out. Second, there are comparisons between objects identified in different ways: results of perspectival identifications can be compared with results of public identifications. Talking about identifications in the latter, non-fundamental sense is out of place if it is meant to convince anyone of the interest of the existence of the two modes in the fundamental sense.

When speaking of the two modes, Hintikka seems to have in mind merely two ways of reasoning about one and the same stock of independently available individuals—in one case in terms of neutral, context-insensitive features and in the other case with reference to context-relative properties. If we take seriously Hintikka's talk of social organization as a framework for public identification, he has in mind something different from physical individuation in the sense discussed in Sect. 3.2. Apparently, he takes the framework to consist of properties that individuals are likely to have over large spans of time and that can be used for uniquely describing them—in particular, properties that people may have as members of a society. Perspectival identification is supposed to be based on one's momentary visual information and to be relative to a framework of positions in an agent's visual space. This view on perspectival identification is problematic for two reasons. First, the notion of visual space (or visual field) is severely ambiguous, as Austen Clark [16] convincingly argues.[16] Second, if the cross-world notion of *same individual* is indeed considered problematic, why should the somewhat more abstract notion of *same spatial position*

[16]Clark distinguishes three senses of 'visual field': (1) the sum of physical things seen; (2) the sum of visual representings: an array of visual impressions organized so that spatial relations obtaining

(in one's visual space) be any less problematic in cross-world settings? Then again, if spatial positions themselves are construed as world lines, it must be possible to get clear about their mode or modes of identification before they can be used in an account of visual objects.

As I see it, the distinction between physically and intentionally individuated world lines is a robust systematic distinction, motivated by the idea that cross-world identity is a notion to be analyzed and the fact that in natural language, we are quite capable of reasoning in terms of both physical and intentional objects. In semantics, we may need world lines of these two types not only to interpret 'first-order quantifiers' in modal contexts but even in more abstract cases: when speaking of *the same time* and *the same spatial location* in many-world settings (cf. Tulenheimo [119]). Hintikka's attempt to formulate a distinction between two modes of identification is unsatisfactory. He mixes epistemic and conceptual considerations without warrant. The notions of cross-world identification he develops do not live up to the standards he himself has set: their meaningfulness presupposes the availability of suitable cross-world links, instead of accounting for the nature of such links.

3.7 Necessitism and World Lines

In his book *Modal Logic as Metaphysics* [127], Williamson argues for *necessitism*: he defends the claim 'necessarily everything is necessarily something'. In terms of first-order quantifiers and the operator $\Box$ for metaphysical necessity, this means holding that $\Box\forall x\Box\exists y\ x = y$ is a valid formula. Here, the range of the quantifiers 'everything' and 'something' is taken to be absolutely unrestricted.[17] That these quantifiers are not contextually restricted means that their range does not vary depending on the possible world in which they are evaluated. Necessitism ensues if unrestricted quantification is combined with the assumption that the things over which we can quantify in one world satisfy the identity formula $x = x$ in all worlds.[18] According to the necessitist, then,

(Footnote 16 continued)
among the impressions resemble the spatial relations among things; and (3) the sum of things *as* represented visually: what the world would be if it were just as it visually appears to be.

[17] Postulating unrestricted quantification is *not* the same as subscribing to necessitism. If among all the things that exist (in the unrestricted sense) there was a thing that is possibly nothing, then necessitism would fail. Cf. Williamson [127, pp. 15–6].

[18] Namely, if w and w' are worlds and the value of x belongs to $range(w)$, then by the mentioned extra assumption, its value satisfies $x = x$ in w'. If quantification is unrestricted, then $range(w) = range(w')$, from which it follows that the value of x satisfies $\exists y\, x = y$ in w'. Note that if we further assume that a value of x cannot satisfy a predicate $P(x)$ in a world without satisfying the identity $x = x$ therein, we may infer what Williamson [127, p. 149] calls the *being constraint*, expressed by the formula $\Box\forall x\Box(P(x) \rightarrow \exists y\ x = y)$.

there is a necessary pool of objects that are possible values of quantified variables in every world and that moreover 'are something' in every world. *Contingentists* accept the negation of the necessitist claim: either they take the range of quantifiers not to be the same in all worlds or they deny that an arbitrary value of a variable 'is something' in all worlds.

Let us say that a set X of objects is a *quantification pool* if $range(w) = X = range(w')$ for all worlds w and w'. It is an *identity pool* if for all w and w', the set of objects satisfying the formula $x = x$ in w equals X, which furthermore equals the set of objects satisfying $x = x$ in w'. Necessitism states that a certain set is both a quantification pool and an identity pool. Kripke semantics rejects the domain constraint and is committed to an identity pool but not to a quantification pool. My world line semantics is committed to neither, independent of whether by 'quantification' we intend physical or intentional quantification. There is no apparent contradiction with the fourth combinatorial possibility either—i.e., holding that there is a quantification pool but no identity pool. This is what we get if the domain of each world is a proper subset of the quantification pool and the identity predicate obeys the domain constraint.

Discussing the Barcan formula $\Diamond\exists x\phi \rightarrow \exists x\Diamond\phi$ understood as a schema, Prior notes that we appear to be able to refute it by interpreting $\Diamond\psi$ as 'it will be the case that ψ' and considering its following instance: 'if it will be the case that someone is flying to the moon, then there exists someone who will be flying to the moon' [96, pp. 26, 29]. The antecedent would have been true in 1900, because in 1969, Neil Armstrong flew to the moon, but we might well be reluctant to admit that the consequent was true in 1900, as well, for Armstrong was yet to be born then. In Prior's understanding, the variable x in $\Diamond\exists x\phi$ has, in 1900, no range of values at all; the truth-value of $\Diamond\exists x\phi$ in 1900 depends on the range of values that x will acquire at later times [ibid. p. 32]. Prior sees postulating a permanent pool of things as the only way to save the validity of the Barcan formula. He notes that while one might be able to invent a metaphysical justification for the pool hypothesis, there is no logical reason to accept it, and therefore, one had better not base one's logic on assuming such a pool [ibid. pp. 29–30]. Now, in order to validate the Barcan formula, it is actually sufficient to postulate a *quantification* pool. No identity pool is needed—unless one's philosophical convictions happen to tie the two issues together. Prior's convictions happen to do so. He declares that 'x exists' means 'there are facts about x'—which in his analysis amounts to x being a subject of a predicate [ibid. p. 31]. If so, x is an object that exists at all times iff at all times some predicates are applicable to x. Among those predicates, there is, according to Prior, always at least the identity predicate: he maintains that if we can frame a statement about x in the first place, then x satisfies the identity predicate (cf. [ibid. p. 59]).

Hintikka fiercely criticizes the idea that in modal semantics, we could take a set of 'prefabricated individuals' as our starting point and define possible worlds as different ways in which various characteristics can be distributed over this set [46, pp. 122, 208–9]. He takes this view to misrepresent the relation of cross-world identity. It was explained in Sect. 1.5 that if a set D was the common domain of individuals of all worlds considered, questions of cross-world identity would be trivially settled.

If w_1 and w_2 were any two worlds with $b_1 \in dom(w_1)$ and $b_2 \in dom(w_2)$, then both b_1 and b_2 would be elements of D, and therefore, one of the relations $b_1 = b_2$ and $b_1 \neq b_2$ would hold. According to the transcendental interpretation of world lines, questions of identity and numerical distinctness cannot be meaningfully posed concerning local objects of distinct worlds. This fact does not by itself mean that there is no identity pool, since values of variables are not local objects but world lines. However, if there was an identity pool, no world line could be realized in one world without also being realized in all other worlds—given that the semantics of identity formulas is kept intact. Yet, neither physically nor intentionally individuated world lines are, generally, realized in all worlds. Since necessitism postulates an identity pool, it follows that the view on which world line semantics is based refutes both necessitism about intentional objects and necessitism about physical objects. Insofar as the set of world lines available to be quantified over is taken to vary depending on the world considered, the view also goes against postulating a quantification pool. Hintikka certainly takes there to be such variation, but his grounds for thinking so mainly stem from the epistemic interpretation: 'methods of identification' that agents have at their disposal are world-dependent.

Is the transcendental interpretation of world lines antithetical to unrestricted quantification? This question must be posed separately for intentional and physical quantifiers. The idea of unrestricted quantification over intentional objects would be utterly incredible. Suppose Macbeth is hallucinating a dagger in 1055 at t_0. He would presumably be able to talk about the dagger at that time: the dagger is an intentional object for him then and, therefore, a possible value of the intentional quantifier $\mathsf{E}_{\mathsf{Macbeth}}$ at t_0. Given the nature of intentional objects (being essentially dependent on the agents whose intentional objects they are), how could anyone defend the idea that the particular hallucinated dagger is available to be talked about at *all times* in our world, let alone at all times in *all possible worlds*? The dagger was not available even to Macbeth himself prior to t_0. Once hallucinated, Macbeth may have forgotten about the dagger, being from that time on unable to think of it. Unrestricted quantification applied to intentional objects would entail that the hallucinated dagger would nevertheless be available to be talked and thought about at those later times. It would still now, almost 1000 years later, be so. Thought by whom? In virtue of what? Excluding the existence of a universal stock of thoughts ever thought, the mere idea is as close to an absurdity as a philosophical proposal can be. Williamson expressly avoids discussing intentionality, thereby ignoring an important variety of objects we resort to in our philosophical and everyday discourse.[19] Of course, any theoretical inquiry must be limited somehow, so this is excusable.

What about unrestricted quantification over physical objects? If we are always bound to a specific world, can we ever be in a position to speak about the totality of world lines physically available in one world or another? Are all elements of the union $\bigcup_{w \in W} \mathcal{P}_w$ in *some* reasonable sense available to be talked about in a fixed world w_0? It would be too quick to reply 'no' just because the semantics I have formulated

[19] He states that his book is about metaphysical modality, not about intentionality [127, p. 217].

does not allow quantifying over non-realized physical objects. The semantics could be reformulated if doing so turned out to be motivated. It would also be too quick to reply 'yes' simply because when describing models of L, we do just that: we talk about the union $\bigcup_{w \in W} \mathcal{P}_w$. When reasoning in terms of models, we adopt a neutral metatheoretic viewpoint. This is not only perfectly legitimate but also unavoidable in a model-theoretic approach to semantics. However, this does not mean that when we are in the midst of actually using language, we have automatic access to the full panorama of all meaning-theoretically relevant items that someone abstractly modeling our language-use wishes to incorporate in her model.

Supposing that all individuals that *would* be available to be talked about under such-and-such circumstances already *are* available to be talked about is to prejudge that counterfactual situations and counterfactual individuals are on a par with situations and individuals already actualized. Countless possible states of affairs *will* or *could* get realized and, with them, countless individuals. It is by no means evident that for this reason, those individuals *already now* lie in the range of our quantifiers. Prior argues against the idea of a quantification pool by insisting that we can speak of mere possibilities only in general terms. In 102 BC, it would have been true to say 'The governor of the province of Asia and his wife will have *a* child who will become the dictator of the Roman Empire', but it would *not* have been possible even to frame the statement 'Julius Caesar will become the dictator of the Roman Empire' [98, pp. 70–1]. This is not a consequence of our epistemic limitations: prior to 100 BC, there *is* no individual Julius Caesar. As Prior explains, before 100 BC, there was no such possibility as Julius Caesar being born to his parents, and yet, this very possibility was realized in 100 BC!

I do not claim to refute necessitism in this book. The way I formulate my semantic framework uses the identity predicate only locally and therefore goes against the postulation of an identity pool.[20] The framework itself is neutral regarding the existence of a quantification pool. However, when applying the framework, I assume that the ranges of physical quantifiers can vary depending on the world considered: my model-theoretic observations assume the truth of contingentism.[21] I take this to be in line with the transcendental interpretation of world lines. It is a structural requirement of our thinking and talking of individuals that they are thought of as world lines, but the question of which specific individuals fit this structural scheme in specific worlds can only be answered a posteriori, and it is certainly not obvious that we should suppose the same set of individuals to be given in all worlds. Besides, necessitist discourse can—to some extent, at least—be simulated in contingentist terms. The necessitist would construe 'Socrates no longer exists' as meaning that

[20] However, this is merely a fact concerning the specific language L. For some purposes, it is desirable to consider an intensional identity predicate (applied to world lines instead of local objects); see Sect. 5.6. The corresponding extended language would be committed to an identity pool.

[21] Some of the models I describe are acceptable to the necessitist only from the viewpoint of a non-standard interpretation of quantifiers. The necessitist could view the sets $\mathcal{P}_w$ I define as subsets of the fixed quantification pool consisting of physical objects that are *concrete* in w. Williamson [127, p. 19] finds it preferable to avoid using the word 'exists' but notes that if a necessitist nevertheless wishes to use this word, it can be construed as an existential quantifier restricted to *concrete* things.

there *is* an individual who was but no longer is concrete. In my framework, this sentence can be understood as expressing at t that there was, at a certain time t' earlier than t, a physical object $\mathbf{I}$ (namely, the individual Socrates) that belonged to $\mathcal{P}_{t'}$, and this individual $\mathbf{I}$ is not realized at t.

One may be tempted by necessitism, because undeniably, we can describe mathematical models in which there are a number of alternatives as to what may happen tomorrow, phrased in terms of possible scenarios, some of which may involve individuals that are not yet actualized. Such a model employs the fact that however things go, in all possible scenarios, some individuals are realized. In my framework, it is perfectly in order to model arbitrary individuals—also those not yet realized—by partial functions. But, it would be a misinterpretation of the mathematical model to think that all such functions represent individuals of which we, in our actual situation, can talk about. The model is based on the idealization that all possibilities have already been realized and, with them, all individuals that ever will or could come into being. The contingentist maintains that there will always remain possibilities yet to be realized and individuals that might yet come into being.

3.8 Meinongianism and World Lines

According to Meinongians, we can talk about non-existent things. In their view, 'exists' is a predicate, and this predicate does not apply to all objects over which our quantifiers range. My analysis agrees with this aspect of Meinongianism: there are worlds in which the claim 'Some things do not exist' is true, provided that 'some' is construed as an intentional quantifier and 'exists' is interpreted in terms of realization. Williamson [127, p. 19] points out that the necessitist can similarly endorse the claim 'Some things do not exist', if the quantifier 'some things' ranges over everything without discrimination but 'exists' applies to concrete things only. What makes Meinongianism problematic is its adherence to the *characterization principle* (CP), according to which there is a very large class $\mathcal{C}$ of precicates (including ones like '— is a round square' and '— is a golden mountain') such that each predicate in $\mathcal{C}$ characterizes an object.[22] Meinongians acknowledge that $\mathcal{C}$ must be limited somehow *and* that it is difficult to say how to delineate it. Yet, they are not sufficiently discouraged to give up the principle. Necessitists are not committed to the idea that for any predicate (of the sort Meinongians may wish to consider), *there is* an object satisfying it. Necessitism does not entail that there is a golden mountain, let alone a round square. What can be said of CP in the context of world line semantics?

It is particularly obvious that CP does not hold for *physical objects*. For example, the predicate '— lives in Baker Street and is a detective of acute powers of observation

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[22]For CP, see, e.g., Priest [95, Sect. 4]. Both Crane [21, pp. 27, 58–9] and Williamson [127, p. 19] identify CP as the source of the problematic consequences of Meinongianism.

and inference' does not characterize a (possible or actual) physical object. This is because ordinary physical objects are temporally extended material bodies, never immutable with respect to all their properties. Presumably, no one is throughout his or her life a detective of acute powers of observation and inference. With *intentional objects*, things are somewhat different. It can indeed well happen that for a specific agent in a specific world, a suitable predicate characterizes an intentional object. Yet, this is not a reason to accept CP for intentional objects: certainly, not every predicate of the kind Meinongians might want to consider characterizes an intentional object for a fixed agent. It all depends on what the agents happen to think. In my analysis, contradictory predicates never characterize an intentional object, and when a predicate indeed characterizes an intentional object, this fact has no *metaphysical* consequences. Any consequences are of a psychological nature.

Priest [95, pp. 84–5] argues that CP holds in full generality if it is modally qualified: for any condition $P(x)$, there is an object that is characterized by $P(x)$ *in at least one world*. The object may satisfy $P(x)$ in many worlds. The actual world need not be one of them.[23] My analysis of intentional objects bears resemblance to Priest's account in several respects. First, Priest takes the characterized object of thought to satisfy the predicates utilized in its characterization in *all worlds* compatible with the way the agent represents things to be. Similarly, in my account, ascribing a predicate $P(x)$ to an intentional object means that it satisfies $P(x)$ throughout its internal modal margin (cf. Example 3.4). Second, Priest notes that the agent's representation is always incomplete: the intentional object will be indeterminate with respect to various predicates. The object of thought Sherlock Holmes will be a detective of acute powers of observation and inference in all relevant worlds but left-handed in some worlds and right-handed in others. Correspondingly, I take intentional objects to be always indeterminate in some respects: there are always a number of predicates an intentional object satisfies in some but not all worlds in which it is realized. Third, Priest takes objects (including objects of thought) to be functions from worlds to what he calls 'identities' [95, p. 43], and I take world lines (including those that are intentionally individuated) to be partial functions from worlds to local objects.

It is not Priest's innovation to analyze objects of thought in terms of functions defined on worlds. Hintikka uses his notion of world line predominantly for the very purpose of discussing such intentional states as belief and perception—though he has not analyzed the distinction between physical and intentional objects along the lines proposed in this book. The novelty in Priest's proposal is rather, in effect, to see CP as applying to world lines enjoying the characterizing properties over a set of worlds—a set that may exclude the actual world.[24] Apart from the formal similarities

[23]Priest postulates impossible worlds and countenances impossible objects, so he is prepared to accept even instances of CP corresponding to contradictory conditions $P(x)$; see [95, pp. 15, 58].

[24]Priest reports that Nicholas Griffin and Daniel Nolan have likewise considered the idea that those objects that have the characterizing properties as postulated by CP have them in suitable non-actual worlds. He refers to the unpublished papers 'Problems in Item Theory' (Griffin) and 'An Uneasy Marriage' (Nolan), both read at the 1998 meeting of the Australasian Association for Logic.

between Priest's account and mine, there are important differences. First, I pointed out in Sect. 3.7 that it would lack all psychological credibility to assume that the set of intentional objects available to be quantified over is the same, independent of the world considered. Priest, however, employs a constant-domain semantics in which the range of quantifiers—including what he calls the objects of thought—indeed is the same in all worlds [95, p. 12]. Second, I distinguish formally two types of world lines, while Priest operates with a single set of objects. Yet, there is no reason to give serious metaphysical weight to intentional objects. They are not there to be quantified over if no one is thinking of them: they are mere correlations of local objects across worlds that are compatible with an agent's thoughts at a given moment. Already for this reason, it serves clarity not to fuse physical and intentional objects into a single stock of objects, even relative to a single world. Besides, as will become clear in Chap. 4, the modal behavior of physical objects is very different from that of intentional objects. Third, from my viewpoint, we could not speak of the cross-world behavior of physical and intentional objects in the first place unless they were construed as world lines. Priest, again, conceptualizes objects as functions defined over worlds, because he wants to discuss different roles that objects may have in different worlds. He does not find anything problematic in the notion of cross-world sameness as such. For me, local objects are strictly world-bound. They provide the ground level at which the simple notion of extensional identity is applicable. Priest does not require that 'identities' be world-bound: he allows a given object to have literally the same 'identity' in distinct worlds [ibid. p. 45]. Fourth, Priest stipulates that the formula $x = y$ holds in a world w relative to a function s mapping variables to objects iff the 'identities' $s(x)(w_0)$ and $s(y)(w_0)$ are indistinguishable in terms of atomic predicates in the actual world w_0 [ibid. pp. 44, 87–8].[25] I take the formula $x = y$ to hold in w relative to an assignment g mapping variables to world lines iff the world lines $g(x)$ and $g(y)$ are realized in this same world w and the local object $g(x)(w)$ equals the local object $g(y)(w)$. In my analysis, the fact that the actual realizations of **I** and **J** satisfy the same predicates in w_0 does not even guarantee that the assignment $x := \mathbf{I}$, $y := \mathbf{J}$ satisfies $x = y$ in w_0. This fact does not provide the slightest indication to the effect that this assignment satisfies $x = y$ in an arbitrary world. By grounding the sameness of objects on properties of their actual identities, Priest in effect trivializes their functional nature and ends up blocking the possibility of distinct but overlapping objects. In my view, worlds and world lines are mutually independent modal unities, and questions about the sameness of world lines cannot generally be answered by inspecting any single world.

If subscribing to a version of CP is a necessary condition for defending Meinongianism, then my semantic account is not Meinongian. My analysis shares with Priest's view the understanding of what it means to characterize an intentional object in terms of a predicate, but it differs from his account both in the formal details and in the philosophical motivation of the associated semantic framework.

[25] In Priest's view, an object cannot lack an actual identity: all objects are defined in all worlds.

Chapter 4
Intentional Objects as World Lines

4.1 Introduction: Features of Intentional Objects

Elizabeth Anscombe took there to be three salient features of intentional objects: indeterminacy, sensitivity to the way in which they are described, and possible non-existence [1, pp. 159, 161, 171]. Relatedly, Tim Crane speaks of accuracy, aspect, and absence as features of intentional states [18, pp. 455–6]. In Edmund Husserl's theory of intentional relations, these or similar features have been termed indeterminacy of characterization, conception-dependence, and existence-independence of intentional relations; cf. [113, pp. 11–7]. Linguistically, these characteristics are reflected in the behavior of verb phrases consisting of an intensional transitive verb and its complement: such verb phrases admit an unspecific reading; substituting a singular term for a coreferential singular term in the complement need not preserve truth; and existential generalization with respect to the complement is not a valid rule of inference when usual ontologically committing quantifiers are employed.[1]

Suppose Alice is thinking of a man. We cannot infer that she is thinking of a man of a particular height, nor that there is a man she is thinking of—in the ontologically committing sense of 'there is'. By contrast, if Alice hits a man, she hits a man of a particular height, and there is a man hit by her. Alice may have a perceptual experience of a unicorn, but she cannot catch a unicorn, supposing none exists. She may think of Patrick Modiano without thinking of the winner of the Nobel Prize in Literature of 2014, but she cannot shake hands with Modiano without shaking hands with the winner of the Nobel Prize in Literature of 2014. Here, *hit*, *catch*, and *shake hands with* are extensional verbs, while *think of* and *have a perceptual experience of* are intensional verbs. Extensional transitive verbs can be analyzed as binary predicates applied to local objects—or as quasi-extensional predicates applied to world lines; cf. Sect. 2.4. Intensional transitive verbs behave differently. Very differently.

I will make use of my semantic framework to discuss intensional transitive verbs in Chap. 6. In the present chapter, I discern two senses of predication that are needed

[1] For an introductory discussion of intensional transitive verbs, see, e.g., Forbes [28].

T. Tulenheimo, *Objects and Modalities*, Logic, Epistemology, and the Unity of Science 41, DOI 10.1007/978-3-319-53119-9_4

when analyzing the two modes of individuation (Sect. 4.2). I illustrate how the three features of description-sensitivity, indeterminacy, and existence-independence can be accounted for in terms of world lines (Sects. 4.3, 4.5 and 4.6). These features are directly related to three questions we can ask about a given world line. First, how can predicates be used to describe the world line? Second, how is its modal margin determined? Third, how are the worlds in which the world line is available related to those in which it is realized? I comment on the property of factiveness that some intentional states have (Sect. 4.4). This property is of interest in relation to states having an existent intentional object, though a state need not be factive to have an existing object, and a state may be factive while its intentional object fails to represent a specific physical object. I take up the question of how contents of intentional states can be analyzed in my framework (Sect. 4.7). Finally, I discuss conditions that must be met in order for an intentional object to be a representation of a physical object (Sect. 4.8).

4.2 Two Modes of Predication

There are different ways in which predicates can be used to describe world lines. As already hinted at in Example 3.4, predication applied to physical objects must be distinguished from predication applied to intentional objects. If Alice talks about a lion living in a local zoo and says that it is sleeping, she is only affirming what this physical object does here and now. She is talking about its *current* realization. By contrast, if Bob is hallucinating a sleeping lion, his object of thought is an entire world line defined over the set of worlds compatible with his perceptual experience. Its being describable as a sleeping lion requires that *all* its realizations satisfy the predicate '— is a sleeping lion'. Intentional objects are by their nature defined over a set of scenarios compatible with an agent's intentional state. Those states are typically compatible with a variety of ways in which the world could be. This is why the modal margin of an intentional object normally contains a number of worlds. Talking about intentional objects makes sense only relative to such a margin, dependent on the agent and the world in which she is located. The unity to be characterized when characterizing an intentional object is the totality of its realizations. We cannot slice up a representation and expect that the slices in isolation continue to serve the intended representative function. Mere realizations of intentionally individuated world lines cannot represent properties of intentional objects independently of the other realizations. By contrast, generally, we can speak of a physical object in a world without thereby being obliged to speak of it in other worlds.

I distinguish two *modes of predication*: the physical and the intentional. Let M be a model with the interpretation function *Int*. Suppose w_0 is a world, $\mathbf{I}$ is a (physically or intentionally individuated) world line, and P is a unary atomic predicate. Further, let $R_{\mathsf{i}}(w_0)$ be the set of worlds compatible with the intentional state i at w_0.

- *Physical predication:* Ascribing P to $\mathbf{I}$ in w_0 under the physical mode is to affirm that $\mathbf{I}$ is realized in w_0 and $\mathbf{I}(w_0) \in Int(P, w_0)$.
- *Intentional predication:* Ascribing P to $\mathbf{I}$ in w_0 under the intentional mode relative to state i is to affirm that $\mathbf{I}(w) \in Int(P, w)$ for all worlds $w \in R_{\mathsf{i}}(w_0) \cap marg(\mathbf{I})$.

Viewing formulas as predicates, these notions can be formulated more generally:[2]

- *Physical predication (general):* Ascribing $\phi(x_1, \ldots, x_n)$ to the tuple of world lines $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ in w_0 under the physical mode is to affirm that $\langle w_0, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in |\phi|^M$.
- *Intentional predication (general):* Ascribing $\phi(x_1, \ldots, x_n)$ to the tuple of world lines $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ in w_0 under the intentional mode relative to state i is to affirm that $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in |\phi|^M$ for all $w \in R_{\mathsf{i}}(w_0) \cap \bigcap_{1 \le j \le n} marg(\mathbf{I}_j)$.

In my logical language, we need not have available separate, syntactically unanalyzed forms for both modes of predication. It is enough that the distinction is expressible. This is the case, since ascribing P to $\mathbf{I}$ under the intentional mode amounts to requiring that the assignment $x := \mathbf{I}$ satisfy the formula $\Box[x = x \rightarrow P(x)]$, where $\Box$ ranges over the worlds compatible with the agent's relevant intentional state. This is to be contrasted with the requirement that the assignment $x := \mathbf{I}$ merely satisfy the formula $P(x)$, which amounts to ascribing P to $\mathbf{I}$ under the physical mode. Satisfaction under the physical mode means, simply, satisfaction in the sense defined by the semantics of L. By contrast, satisfaction of $\phi(x_1, \ldots, x_n)$ in a world w under the intentional mode means that the modalized conditional formula $\Box[(x_1 = x_1 \wedge \ldots \wedge x_n = x_n) \rightarrow \phi(x_1, \ldots, x_n)]$ is satisfied in w in the sense of L-semantics.

It should be noted that a *local object* satisfies necessarily all extensional predicates it satisfies at all: if a local object b that belongs to $dom(w)$ is P in w—that is, if we have $b \in Int(P, w)$—this fact prevails no matter from which perspective we look at it. This is comparable to the fact that if the sentence 'It is raining' is true in a structured world w at t in location s, then the contextualized sentence 'It is raining in w at t in location s' is true in any context whatsoever.[3] Consequently, if in w a *world line* $\mathbf{I}$ satisfies the intensional predicate $P(x)$ in the sense of physical predication, then this fact holds necessarily, since it simply means that the local object $\mathbf{I}(w)$ satisfies the extensional predicate P in w. However, this does not destroy the contingency of physical predication in the relevant sense. To say that a world line $\mathbf{I}$ satisfies an intensional predicate $\phi(x)$ *contingently* in the sense of physical predication means that $\mathbf{I}$ satisfies $\phi(x)$ in some but not all worlds in which $\mathbf{I}$ is realized. More specifically, to say that a physically individuated world line $\mathbf{I}$ satisfies $\phi(x)$ contingently *in world* w means that $\mathbf{I}$ satisfies $\phi(x)$ in w—a fact that itself is not contingent—and that in certain other worlds belonging to the modal margin of $\mathbf{I}$, the world line $\mathbf{I}$ does not satisfy $\phi(x)$. Recalling from Sect. 2.7.4 what it means for a world line to necessarily satisfy a predicate, it follows that $\mathbf{I}$ satisfies $\phi(x)$ contingently iff $\mathbf{I}$ satisfies neither $\phi(x)$ nor $\neg\phi(x)$ necessarily.

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[2] As noted in Sect. 2.4, formulas with n free variables can be viewed as intensional n-ary predicates.

[3] The proposition (content) expressed by the latter sentence is *perfect* in the sense of Kaplan [61, p. 503].

Whereas a world line may well fail to satisfy necessarily a predicate in the sense of physical predication, the situation is different in connection with intentional predication. Recall that a world line **I** satisfies $\phi(x)$ under the intentional mode iff **I** satisfies $\phi(x)$ in *all* worlds belonging to the internal modal margin of **I**. Therefore, whatever predicates an intentional object satisfies under the intentional mode, it satisfies necessarily, not contingently. This is as it must be: intentional objects have their characterizing properties essentially. Intentionally individuated world lines are characterized by ascribing to them predicates in the sense of intentional predication, whence they satisfy all those predicates 'necessarily' in the sense specified. By contrast, an intentional object satisfies 'contingently' any of those predicates with respect to which it is *indeterminate* (cf. Sect. 3.5). Namely, these are predicates $\phi(x)$ such that the intentional object satisfies neither $\phi(x)$ nor $\neg\phi(x)$ under the intentional mode—which means that some of its realizations satisfy $\phi(x)$ and others do not.

Predicates ascribed under the intentional mode have, in my analysis, the role that *pleonastic* properties have in Crane's discussion of non-existent objects (cf. Sect. 3.5). Intentional predication is metaphysically neutral: ascribing $\phi(x)$ to a world line **I** under the intentional mode does not entail that $\phi(x)$ applies to something in the actual world.[4] Whether $\phi(x)$ is an existence-entailing predicate or not, intentionally predicating $\phi(x)$ of **I** at w_0 means affirming that **I** satisfies $\phi(x)$ in all worlds belonging to the intersection $R_{\mathsf{i}}(w_0) \cap marg(\mathbf{I})$—and in all these worlds, **I** exists. However, it does not follow that **I** exists in w_0, unless w_0 belongs to this intersection. By contrast, physical predication simply amounts to satisfaction in the sense specified by the semantics. Physically predicating an existence-entailing predicate $\phi(x)$ of **I** in w_0 indeed commits us to the existence of **I** in w_0. Yet, physical predications need not be existence-entailing: for example, a value of x may satisfy the predicates $\Diamond P(x)$ and $\neg P(x)$ in w_0 without being realized in w_0. Physical predication of existence-entailing predicates resembles attribution of *substantial* properties in Crane's sense.

Physical and intentional predication can be characterized in terms of the relations of local and uniform support spelled out in Definition 2.5. Consider a situated content $C = \langle W, \mathbf{I}_1, \ldots, \mathbf{I}_n, w_0 \rangle$, where $W = R_{\mathsf{i}}(w_0) \cap marg(\mathbf{I}_1) \cap \ldots \cap marg(\mathbf{I}_n)$. Physically predicating $\phi(x_1, \ldots, x_n)$ of the tuple $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ in w_0 means that $C \Vdash_{loc} \phi$, whereas intentionally predicating $\phi(x_1, \ldots, x_n)$ of $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ in w_0 means that $C \Vdash_{uni} [(x_1 = x_1 \wedge \ldots \wedge x_n = x_n) \rightarrow \phi]$. The reason why we must use here the conditional formula $(x_1 = x_1 \wedge \ldots \wedge x_n = x_n) \rightarrow \phi$ rather than the formula ϕ itself is that in the definition of intentional predication, attention is confined to the subset of $R_{\mathsf{i}}(w_0)$ consisting of *those* worlds in which all world lines $\mathbf{I}_1, \ldots, \mathbf{I}_n$ are realized.

Example 4.1 Suppose Alice hallucinates a sleeping lion at w_0. Let $R_{\mathsf{a}}(w_0)$ be the set of scenarios compatible with her perceptual experience at w_0, and let **J** be the intentional object of her experience. In this case, $marg(\mathbf{J}) = R_{\mathsf{a}}(w_0)$. First, in order for her experience to be a hallucination, Alice must take **J**'s existence seriously. There can be no world $w \in R_{\mathsf{a}}(w_0) \setminus marg(\mathbf{J})$. Conversely, hallucinations are exclusively

[4]Recall the convention about the use of the expression 'the actual world' agreed upon in footnote 23 in Sect. 1.4. This expression does not refer rigidly to any distinguished world. It refers to whatever world is considered as the circumstance of evaluation or the scenario in which an agent is located.

concerned with the way the agent takes the world to be (as opposed to illusions, which are grounded in how things in fact are): there is no world $w \in marg(\mathbf{J}) \setminus R_{\mathsf{a}}(w_0)$. Note that $w_0 \notin R_{\mathsf{a}}(w_0)$: the object of hallucination must be non-existent.[5] Now, let $L(x)$ stand for the extensional predicate 'x is a sleeping lion'. That Alice hallucinates a sleeping lion at w_0 means that at w_0, her object of experience $\mathbf{J}$ can be ascribed the predicate '— is a sleeping lion' under the intentional mode: the predicate applies individually to all realizations $\mathbf{J}(w)$ of $\mathbf{J}$, where $w \in R_{\mathsf{a}}(w_0) \cap marg(\mathbf{J}) = R_{\mathsf{a}}(w_0)$. That is, it is required that $w_0, x := \mathbf{J} \models \Box_{\mathsf{a}} L(x)$. By contrast, that a lion is sleeping at w_0 means that at w_0, the predicate '— is a sleeping lion' can be ascribed to a physical object $\mathbf{I}$ under the physical mode, so that we have $w_0, x := \mathbf{I} \models L(x)$. □

Example 4.2 Suppose Bob thinks of a winged horse at w_0. Let $R_{\mathsf{b}}(w_0)$ be the set of scenarios compatible with Bob's beliefs at w_0, and let $\mathbf{I}$ be his object of thought. (Throughout this book, I take it that the semantics of the intensional transitive verb *think of* can be analyzed in terms of the set of worlds compatible with what the relevant agent believes.) There can be worlds w in $R_{\mathsf{b}}(w_0)$ in which $\mathbf{I}$ is not realized: perhaps Bob is just reflecting on what he has heard about Pegasus, without taking its existence seriously. Let $H(x)$ stand for 'x is a winged horse'. That Bob thinks of a winged horse at w_0 means that at w_0, the predicate '— is a winged horse' can be ascribed to his object of thought $\mathbf{I}$ under the intentional mode. That is, the predicate applies throughout the internal modal margin of $\mathbf{I}$: it applies individually to all realizations $\mathbf{I}(w)$ of $\mathbf{I}$ with $w \in R_{\mathsf{b}}(w_0) \cap marg(\mathbf{I})$. In other words, it is required that $w_0, x := \mathbf{I} \models \Box_{\mathsf{b}}[x = x \to H(x)]$. It is *not* required that $w_0, x := \mathbf{I} \models \Box_{\mathsf{b}} H(x)$, because it is not assumed that Bob believes that a winged horse exists. □

It was noted in Sect. 3.8 that my account of intentional predication bears resemblance to what Priest says of predicates characterizing objects of thought: in both cases, an intentional object is taken to be characterized by a predicate $P(x)$ when the object satisfies—in the basic semantic sense of satisfaction—$P(x)$ in all worlds compatible with the agent's intentional state. Priest does not observe the systematic usefulness of the distinction between satisfaction and intentional predication. He claims that it is wrong to think that the properties of Sherlock Holmes are exhausted by what Conan Doyle has told us about him, because in some worlds compatible with our representation, Holmes is left-handed and in others right-handed [95, p. 88].[6]

[5]For the possibility of actually realized objects of hallucination, see Sect. 4.6 and footnote 12 in Sect. 4.4.

[6]Fictional objects such as Sherlock Holmes are possible objects of intentional states. Therefore, they count as intentional objects. Priest analyzes fictional objects modally, in terms of worlds that 'realize the way the agent represents things to be': these are the worlds compatible with the agent's representation. Similarly, intentional objects are in my account considered in relation to the set of worlds compatible with the agent's intentional state. Non-existent intentional objects do not exist (are not realized) in the world w_0 in which the agent having representations or intentional states is situated, but this fact does not render a modal analysis any less applicable. An intentional object is a world line realized over a set of worlds, whether the world w_0 belongs to that set or not. In particular, intentional objects may be possible values of intentional quantifiers in w_0 without being realized in w_0.

While variation in Holmes's handedness over the relevant set of worlds superficially looks like information that adds to what Conan Doyle says, this is not so. Precisely because the totality of what the author says provides a list of predicates intentionally ascribed to Holmes *and* serves to specify that no further predicates are intentionally ascribed to him, Holmes the intentional object is indeterminate with respect to the latter predicates, which *means* that Holmes satisfies (in the basic semantic sense of satisfaction) any such predicate in some but not all worlds considered. Priest does not fully appreciate the fact that the sense of predication involved in saying that Holmes is left-handed is not the same as the sense in which Holmes is characterized as being a detective. Holmes the intentional object is not realized in w_0. Thus, he does not satisfy any existence-entailing predicate (such as '— is a detective' and '— is left-handed') in w_0. Yet, Holmes is characterized by the predicate '— is a detective' in w_0 in the sense of intentional predication and satisfies the predicate '— is left-handed' in some but not all worlds compatible with the agent's representation.

As noted in Sect. 3.1, I do not postulate impossible objects, intentional or otherwise. I do not take having contradictory beliefs about a given object to involve impossible objects (whatever that would mean)—in particular, not in the sense that a given intentional object would simultaneously satisfy and fail to satisfy a predicate Q. By contrast, I will explain in Sect. 6.3 how to analyze contradictory beliefs about a physical object. In such cases, there are three world lines involved: one physical object $\mathbf{I}$ and two intentional objects $\mathbf{J}_1$ and $\mathbf{J}_2$. These world lines share their realization in the actual world—that is, we have $\mathbf{J}_1(w_0) = \mathbf{I}(w_0) = \mathbf{J}_2(w_0)$. This is why the intentional objects are objects of thought concerning a physical object. Yet, the world lines $\mathbf{I}$, $\mathbf{J}_1$, and $\mathbf{J}_2$ are pairwise distinct: for any two of them, there is a non-actual world w in which they do not have extensionally identical realizations (either because their realizations in w are distinct or because only one of these world lines is realized in w). Finally, one of the intentional objects satisfies the predicate Q in the sense of intentional predication, while the other does not. No logical contradiction is, then, involved in an agent's having contradictory beliefs about a physical object. It is not assumed that a single object satisfies and fails to satisfy Q. Instead, there are distinct intentional objects that happen to share their realizations with a physical object in the actual world. One of these intentional objects satisfies Q, whereas the other does not.

4.3 Description-Sensitivity

Anscombe notes that an action can often be described in a number of ways, while only 'under certain of its descriptions will it be intentional' [1, p. 159]. Alice may intend to use a pen, which is Bob's pen, without intending to use Bob's pen. Cecile may think of a certain parliament member, who in fact is a former spy, but we cannot unproblematically describe her object of thought as a former spy. Anscombe says that an intentional object is introduced by a word or phrase that gives a 'description under which' [ibid. p. 166]. She explains that in her usage, the expression 'under a description' is a modifier of a predicate and not of the subject term [2, p. 219]: when she says 'x under the description d is intentional', she is not suggesting that 'x under the description d' is a noun phrase standing for some peculiar entity distinct from x. Instead, she wants to call attention to the complex predicate '— is intentional under the description d'. Crane [21] speaks of representing an object under an aspect, where Anscombe speaks of the object being intentional under a description. Smith and McIntyre [113, p. 14] describe the same idea by saying that intentional relations are relations between a person and an object under a particular conception of the object; object-directed intentional states are *of* something *as* conceived in a specific way.[7] Anscombe takes it that an action may be intentional under one description without being intentional under another description, and she stresses that this violates Leibniz's law of indiscernibility of identicals as little as does the fact that Socrates may be taller than Theaetetus but not taller than Plato [2, p. 220].

An example from Anscombe [1, p. 177] illustrates the phenomenon of sensitivity to a description in connection with objects of thought. If Bob is looking at a girl with a mole between her shoulder-blades, having no idea that the girl has a mole on her back, there is a sense in which the sentence 'Bob sees a girl who has a mole between her shoulder-blades' is true, and there is another sense in which it is false. Suppose the girl is Alice. Alice can be described in a number of ways: various predicates can be physically ascribed to her. In addition to having a mole between her shoulder-blades, perhaps she likes jogging and is Cecile's unique second-degree cousin. Now, if $\mathbf{I}$ is the intentional object of Bob's sight and $R(w_0)$ is the set of worlds compatible with what Bob sees in w_0, there are worlds $w, w' \in R(w_0)$ such that $\mathbf{I}(w)$ satisfies the predicate '— has a mole on her back', but $\mathbf{I}(w')$ fails to satisfy it. This is either because Bob does not even pose the question of an eventual mole on Alice's back or because what he sees, combined with what he independently believes, provides no sufficient ground for having a definite opinion on the existence of such a mole. Bob's intentional object is indeterminate with respect to this predicate. If $M(x)$ stands for 'x has a mole between her shoulder-blades' and $G(x)$ stands for 'x is a girl', we have

1. $\mathsf{E}_{\mathsf{Bob}} x\, [M(x) \ \wedge \ \Box G(x)]$,

but we do not have

2. $\mathsf{E}_{\mathsf{Bob}} x\, \Box[M(x) \ \wedge \ G(x)]$.

Actually, we have

3. $\mathsf{E}_{\mathsf{Bob}} x\, [M(x) \ \wedge \ \Box G(x) \ \wedge \ \Diamond M(x) \ \wedge \ \Diamond\neg M(x)]$.

[7] Formally the structure 'of — as —' of intentional states is reminiscent of Kant's analysis of objects of cognition in terms of *intuitions* (representations through which objects are given to us) and *concepts* (through which we think about objects); cf. Gardner [32, pp. 43–4]. While people undeniably have conception-dependent object-directed intentional states, accepting them is at least *prima facie* compatible with realism about physical objects. For Kant, objects of cognition are of course conceptually conditioned in a much more fundamental way: objects of empirical knowledge are appearances given via intuitions and filtered through concepts, and nothing beyond these appearances is knowable. The concepts relevant for Kant's analysis are ones like *substance* and *causality*; aspects under which an object is thought can be *frying onions* or *listening to the radio*.

In Anscombe's terminology, Alice is the *material object* of Bob's sight [1, pp. 160–1, 167]. We can infer that the material object of Bob's sight is Cecile's unique second-degree cousin and has a mole between her shoulder-blades. Trivially, any predicates ascribable to Alice in the sense of *physical* predication are ascribable to the material object of Bob's sight—because Alice *is* the material object of Bob's sight and the mode of predication appropriate for talking about the material object of perceptual experience is the physical mode. These predicates are satisfied by the actual realization $\mathbf{I}(w_0)$ of $\mathbf{I}$. By contrast, the relative clause 'who has a mole between her shoulder-blades' does not describe the intentional object $\mathbf{I}$ of Bob's sight, since Bob is not, on the basis of what he sees, aware of Alice as a girl who has a mole between her shoulder-blades. The *description under which* Bob sees Alice is not '— has a mole between her shoulder-blades'. It is the *intentional* mode of predication that is appropriate for talking about the intentional object of perceptual experience. We cannot transfer to $\mathbf{I}$ an arbitrary predicate true of $\mathbf{I}(w_0)$: in order for $\mathbf{I}$ to qualify as a girl with a mole between her shoulder-blades, the predicate '— has a mole between her shoulder-blades' should be true of *all* realizations $\mathbf{I}(w)$ of $\mathbf{I}$ in worlds w belonging to the internal modal margin of $\mathbf{I}$.

Whenever an agent has an intentional state with a material object, there is an intentionally individuated world line $\mathbf{I}$ (an intentional object of the state) and a physically individuated world line $\mathbf{J}$ (the material object of the state) that coincide in the current circumstances: $\mathbf{I}(w_0) = \mathbf{J}(w_0)$. Descriptions of the intentional object $\mathbf{I}$ are predictably ambiguous: such a description can be meant either as a physical predication or as an intentional predication. The former reading is available because $\mathbf{I}$ is realized in w_0. The descriptions need not be *definite* in either case. A description of the material object need not be a predicate that the object $\mathbf{J}(w_0)$ alone satisfies in w_0. Further, in order for P to be a description of an intentional object $\mathbf{I}$, it is only required that $\mathbf{I}$ satisfy P throughout its internal modal margin. In a given world w, there may be other local objects beside $\mathbf{I}(w)$ that satisfy P, and the agent may even have other intentional objects $\mathbf{I}'$ that likewise satisfy P throughout their internal modal margin.

The fact that intentional objects need not be characterized in terms of definite descriptions is worth stressing. The problem of representing logical forms of sentences about intentional objects is needlessly muddled by typical linguistic examples formulated in terms of *definite* descriptions or seeming proper names. It is an interesting question in its own right how to understand the logical form of such sentences as 'Ponce de León looks for the fountain of youth', 'Schliemann seeks the site of Troy', and 'Le Verrier is thinking about Vulcan',[8] but insofar as our primary interest is in the logical behavior of expressions for intentional objects, the sentences 'John looks for a fountain of youth', 'Mary seeks a griffin', and 'Jane is thinking about a flying saucer' would be better points of departure. These latter examples utilize *indefinite* descriptions. The crucial question is how to understand the possibility of talking about non-existent objects. More generally, the problem is to understand what it means to talk about intentional objects, existent or not. It is highly

[8]For these examples, see, e.g., Church [15, pp. 8–9], Kripke [72, Lecture III], Recanati [103, Chap. 13], and Crane [21, Sects. 1.3–4].

counterproductive to render answering these questions more difficult by pretending that they must be formulated as questions about the semantics of proper names or definite descriptions. Since intentional objects are normally tied to a way in which they are characterized, talking about intentional objects will involve descriptions but need not involve definite descriptions.

Since intentional objects need not 'exist', an intentional state need not have a material object. This is why usual logical formalisms with ontologically committing quantifiers cannot represent the general form of a statement expressing that an intentional object is P.[9] It would be of no help to write $\Box \exists x\, P(x)$, as this would allow for $\exists x$ to have distinct witnesses in distinct accessible worlds. The desired condition is captured by using intentional quantifiers with wide scope: the formula $\mathsf{E}x \Box P(x)$ means that an intentional object is describable as P.

Discussing Anscombe's paper, Prior assimilates description-sensitivity to referential opacity [99, p. 126]. It is misleading to speak of *referential* opacity here. It makes sense to speak of reference in connection with singular terms, but only *definite* descriptions could be considered as singular terms. Anscombe speaks of non-substitutability of different descriptions of an object [1, p. 161], which is less misleading—provided that one's notion of substitution is not limited so as to apply to singular terms only. This said, referential opacity in effect formally resembles description-sensitivity. Alice could believe that Molière is a playwright without believing that J. B. Poquelin is a playwright, notwithstanding the fact that Molière and Poquelin are one and the same person: from the premises $a = b$ and $\Box P(a)$, we cannot infer $\Box P(b)$. This is analogous to the fact that $\mathsf{E}_{\mathsf{a}}x\, [P(x) \wedge Q(x) \wedge \Box P(x)]$ does not entail $\mathsf{E}_{\mathsf{a}}x\, [P(x) \wedge Q(x) \wedge \Box Q(x)]$: even if the material object of a state is describable as P and Q and the intentional object is describable as P, it does not follow that the intentional object is describable as Q. On the other hand, if Alice believes *de re* about Molière that he is a playwright, then she believes *de re* also about Poquelin that he is a playwright: from the premises $a = b$ and $\exists x(x = a \wedge \Box P(x))$, we can indeed infer $\exists x(x = b \wedge \Box P(x))$. Similarly, from 'Bob sees a girl who has a mole between shoulder-blades', we can infer 'Bob sees a girl who likes jogging', provided that the material object of Bob's vision is a certain girl who likes jogging and given that the relative clauses in these sentences are intended as applying to the material object of Bob's vision. More precisely, the premises $\mathsf{E}_{\mathsf{b}}x\, [M(x) \wedge \Box G(x)]$ and $\exists y[L(y) \wedge \mathsf{A}_{\mathsf{b}}x\, (\Box G(x) \rightarrow x = y)]$ allow us to infer $\mathsf{E}_{\mathsf{b}}x\, [L(x) \wedge \Box G(x)]$. Let us see why. Let $\mathbf{J}$ be an intentional object witnessing the intentional existential quantifier of the first premise in w_0: $\mathbf{J}$ is a girl that Bob sees. Further, let $\mathbf{I}$ be a physical object that witnesses the physical existential quantifier of the second premise in w_0: $\mathbf{I}$ is a physical individual who in fact likes jogging. By the first premise, we have $w_0,\ x := \mathbf{J} \models \Box G(x)$. That is, $\mathbf{J}$ is an intentional object describable as a girl in the sense of intentional predication. The second premise entails $w_0,\ y := \mathbf{I},\ x := \mathbf{J} \models L(y) \wedge (\Box G(x) \rightarrow x = y)$. It follows that $w_0,\ y := \mathbf{I},\ x := \mathbf{J} \models L(y) \wedge x = y$. That is, the world lines $\mathbf{I}$

[9] Cf., however, the possibility of using informational independence to represent *de objecto* attitudes, as discussed in Rebuschi and Tulenheimo [102].

and $\mathbf{J}$ coincide in the actual world: $\mathbf{I}(w_0) = \mathbf{J}(w_0)$, and the physical object who likes jogging is the material object of Bob's intentional state. Consequently, we have $w_0, x := \mathbf{J} \models L(x) \wedge \Box G(x)$, and we may conclude that $\mathsf{E}_{\mathsf{b}}x\,[L(x) \wedge \Box G(x)]$ is true at w_0.

4.4 Factiveness and Material Objects of Intentional States

An intentional state is factive if its correlated accessibility relation is reflexive. To put it otherwise, a state is factive if its propositional content cannot fail to be satisfied by the actual circumstances.[10] I point out in this section that given hypotheses H3 and H4 about local interrelations of intentionally and physically individuated world lines (cf. Sect. 3.4), factiveness of an intentional state with an intentional object suffices for its having a material object. By contrast, in order for an intentional state to *represent a physical object*, much stronger requirements must be met (see Sect. 4.8).

Let V and S be the accessibility relations triggered by Alice's visual experience and veridical visual experience, respectively. That is, $V(w, v)$ iff for all Alice can tell on the basis of her visual perceptual experience in w, she is in v, and $S(w, v)$ iff for all Alice can tell based on what she sees in w, she is in v. The relation S is reflexive but V is not. Seeing is factive; visual perceptual experience is not. Alice can visually experience that things are a certain way, while in fact they are not. For example, she might be hallucinating rain: it could be raining in all scenarios $w \in V(w_0)$ but not in w_0. If so, $w_0 \notin V(w_0)$. Let the operators $\Box_{\mathsf{V}}$ and $\Box_{\mathsf{S}}$ be interpreted in terms of V and S, respectively. Generally, $V \subsetneq S$: the formula $\Box_{\mathsf{S}}\phi \to \Box_{\mathsf{V}}\phi$ is valid, but $\Box_{\mathsf{V}}\phi \to \Box_{\mathsf{S}}\phi$ is not. Other examples of factive states are *correctly believing* and *knowing*, while *believing* is non-factive.

Taking intentional states to be modalities allows speaking of their propositional contents. For example, the sentence 'It is raining' expresses a propositional content when uttered in a specific location at a specific time. Now, typically, intentional states involve intentional objects (e.g., tomatoes, winged horses). I take them to constitute an aspect of intentional states irreducible to propositional contents.[11] Let us consider objects of visual perceptual experience. Objects of sight can be viewed as intentionally individuated world lines. Here is the minimum condition allowing us to say that Alice sees a tomato:

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[10] I am not assuming that the content of an intentional state is always simply a *propositional* content. However, in Sects. 4.7 and 6.2, I will argue that all intentional states involve a propositional component (a world representation), while some intentional states involve world line components, as well (object representations). Propositional contents will be discussed in Sect. 6.4.

[11] Actually, if the content expressed by the sentence 'It is raining', when uttered in world w at time t in location s, is supposed not to be tensed and spatially indeterminate but temporally and spatially specific (*that it rains at t in s*), then even this content is not purely propositional but involves intentionally individuated *temporal* and *spatial world lines*: it requires the possibility of speaking of the same time and the same spatial location in relation to several worlds. For a discussion, cf. Sect. 3.6 and see Tulenheimo [119].

4. $\mathsf{E}_{\mathsf{a}}x\ \Box_{\mathsf{S}}\ T(x)$.

Since seeing is a factive modality, it ensues that

5. $\mathsf{E}_{\mathsf{a}}x\ T(x)$,

i.e., the intentional object of Alice's sight is actually realized and is in fact a tomato. Assuming that realizations of intentional objects are local objects (H3) and that every local object is the realization of some physical object (H4), it follows that

6. $\exists x\, T(x)$,

and more specifically that

7. $\mathsf{E}_{\mathsf{a}}x\ \exists y\ [x = y\ \wedge\ \Box_{\mathsf{S}}\ T(x)]$.

That is, Alice's experience has a material object. The intentional object $\mathbf{J}$ of Alice's sight (witnessing $\mathsf{E}_{\mathsf{a}}x$) coincides with a physical object $\mathbf{I}$ (witnessing $\exists y$) locally, in the actual world. The two world lines satisfy $\mathbf{I}(w_0) = \mathbf{J}(w_0)$. Note that if $\Box$ stands for a factive state, from $\mathsf{E}x[P(x) \wedge \Box Q(x)]$, we can infer $\mathsf{E}x\ Q(x)$ but not $\mathsf{E}x\ \Box P(x)$. Seen from a modal-logical perspective, this is trivial. It is nevertheless of interest from the viewpoint of intentional objects. Intentionally ascribed predicates of an intentional object of a factive state are automatically physically ascribable to the material object of that state, but the converse fails: predicates of the material object need not be predicates of the intentional object.

Regarding mere visual experience, it is evident that from the premise that it visually appears to Alice that something is a tomato, it does not follow that something in fact is a tomato. That is, from

8. $\mathsf{E}_{\mathsf{a}}x\ \Box_{\mathsf{V}}\ T(x)$,

we cannot infer $\exists x\ T(x)$. The fact that Alice has an object of visual experience—a certain intentional object characterizable as a tomato—does not guarantee that some physical object is a tomato. Alice's visual experience may be hallucinatory. For this, it suffices that none of the intentional objects of her visual experience is realized actually.[12] Formula (8) is compatible with (9):

9. $\mathsf{A}_{\mathsf{a}}x\ [\Box_{\mathsf{V}}\, T(x)\ \rightarrow\ \neg x = x]$.

Formula (9) entails that Alice does not *see* a tomato:

10. $\neg\,\mathsf{E}_{\mathsf{a}}x\ \Box_{\mathsf{S}}\ T(x)$.

[12]It appears reasonable to think that the object of a hallucinatory experience *must* fail to be actually realized. Given that hallucinatory states lack a material object, it is, in any event, clear that a hallucinatory intentional object cannot share a realization with a physical object in the actual world. In Sect. 4.6, I consider in passing the possibility that an intentional object is actually realized without coinciding with the realization of any physical object, so that the intentional state lacks a material object. (This is possible if realizations of intentionally individuated world lines are *wholes* composed of realizations of physically individuated world lines and unrestricted world-internal composition of local objects is not assumed.) However, if there are such intentional states, they can presumably always be construed as cases of *illusion* rather than hallucination.

Namely, if we had $\mathsf{E}_{\mathsf{a}} x\, \Box_{\mathsf{S}}\, T(x)$, we would have $\mathsf{E}_{\mathsf{a}} x\, [x = x \wedge \Box_{\mathsf{V}}\, T(x)]$—because S is reflexive, $T(x)$ is existence-entailing, and $V \subsetneq S$. This would contradict (9). Note that (10) is compatible with the falsity of (6): the perceptual situation need not involve any tomato.

In order for an agent's object-directed intentional state to have a material object, it is enough that the current world w_0 is compatible with the state: by H3 and H4, having $R(w_0, w_0)$ *suffices* to validate the inference from the truth of $\mathsf{E} x\, \Box\, T(x)$ at w_0 to the truth of $\mathsf{E} x\, \exists y\, [x = y \wedge \Box T(x)]$ at w_0, given that R is the accessibility relation corresponding to $\Box$. It is *not necessary* that the relation R be reflexive. Actually, it is not even necessary that $R(w_0, w_0)$. Certain types of illusory experiences provide examples of cases in which the current circumstances are not compatible with the agent's intentional state, which nevertheless has a material object. For example, Alice might be looking at a bear, though it appears to her that the object is a stone:

11. $\mathsf{E}_{\mathsf{a}} x\, [B(x) \wedge \Box_{\mathsf{V}} S(x)]$.

If (11) is true in w_0, it is excluded that $V(w_0, w_0)$. Otherwise, the actual realization of the object of the state (the witness of $\mathsf{E}_{\mathsf{a}} x$) would be both a bear and a stone, which is impossible. In illusions of this sort, the modal margin of the illusory intentional object must extend beyond the set $V(w_0)$. The state has a material object, which is why the modal margin of the intentional object comprises the world $w_0 \notin V(w_0)$. In Sect. 4.6, we will encounter examples of illusions of another type: ones that lack a material object in the sense discussed here.

4.5 Indeterminacy

According to Kant [A 572–3/B 600–1], everything that exists is subject to the principle of complete or thoroughgoing determination (*Grundsatz der durchgängigen Bestimmung*): given any existing object, a list of all possible predicates, and any pair of contradictorily opposed predicates in this list, exactly one of the predicates of the pair belongs to the object. This way of phrasing the principle risks to reduce it to a triviality, since 'P and Q are contradictorily opposed' means 'nothing satisfies both P and Q but everything satisfies one or the other', whence the principle holds by definition. Besides, the applicability of the principle to concrete cases would be problematic: P and Q are contradictorily opposed iff $\forall x[P(x) \leftrightarrow \neg Q(x)]$ is analytically true, but it is notoriously very difficult to make precise sense of the latter requirement. A better formulation of the principle would be as follows: for any existing object x and any predicate P, either $P(x)$ or else $\neg P(x)$. Thus construed, the principle is a statement of the *law of excluded middle* for all predicates relative to all existent objects.[13] For Kant, this principle does not have the status of a trivial semantic fact. It is supposed to hold for empirical objects, thanks to the cognitive operation of

[13]Kant took negated concepts to be derived from positive ones [A 575/B 603]: he had his reasons for speaking of contradictorily opposed pairs rather than using negation applied to predications.

synthesis giving rise to the experience of objects. It states that the synthesis must specify in connection with every empirical object and every predicate, whether the object has the predicate or not. In this way, the totality of all predicates becomes a transcendental precondition of empirical objects (cf. Gardner [32, p. 154]).

The notion of complete determination can be usefully employed when discussing the difference between physical and intentional objects. Physical objects are subject to the following variant of this principle: if $\phi(x_1, \ldots, x_n)$ is any n-ary predicate, $\mathbf{I}_1, \ldots, \mathbf{I}_n$ are physically individuated world lines, and w_0 is a world in a model M, then either ϕ or $\neg\phi$ is ascribable to the tuple $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ in w_0 under the physical mode. We have $M, w_0, x_1 := \mathbf{I}_1, \ldots, x_n := \mathbf{I}_n \models \phi \vee \neg\phi$. This principle holds trivially. The corresponding principle does *not* hold for intentional objects and for the intentional mode of predication. If it did, it would follow in particular that for all unary predicates P, intentional objects $\mathbf{I}$, and worlds w_0 in a model M, either $P(x)$ or $\neg P(x)$ is ascribable to $\mathbf{I}$ in w_0 under the intentional mode. That is, we would have $M, w_0, x := \mathbf{I} \models \Box[x = x \rightarrow P(x)] \vee \Box[x = x \rightarrow \neg P(x)]$. The principle of complete determination for intentional objects would mean that if $\mathbf{I}$ is an intentionally individuated world line, then either in all worlds belonging to its internal modal margin, $\mathbf{I}$ satisfies P or else in all those worlds, $\mathbf{I}$ fails to satisfy P. There is no reason why one of these two conditions should hold. An intentional object may well be *indeterminate* or *unspecific* with respect to P—satisfying P in some but not all worlds of its internal modal margin. It is a psychologically reasonable assumption that intentional objects are always to some extent indeterminate. They are always conceived in terms of a limited range of predicates, being consequently indeterminate with respect to any further predicates.[14]

In practice, the internal modal margin of an intentional object always contains a multiplicity of worlds. This is what makes it possible for intentional objects to be indeterminate. In order to discuss this issue systematically, I will make use of the following relation of internal indistinguishability. Provided that M is a model with an interpretation function *Int* and $w, w' \in dom(M)$, let us write $f : w \cong w'$ if $f : dom(w) \rightarrow dom(w')$ is a bijective map such that for all $n > 0$, n-ary predicate symbols Q, and local objects $a_1, \ldots, a_n \in dom(w)$:

$$\langle a_1, \ldots, a_n \rangle \in Int(Q) \text{ iff } \langle f(a_1), \ldots, f(a_n) \rangle \in Int(Q).$$

If there exists a map f such that $f : w \cong w'$, I say that the worlds w and w' are *internally indistinguishable* and that they are of the same *world type*, in symbols $w \cong w'$. To clarify the phenomenon of indeterminacy, note that if $\langle W, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ is a content over M, there can be in W internally indistinguishable worlds w and w'. Such worlds may of course be differently related to further worlds. For example, w

(Footnote 13 continued)
Yet, the task of delineating the relevant positive/negative distinction is rather desperate, and even if it was manageable, the problem of delineating the analytic/non-analytic distinction would remain.

[14] According to Husserl, intentional acts pertaining to a physical object—e.g., perception—are necessarily indeterminate. Any conception under which a physical object is ‘intended’ captures only a small part of all that is actually true of it; cf. [113, p. 16].

could be R-maximal while w' has exactly 27 immediate R-successors. What is more, they might behave differently vis-à-vis the world line components of the content. The fact that $\mathbf{I}_j(w) = a$ and $\mathbf{I}_j(w') = b$ does not entail that $b = f(a)$. In short, we may have $f(\mathbf{I}_j(w)) \neq \mathbf{I}_j(w')$.[15]

Suppose Alice is thinking of a man of no definite height. There are two jointly acting but separate reasons why there are many worlds compatible with her intentional state. First, in practice, Alice's thoughts will inevitably fail to determine the *current circumstances* in a unique way: for all Alice can tell, the world could be slightly or even grossly different from w_0. That is, the set $R(w_0)$ of worlds compatible with her thoughts in w_0 contains many pairwise non-isomorphic worlds. Second, in practice, it will be indeterminate how Alice's intentional object is realized relative to a given world type. There will be predicates P and internally indistinguishable worlds w and w', both lying in the internal modal margin of Alice's object of thought $\mathbf{J}$, such that $\mathbf{J}(w)$ satisfies P but $\mathbf{J}(w')$ does not satisfy P. Consequently, for any map $f : w \cong w'$, we have $f(\mathbf{J}(w)) \neq \mathbf{J}(w')$. In this way, the world line $\mathbf{J}$ creates a difference between w and w', even though these worlds cannot be distinguished in terms of atomic predicates distributed over their local objects. Various worlds $w \in R(w_0)$ will have isomorphic copies $w' \in R(w_0)$ such that the realization of the world line $\mathbf{J}$ in w is qualitatively different from its realization in w'.

Given that Alice's object of thought is intentional under the description '— is a man', this predicate is true of all realizations of $\mathbf{J}$. By contrast, as Alice's object of thought has no definite height, there is no X such that all realizations of $\mathbf{J}$ satisfy the predicate '— is X cm tall'. Distinct realizations of $\mathbf{J}$ differ with respect to height. There are scenarios w_1, w_2, and w_3 in the internal modal margin of $\mathbf{I}$ such that the height attributed to $\mathbf{J}(w_1)$ is 170 cm, while the heights of $\mathbf{J}(w_2)$ and $\mathbf{J}(w_3)$ are, respectively, 173 cm and 176 cm. In particular, such scenarios w_1, w_2, and w_3 may well be pairwise internally indistinguishable.

While a sentence such as 'Alice hits a man' is extensional and has the form $\exists x\, [M(x) \wedge H(\mathsf{a}, x)]$, the form of 'Alice thinks of a man' is $\mathsf{E}_{\mathsf{a}} x \,\Box\, M(x)$.[16] The availability of an intentional object is stated using an intentional quantifier. Here, the modal operator $\Box$ triggers a set of worlds functioning as the internal modal margin of the intentional object. In the scope of $\Box$, the predicate 'M' is used to describe the

[15] If distinct worlds can indeed be internally indistinguishable, the notion of 'same world' cannot be understood in terms of (atomic) predicates distributed over local objects in these worlds or in terms of propositions true in these worlds. I do not wish to base this notion on structural considerations either—i.e., on possible differences in how the worlds are related to other worlds in terms of accessibility relations or how their local objects are related to local objects of other worlds in terms of world lines. Rather, I take it to be a consequence of the nature of local objects that they are partitioned into cells such that elements of any one cell can be compared in terms of numerical identity but elements coming from distinct cells cannot. The relation of numerical identity among local objects gives rise to the notion of 'same world': worlds are distinct when their local objects cannot be compared in terms of numerical identity.

[16] As hinted at in footnote 12 in Sect. 3.5, the notion of logical form is not well behaved in L. Strictly speaking, we must resort to a schematic version S_L of this language to obtain a notion of logical form that behaves in the expected way; see Sect. 5.6 for details. Intensional constructions, such as *think of*, are discussed in more detail in Sects. 6.6–6.7.

realizations of the intentional object. We can express that Alice's object of thought is indeterminate with respect to the property of being 170 cm tall by the formula

12. $\mathsf{E}_{\mathsf{a}} x\, [\Box M(x) \wedge \Diamond T_{170}(x) \wedge \Diamond\neg T_{170}(x)]$.

The phenomenon of indeterminacy is by no means restricted to non-factive intentional states. Consider again Alice, who sees a tomato. The intentional object of her vision need not have a definite size, definite weight, or definite shade of redness. Nevertheless, seeing is a factive modality, and Alice's veridical visual experience has a material object—a certain physical tomato with a definite size, weight, and shade of redness. Alice's visual perception need not yield a precise grip of the real tomato.

I take it that *physically* individuated world lines have by their very nature a uniform behavior over internally indistinguishable worlds: if w and w' are internally indistinguishable, then their internal indistinguishability can be witnessed by an isomorphism that maps the realization of any physical object in w to the realization of that same physical object in w'. The following hypothesis may be added to the list of hypotheses H1 through H4 laid down in Sect. 3.4:

H5. If there is a map $g : w \cong w'$, then $\mathcal{P}_w = \mathcal{P}_{w'}$, and there is in particular a map $f : w \cong w'$ such that $f(\mathbf{I}(w)) = \mathbf{I}(w')$ for all $\mathbf{I} \in \mathcal{P}_w$.

In the presence of hypotheses H1 and H4, the map f mentioned in H5 is actually uniquely determined by the set $\mathcal{P}_w$: it is defined by the condition $f(a) = \mathbf{I}_a(w')$ for all $a \in dom(w)$, where $\mathbf{I}_a \in \mathcal{P}_w$ satisfies $\mathbf{I}_a(w) = a$. For an explication, see Sect. A.1 of the Appendix A. The fact that physical objects satisfy H5 but intentional objects fail to satisfy a corresponding principle must be taken into account when discussing the conditions under which an intentional object represents a specific physical object (Sect. 4.8).

4.6 Non-Existent Objects and States with Material Objects

The logical form of sentences such as 'Bob sees a lion' or 'Alice thinks of a tomato' is in no way dependent on whether the relevant intentional object exists. These sentences are of the form $\mathsf{E}_{\mathsf{a}} x \,\Box\, Q(x)$. They both report that an agent relates in a certain way to a world line individuated according to the intentional mode. Suppose an agent's intentional state at w_0 has an intentional object $\mathbf{J}$. There are at least three questions we may ask, all related to the property of *existence-independence*:

(a) Is $\mathbf{J}$ realized in w_0?
(b) Does the state have a material object?
(c) Under what conditions does $\mathbf{J}$ represent a specific physical object?

I have taken 'existence' of a world line to mean that it is realized. Intentional objects are 'existence-independent': they need not be realized while being available to be talked about. The answer to Question (a) may well be ‘no’. Objects of hallucinations, fictional objects, and, arguably, past individuals are examples of non-existing intentionally individuated world lines. From ‘Alice thinks of a tomato’ or

13. $\mathsf{E}_{\mathsf{a}}x\, \Box T(x)$

we cannot infer ‘A tomato Alice thinks of exists’ or

14. $\mathsf{E}_{\mathsf{a}}x\, [x = x \,\wedge\, \Box T(x)]$.

This is because *thinking* is not a factive intentional state. Formula (14) being true in w_0 means that there is $\mathbf{J} \in \mathcal{J}^{\alpha}_{w_0}$ such that for all $w \in R(w_0)$, $\mathbf{J}$ is realized in w and $\mathbf{J}(w)$ is T. Since R is not reflexive, we cannot infer that $\mathbf{J}$ is realized in w_0.

In Sect. 4.4, we saw that under hypotheses H3 and H4, Question (b) reduces to Question (a). However, generally, Question (b) is more complicated than appears at first sight. What if we drop hypothesis H3, according to which any realization of an intentional object is a local object, still retaining H4, which states that every local object is the realization of some physical object? Given H4, hypothesis H3 can be formulated by stating that (15) is valid:

15. $\mathsf{A}x\, \exists y\, (x = x \rightarrow x = y)$.

The notion of model can be modified so as to make (15) refutable. We may think of the domain of a world as being constituted of a totality that is sliced up in one way by realizations of physical objects and in a possibly different way by realizations of intentional objects. One way of implementing this idea would be as follows. Let every world w have a domain $dom(w)$ consisting of local objects as before. Interpret these local objects as realizations of physical objects. Define an irreflexive part-whole relation $<$ on $dom(w)$ with local objects as atoms ($<$-minimal elements). Finally, let realizations of intentional objects be wholes composed of local objects—i.e., wholes having realizations of physical objects as their atomic parts.[17]

With models defined in this way, (15) becomes refutable, which amounts to giving up hypothesis H3. Consider Bob, who is surveying his surroundings in w_0. Suppose in the direction Bob is looking at there are two physical objects: a red and rather big one far away ($\mathbf{I}_1$) and a black and small one rather near ($\mathbf{I}_2$). As it happens, the intentional object $\mathbf{J}$ of Bob’s vision is, however, a single two-colored object—an object with a small black part surrounded by a large red part. Bob’s intentional object is realized actually: its realization $\mathbf{J}(w_0)$ is a whole composed of two parts—namely, $\mathbf{I}_1(w_0)$ and $\mathbf{I}_2(w_0)$. Yet, there is no single physical object whose realization would coincide with the realization of $\mathbf{J}$. By the earlier definition, Bob’s perceptual experience does

[17]More generally, it could be assumed that world lines of neither type need have local objects as their realizations. This could, then, be implemented either by letting arbitrary realizations to be wholes composed of local objects or by dispensing with local objects altogether, letting realizations be atomless wholes. Instead of wholes in the sense of traditional mereology, we could alternatively consider rigid embodiments in the sense of Fine: in this way, realizations of physical and intentional world lines would themselves be, in effect, world lines defined over local objects (first option) or over portions of ‘stuff’ (second option). For a metaphysics of stuff, see, e.g., Prior [97, p. 174], [98, p. 80], [100, pp. 181–6] and Jubien [58].

not have a material object, though the experience is, so to say, physically grounded. Using a suitably extended syntax, we can describe this case as follows:

16. $\mathsf{E_b}x\,\exists y\,\exists z\,\big[R(y) \wedge B(z) \wedge y < x \wedge z < x \wedge \mathsf{A_b}v(v < x \rightarrow [v = y \vee v = z]) \wedge \Box C(x)\big]$,

where $R(x)$, $B(x)$, and $C(x)$ stand for 'x is red', 'x is black', and 'x is two-colored', respectively. Bob's perceptual experience *lacks* a material object, but still, we would not say that his experience is hallucinatory. The intentional object of his experience *is* realized actually. His experience is illusory: it represents two physical objects as one. Whereas 'numerical illusions' of this kind can well appear in connection with factive modalities, 'qualitative illusions', discussed in Sect. 4.4, cannot.

Not all intentional states lacking a material object are cases of illusion or hallucination. Perfectly normal perceptual experiences may have intentional objects whose actual realization is not the realization of any physical object (so the state has no material object), but still, the actual realization is composed of realizations of physical objects. Suppose Alice is thinking of a bookcase with 900 books distributed over its shelves. This may be considered as a single object of thought, though its actual realization is composed of realizations of 901 physical objects. If the realization of the intentional object is taken not to have further parts in addition to the realizations of the mentioned 901 objects, this situation can be described by the formula

17. $\mathsf{E_a}x\,\exists y_1 \ldots \exists y_{901}\big[(\bigwedge_{1 \le i \le 901} y_i < x) \wedge \mathsf{A_a}z\,(z < x \rightarrow \bigvee_{1 \le i \le 901} z = y_i)\big]$.

Having addressed Questions (a) and (b), we may note that the issue of what can be meant by attributing existence to an intentional object is potentially trickier than what is suggested by the idea of existence as actual realization: unless hypothesis H3 is assumed, we cannot take it for granted that whenever an intentional object of a factive intentional state is actually realized, the state has a material object. To see why this is so, recall that if the intentional object of an object-directed intentional state is $\mathbf{I}$ (a certain physically individuated world line), then the *material* object of the state is, by definition, a physical object $\mathbf{J}$ (a certain physically individuated world line) such that $\mathbf{I}$ and $\mathbf{J}$ coincide in the current circumstances: $\mathbf{I}(w_0) = \mathbf{J}(w_0)$. Now, what does it mean that an intentional object $\mathbf{I}$ of a factive intentional state is actually realized? It means that the intentionally individuated world line $\mathbf{I}$ has a realization in the current circumstances w_0. Without hypothesis H3, we cannot infer that this realization $\mathbf{I}(w_0)$ is a local object, and without this information, there is no way of connecting this realization to any physically individuated world line. By contrast, if $\mathbf{I}(w_0)$ was a local object, then hypothesis H4 would allow us to conclude that $\mathbf{I}(w_0)$ is the realization of some physical object, and such a physical object would be the material object of the state. In short, in the absence of H3, the requirement that a state have a material object is stronger than the requirement that its intentional object be actually realized.

Further, in a framework like mine, in which we have both intentional and physical objects, both understood as world lines, the idea of an intentional state pertaining to a 'real object' admits an even more demanding formulation. It is not evident that a state's having a material object is *sufficient* for its being about a real object. Indeed, what does it take for an intentional object to *represent* a specific physical object? I said in Sect. 3.2 that intentional states are representational in an intrinsic sense. They need not stand in a relation to a physical object. However, in suitable circumstances, a representation may be relational: an intentional object may represent a physical object. Under what conditions can such a relational representation be accomplished?

The simple requirement that an intentional state have a material object would be highly dubious as an analysis of the idea that the state represents a specific physical object. If the material object of Alice's perceptual experience is a tomato that weighs 125 grams, is oblong, and has a reddish brown color but her intentional object is a tomato grossly indeterminate in weight, size, and chromaticity, we might not be happy to say that Alice's intentional object represents the tomato before her eyes. We might be reluctant to say that her intentional state concerns a specific physical object in an epistemologically interesting sense. There is a representation of a tomato alright—a representation in an intrinsic sense characterizable as a tomato—but the representation is too indeterminate to count as pertaining to a specific physical object. Or, take the example of Alice and the bear from Sect. 4.4: the illusory object of Alice's perceptual experience is actually realized, and her experience has a material object, but her intentional object is characterized as a stone and consequently does not represent the bear causally responsible for her experience.

Representation is always representation under an aspect, but aspects go in clusters so that a representation cannot be relational (it cannot pertain to a specific physical object) and represent something as P unless it represents its object as Q, as well. Evidently, no general recipe can be given for telling which aspects require which other aspects. The problem is not only conceptual (analyticity) but also empirical: as a matter of fact, physical objects having certain characteristics tend to have certain other characteristics, as well. Equally evidently, any representation is indeterminate in various respects. As an extreme case, we could have an intentional object $\mathbf{J}$ and a physical object $\mathbf{I}$ such that $\mathbf{I}(w_0) = \mathbf{J}(w_0)$, but for every atomic P physically ascribed to $\mathbf{I}$ in w_0, there is a realization of $\mathbf{J}$ not satisfying P, and for every atomic P that fails to be physically ascribed to $\mathbf{I}$ in w_0, there is a realization of $\mathbf{J}$ satisfying P. Even in this case, $\mathbf{I}$ would be the material object of the intentional state, but the intentional object $\mathbf{J}$ would not represent $\mathbf{I}$ under any (non-trivial) aspect—being indeterminate with respect to all atomic predicates that the material object has or lacks. In Sect. 4.8, I address Question (c) and discuss in detail how to understand relational representations—i.e., the idea that an intentional object represents a physical one.

4.7 Intentional States and Their Contents

For the purposes of semantic theorizing, agent α's intentional states can be modeled as triples $\langle W, \mathfrak{J}, w_0\rangle$, where $W = R(w_0)$ is the set of worlds compatible with α's attitude or experience in w_0, and $\mathfrak{J}$ is a list consisting of the elements of the set $\mathcal{J}^{\alpha}_{w_0}$. The triple $\langle W, \mathfrak{J}, w_0\rangle$ is an R-situated content over a certain model M, the pair $\langle W, \mathfrak{J}\rangle$ being a content in the sense of Definition 2.3. I suggested in Sect. 3.2 that such contents are useful for discussing the semantics of sentences ascribing intentional states to agents. If $\mathbf{J} \in \mathfrak{J}$, we may but need not have $marg(\mathbf{J}) \subseteq W \subseteq marg(\mathbf{J})$. Illusions of the bear-seen-as-a-stone type demonstrate that the first inclusion may fail. To see that the second inclusion need not hold either, consider the sentence 'Alice believes that a winged horse flies faster than a space shuttle'.[18] It has a reading according to which Alice has an opinion about what she expressly takes to be a mythological creature. The setup of this belief can be described by formula (18):

18. $\mathsf{E_a}x(\neg x = x \wedge \Box\big(x = x \rightarrow [H(x) \wedge F(x)]\big) \wedge \Diamond\neg x = x \wedge \Diamond x = x)$.

That is, not only does Alice's intentional object fail to be actually realized, it is realized only in some but not all of Alice's doxastic alternatives. In those worlds in which it is realized, it satisfies the predicates '— is a winged horse' (H) and '— flies faster than a space shuttle' (F). If this is how the content of Alice's intentional state is to be described, she does not believe that her intentional object exists.

Intentional states $\langle W, \mathfrak{J}, w_0\rangle$ are *representational* in an intrinsic sense. Confining attention to states with a direction of fit toward the world, the set W is a *world representation*, and the members of the list $\mathfrak{J}$ are *object representations*.[19] The set W can be viewed as a representation of the world w_0 in which the agent is currently located, though such a representation need not be accurate in any sense. In connection with most object representations, there is no obvious contextually provided physical object to which the object representation could be related. In fact, there is only one exception: the agent's self-representation automatically represents a specific physical object—namely, the agent herself. As for the general case, typically, the agent construes any of her intentional objects as a representation of one physical object or another, but in order for such an intentional object to *be* a representation of a specific external object, certain rather strong conditions must be met. These conditions are discussed in Sect. 4.8. As noted in Sect. 4.5, because in practice the world representation is always indeterminate, the set W contains a number of pairwise non-isomorphic worlds. Moreover, because the object representations are also always indeterminate, for every world line $\mathbf{J} \in \mathfrak{J}$, there are internally indistinguishable worlds w and w' in the internal modal margin of $\mathbf{J}$ such that no isomorphism $f : w \cong w'$ satisfies $f(\mathbf{J}(w)) = \mathbf{J}(w')$. The plurality of isomorphic copies of a given world w in W reflects the plurality of different possible realizations that the object representations may collectively have in a world like w.

I said that the component W of a state $\langle W, \mathfrak{J}, w_0\rangle$ represents the actual world w_0.[20] If so, under what aspects does W represent w_0? Let Ψ be the set of all sentences ψ such that $W \subseteq |\psi|^M$. Restricting attention to conditions expressible in terms of L-sentences, the answer is: W represents w_0 as being ψ iff $\psi \in \Psi$. The set Ψ consists of sentences ψ satisfying $\langle W, \mathfrak{J}, w_0\rangle \Vdash_{uni} \psi$. Among the sentences in Ψ, there are

[18] Cf. footnote 13 in Sect. 3.5.

[19] Beliefs, for example, have the direction of fit mind-to-world (one's beliefs are intended as fitting the world), whereas desires have the direction of fit world-to-mind (it is intended that the world fits one's desires). For the notion of direction of fit, see Searle [110] and Humberstone [54].

[20] Recall, once more, that by the expression 'the actual world', I mean the world in which the agent is by hypothesis located; cf. footnotes 23 and 4 in Sects. 1.4 and 4.2, respectively.

the negations $\neg\chi$ of those sentences χ that satisfy $W \cap |\chi|^M = \varnothing$. By contrast, for sentences θ such that $W \nsubseteq |\theta|^M$ and $W \cap |\theta|^M \neq \varnothing$, there are worlds $w, w' \in W$ such that θ is false in M at w but true in M at w'. With respect to these sentences θ, the representation W is indeterminate. Generally, world representations can be arbitrarily bad: they need not reflect the features of w_0 in any reasonable way. In particular, they may be incompatible with w_0: the world w_0 need not belong to W.

I also said that elements of $\mathfrak{J}$ are representational—without claiming that they generally represent specific physical objects. That is, they are intrinsically representational but need not be relational representations. Suppose $\mathfrak{J} = \langle \mathbf{J} \rangle$. When speaking of $\mathbf{J}$ as a representation, attention is confined to its internal modal margin $marg(\mathbf{J}) \cap W$. Let Φ be the set of formulas $\phi(x)$ intentionally ascribed to $\mathbf{J}$ at w_0. Now, $\mathbf{J}$ is a representation under the aspect ϕ iff $\phi \in \Phi$. The set Φ consists of formulas $\phi(x)$ of one free variable satisfying $\langle W, \mathfrak{J}, w_0 \rangle \Vdash_{uni} \phi(x)$. The world line $\mathbf{J}$ is indeterminate relative to the predicates $\theta(x)$ such that $\langle W, \mathbf{J}, w_0 \rangle$ uniformly supports neither θ nor $\neg\theta$. The definition is a bit more involved in the general case of a state with n intentional objects. If $\mathfrak{J} = \langle \mathbf{J}_1, \dots, \mathbf{J}_n \rangle$, there are $2^n - 1$ ways of choosing at least one of them for consideration. If the possible selections of objects are $S_1, \dots, S_{2^n-1}$ and $S_j = \{\mathbf{J}_{i_1}, \dots, \mathbf{J}_{i_m}\}$, let Υ_j be the set of all formulas $\upsilon(x_1, \dots, x_m)$ of m free variables satisfying $\langle W, \mathbf{J}_{i_1}, \dots, \mathbf{J}_{i_m}, w_0 \rangle \Vdash_{uni} \upsilon(x_1, \dots, x_m)$. The state $\langle W, \mathbf{J}_1, \dots, \mathbf{J}_n, w_0 \rangle$ represents the tuple $\langle \mathbf{J}_{i_1}, \dots, \mathbf{J}_{i_m} \rangle$ under the aspect υ iff $\upsilon \in \Upsilon_j$. The tuple is indeterminate relative to any predicates $\theta(x_1, \dots, x_m)$ such that neither θ nor $\neg\theta$ belongs to Υ_j. The state $\langle W, \mathfrak{J}, w_0 \rangle$ gives rise to the family of sets of aspects $\Upsilon = \{\Upsilon_j : 1 \leq j \leq 2^n - 1\}$.

Using the terminology of Definition 2.3, the world representation W of an intentional state $\langle W, \mathfrak{J}, w_0 \rangle$ is its propositional component, and the object representations $\mathbf{J} \in \mathfrak{J}$ are its world line components. Discussing Husserl's theory of intentionality, Smith and McIntyre [113, pp. 6–9] take up what they call propositional acts. In such acts, an agent 'intends' that so-and-so is the case (e.g., Alice sees that it is raining). Objects of these acts are propositions (or states of affairs). According to Husserl, some propositional acts are intentional in two ways: their primary object is a state of affairs, their secondary object being a physical object. Suppose Alice sees that a certain lion is going to attack her. Primarily, her belief is about the state of affairs that the lion is going to attack her; in a secondary sense, it is about the lion. Structurally, such propositional acts resemble intentional contents: the primary object corresponds to a propositional component and the secondary object to a world line component of an intentional content. However, this analogy is partially misleading. First, world line components are intentional objects; these may or may not represent physical objects, but they are not, literally, physical objects. Second, if Alice hallucinates a lion, in Husserl's sense, her intentional act has no object at all.

Given the way in which I use the expression 'intentional object' in this book, if Alice hallucinates a pink elephant, her visual experience has a non-existent intentional object. In Husserl's terminology, Alice's hallucinatory perceptual act has *no object*—it would only have one if Alice was looking at a pink elephant, a certain physical individual. Smith and McIntyre [ibid. pp. 40–61] discuss at length what they call 'object theories' of intentionality and explain that Husserl rejected any such theory. The object theories postulate a domain of objects of a special kind, ontologically different from ordinary objects. Intentional relations are then construed as ordinary relations that an agent bears to these extraordinary objects.

The object-theoretic approach concerning intentional objects is essentially like the Meinongian approach concerning non-existent objects. A big stock of objects is postulated. This stock is assumed to be organized in different compartments capable of accommodating objects of different varieties. Elements of distinct compartments are distinguished by the fact that they are qualitatively different—they satisfy mutually incompatible predicates. In the Meinongian case, objects of one compartment satisfy the existence predicate and those of another compartment do not. According to the object theories, objects of one compartment are 'ordinary', while those of another are 'intentional'. Crucially, the relationship between objects of these two varieties is not characterized in any insightful manner—the distinction is simply assumed as a brute fact. In particular, no reference is made to a potential internal structure of objects of the two kinds in order to account for their differences.

If, in an approach based on the recognition of local objects, one wished to adopt an object theory of intentional objects, this would amount to postulating *local* objects of two kinds—'ordinary' and 'intentional'. I do not subscribe to an object theory. I do not introduce local objects of different types. Further, I do not ascribe any metaphysical reality to intentionally individuated world lines. They are agent-relative correlations between local objects of distinct worlds—ways of structuring a set of scenarios compatible with an agent's intentional state—and while they have a psychological reality, they have no metaphysical role to play. In the same vein, Crane [21] stresses that intentional objects in his sense are not, generally, entities or things of any kind.[21] As he puts it, if the ordinary relatum of a psychological relation does not exist, this does not mean that a 'non-ordinary' relatum must fill the gap [ibid. p. 105].

It is not my goal (and neither is it Crane's) to suggest that 'intentional' is a privative modifier so that the truth of the sentence 'No intentional object is an object' would follow from the meanings of the words involved. In my account, both physical and intentional objects are objects—but objects of neither type are *local* objects. In Crane's analysis, some intentional objects are existent objects and others are not. For him, existent intentional objects are, literally, physical objects in the world, whereas non-existent intentional objects are not real—they are not in the world to begin with. In my view, as according to Crane, there are both existent and non-existent intentional objects. Being capable of 'existing' is not a feature that distinguishes physical objects from intentional objects. An intentionally individuated world line may but need not exist—i.e., be realized in the current circumstances. Moreover, an intentional object $\mathbf{J}$ may but need not share realizations with a physical object $\mathbf{I}$: there can be a non-empty set of worlds X such that X is included in the modal margins of $\mathbf{I}$ and $\mathbf{J}$, and for all worlds w in X, we have $\mathbf{I}(w) = \mathbf{J}(w)$. By contrast, according to my analysis,

[21] In Crane's analysis, only what *exists* can be an entity, and existent intentional objects are ordinary physical objects. In my analysis, we may only quantify over physical objects in worlds in which they exist. It is a separate question, not to be addressed in this book, whether past individuals qualify as entities or whether we should think of them as intentional objects.

an intentional object $\mathbf{J}$ cannot in practice literally *be* a physical object $\mathbf{I}$ in the sense that we would have $marg(\mathbf{I}) = marg(\mathbf{J})$ and *all* worlds w belonging to this set would satisfy $\mathbf{I}(w) = \mathbf{J}(w)$. (For a discussion, see Sect. 4.8.)

Crane grounds his analysis on the basic notion of representation. For him, an intentional object is what is represented by the mind [ibid. p. 93]. I take intentional states to be analyzable as structures consisting of a world representation and a number of object representations, and my goal is to characterize the difference between object representations and physical objects in terms of the modal behavior of the two types of world lines—instead of simply postulating an unanalyzed qualitative difference between the two types of objects. By contrast, Crane's account merely consists of postulating representations along with physical objects. Another difference between the two views is that mine relies on a theoretical, semantic conception of representation, while Crane's notion is concrete and phenomenological in the sense discussed in Sect. 3.2. What is important, however, is that both analyses render it meaningful to speak of intentional objects without getting committed to a peculiar ontological category of intentional objects construed in the sense of object theories.

In my account of intentional objects, Husserl's idea of two types of objects of *propositional* acts applies even to states that, by Husserl's standards, lack an object. Moreover, it applies not only to propositional acts but also to what Smith and McIntyre [113, p. 7] call *direct-object* acts. In a direct-object act, the agent intends so-and-so (e.g., Alice sees a lion). Generally, if intentional states can be analyzed as structures $\langle W, \mathcal{J}, w_0 \rangle$, they are systematically combinations of two types of objects: a world representation (a proposition) and a number of object representations (intentional objects). These two aspects of intentional states will be further discussed in Sect. 6.2.

Crane [21, p. 117] notes that one of the reasons why many philosophers find the idea of representation problematic is that representation does not seem to be a relation in any straightforward sense: we can represent non-existent things. It is not generally a 'real relation'. For Crane, the character of representations as *representations of so-and-so* is a feature that cannot be further analyzed. He defines the content of a representation as the way the object of the representation is represented [ibid. p. 99]. He stresses that the content need not be propositional—i.e., it need not be a representation according to which things are a certain way. A content, in my sense, is a complex consisting of a world representation and a number of object representations. If the list of object representations happens to be empty, the content is a propositional representation. Such a propositional representation can be seen as a representation of the world in which the agent is located (the actual world), although the representation can be mistaken to the point that the actual world is incompatible with the representation. If an intentional content involves object representations, the content is a really or apparently relational representation, depending on whether the object representations represent specific physical objects.

I close this section by noting that an intentional state $\langle W, \mathcal{J}, w_0 \rangle$ may involve *overlapping* object representations: there can be a world $w \in W$ and distinct intentionally individuated world lines $\mathbf{J}_1, \mathbf{J}_2 \in \mathcal{J}$ such that $\mathbf{J}_1(w) = \mathbf{J}_2(w)$. I describe here two examples: one about not having a definite opinion as to whether given intentional objects are the same, the other about the phenomenon of double vision.[22]

Example 4.3 Suppose Alice is thinking of a spy and a parliament member. She has begun to suspect that they might be one and the same person, but she recognizes that the evidence she has at her disposal leaves equally open the possibility that her suspicions are unfounded. She believes neither that the spy and the parliament member are one and the same individual, nor that they are not one and the same individual. The content of her belief can be described as follows:

19. $\mathsf{E_a} x\, \mathsf{E_a} y\, \big(\Box S(x) \;\wedge\; \Box P(y) \;\wedge\; \Diamond x = y \;\wedge\; \Diamond \neg x = y\big).$

That is, Alice has intentional objects **I** and **J** such that the predicate '— is a spy' is ascribed to **I** and the predicate '— is a parliament member' is ascribed to **J** under the intentional mode, and in some worlds compatible with Alice's beliefs, the two world lines overlap, while in others, they have distinct realizations.[23] □

Example 4.4 Hintikka suggests that the phenomenon of seeing double can be analyzed in terms of overlapping world lines [46, pp. 121–3, 210, 222]. In cases of double vision, the agent is subject to the illusion of seeing two things, this illusion being in fact triggered by a single physical object. If, for instance, Bob has a double vision of a tomato, the perceptual situation could be described by the formula

20. $\exists x_0\, \mathsf{E_b} x\, \mathsf{E_b} y\big(x = x_0 \;\wedge\; y = x_0 \;\wedge\; T(x_0) \;\wedge\; \Box_{\mathsf{V}}[T(x) \;\wedge\; T(y) \;\wedge\; \neg x = y]\big).$

Here, the witness of $\exists x_0$ is the physical tomato giving rise to the illusion of separate intentional objects (the witnesses of $\mathsf{E_b} x$ and $\mathsf{E_b} y$). The latter have distinct realizations over the whole set of scenarios compatible with Bob's visual experience. The subformula $T(x) \wedge T(y) \wedge \neg x = y$ serves to describe the content of Bob's experience. A logical analysis of diplopia would be difficult to envisage in the absence of world lines, in general, and without the two types of world lines, in particular.[24] □

4.8 Representations of Physical Objects

I pointed out in Sect. 4.6 that a state's having a material object is not enough for its intentional object to represent a specific physical object. A suitable object representation must be relevantly *accurate*, and it must represent an object under a relevant

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[22]Earlier in this book, we have encountered overlapping world lines, of which one is intentionally and the other is physically individuated (the notion of material object of an intentional state). Further, I have assumed that no two *physically* individuated world lines share a realization.

[23]Since, in (19), the predicates S and P are existence-entailing, the formula $\Box S(x) \;\wedge\; \Box P(y) \;\wedge\; \Diamond \neg x = y$ entails $\Diamond[x = x \;\wedge\; y = y \;\wedge\; \neg x = y]$. That is, if (19) is true and the values of x and y are, respectively, the witnesses of $\mathsf{E_a} x$ and $\mathsf{E_a} y$, then the reason why these values satisfy $\Diamond \neg x = y$ cannot be that at least one of them fails to be realized in an accessible world.

[24]Hintikka does not phrase his example using two types of quantifiers but simply in terms of splitting world lines. (For splitting, see Sect. 5.2.) In reality, we need two types of world lines: we have two intentional objects of visual experience that pertain to one and the same physical thing.

aspect. The representation is under a relevant aspect if it is in terms of a suitable cluster of predicates that the material object of the intentional state actually satisfies. A necessary condition for its accuracy is that it is under a relevant aspect and does not allow too much indeterminacy with respect to those predicates of the material object that do *not* characterize the intentional object. My example of Alice having as her intentional object a tomato, wildly indeterminate in weight, size, and color, was meant to illustrate how accuracy may fail. It was also meant to illustrate that *being a tomato* is not alone a sufficient aspect under which to represent a physical object; it should be augmented by further features, like *having a weight between 110 and 140 grams* and *being brownish*. The example of an illusion where Alice takes a bear for a stone provides an example of a less sophisticated violation of accuracy due to a mistaken choice of an aspect (Sect. 4.4).

To keep the discussion more manageable, I assume that hypotheses H3 and H4 are in force: I operate with models of the original sort. Further, I restrict my attention to cases in which an intentional state is supposed to represent a physical object *as it is in the actual world at an instant*—the state is not supposed to represent the temporal and modal behavior of its material object. As will become clear, it does not follow from this latter assumption that we can completely ignore how the material object of the state behaves in counterfactual situations. It just means that our goal is not to represent this behavior. Finally, I assume that to discuss the actual behavior of the material object, attention may be confined to intentional states with a single intentional object described in terms of quasi-extensional predicates and therefore indeed in terms of atomic extensional predicates (cf. Sect. 2.4). Under this assumption, distinct aspects are logically independent, and we need not consider predicates applying to (realizations of) several intentional objects. In sum, I consider cases in which an intentional object is supposed to represent, in terms of unary atomic predicates, the momentary behavior of a physical object in a single scenario.

Let us come back to Question (c) of Sect. 4.6. Should we perhaps say that an intentional object $\mathbf{J}$ represents a physical object $\mathbf{I}$ when $\mathbf{J}$ *coincides* with $\mathbf{I}$ in the sense that $\mathbf{I}(w) = \mathbf{J}(w)$ for all worlds w in the internal modal margin of $\mathbf{J}$? If this was correct, Alice's thought pertaining to a real tomato could be described by (21):

21. $\mathsf{E}_{\mathsf{a}} x\, \exists y\, [T(y) \wedge \Box(x = y \wedge T(x))]$.

Now, (21) means that there is a physical object $\mathbf{I}$ (witness of $\exists y$) that coincides with the intentional object $\mathbf{J}$ of Alice's thought (witness of $\mathsf{E}_{\mathsf{a}} x$) over the set $R(w_0)$. This condition is too strong. The intentional object $\mathbf{J}$ manifests indeterminacy, and in particular, this entails that in $R(w_0)$, there are internally indistinguishable worlds w and w' such that the realizations $\mathbf{J}(w)$ and $\mathbf{J}(w')$ are qualitatively different. It would follow from (21) that the realizations $\mathbf{I}(w) = \mathbf{J}(w)$ and $\mathbf{I}(w') = \mathbf{J}(w')$ of the physical object $\mathbf{I}$ are qualitatively different, as well, even if w and w' are internally indistinguishable—contradicting hypothesis H5 (see Sect. 4.5). This is a serious problem, as indeterminacy is unavoidable for representation. Note that the problem with the proposal is *not* simply that physical objects obey the principle of complete determination while intentional objects do not. The fact that intentional objects are not subject to determination in the sense of intentional predication does not by itself render impossible coincidence with a physical object, since such a coincidence would not entail that the physical object is indeterminate in the *relevant* sense—i.e., in the sense of physical predication.[25] The problem comes from there being internally indistinguishable worlds in which the respective realizations of an intentional object are mutually different qualitatively.

To spell out what it takes for an intentional object $\mathbf{J}$ to represent a physical object $\mathbf{I}$, we need, then, something less than coincidence over the set $R(w_0)$. Still, representing a physical object in terms of an intentional state is *demanding*: the set $R(w_0)$ of scenarios compatible with Alice's intentional state is subject to certain limiting conditions. Generally, a scenario could be impossible by physical standards but not impossible for Alice to think of. However, insofar as Alice's intentional state is to represent a physical object, the set $R(w_0)$ must rule out scenarios that are physically impossible. What is more, generally, a scenario w could be both physically possible and compatible with Alice's intentional state and yet diverge from the actual circumstances to the point that the realization $\mathbf{I}(w)$ of the physical object $\mathbf{I}$ in w would be qualitatively grossly different from the realization $\mathbf{I}(w_0)$ of $\mathbf{I}$ in w_0. Thus, it could happen that $\mathbf{J}$ is a representation under an aspect P, with $\mathbf{J}(w_0) = \mathbf{I}(w_0)$ indeed satisfying P, and still, $R(w_0)$ might contain scenarios in which $\mathbf{I}$ does not satisfy this predicate. This would block $\mathbf{J}$ from being a representation of $\mathbf{I}$ under the aspect P. I take it that in order for $\mathbf{J}$ to represent $\mathbf{I}$, the set $R(w_0)$ must contain exclusively worlds in which $\mathbf{I}$ is realized and satisfies all the predicates characterizing $\mathbf{J}$.

Let us consider intentional states $\langle W, \mathbf{J}, w_0\rangle$ with a single object representation, letting P be a unary atomic predicate. What does it take for $\mathbf{J}$ to represent a physical object $\mathbf{I}$ under the aspect P? The minimal requirements concerning the world representation W and the object representation $\mathbf{J}$ are:

(i) The actual scenario is compatible with the agent's intentional state: $w_0 \in W$.
(ii) The physical object $\mathbf{I}$ is the material object of the state: it is available in w_0 and satisfies $\mathbf{J}(w_0) = \mathbf{I}(w_0)$.
(iii) The predicate P is intentionally ascribed to $\mathbf{J}$ in w_0.

As noted in Sect. 4.4, it is sufficient but not necessary for clause (i) that the agent's intentional state is factive. (For example, belief is not factive, but still, the actual circumstances can of course be compatible with one's beliefs.) From (i) and (ii), it follows that w_0 belongs to the internal modal margin of $\mathbf{J}$. Further, these clauses guarantee that if Φ is the set of atomic predicates intentionally ascribed to $\mathbf{J}$, all these predicates are satisfied by $\mathbf{I}(w_0)$: all predicates in Φ are physically ascribed to $\mathbf{I}$ in w_0. It of course does not follow that conversely, all atomic predicates physically ascribed to $\mathbf{I}$ in w_0 are intentionally ascribed to $\mathbf{J}$: the set Φ of aspects under which $\mathbf{J}$ represents $\mathbf{I}$ in w_0 need not be exhaustive in any way. The conjunction of (i) and (ii) is compatible with the set Φ being empty. Clause (iii), however, specifies that $P \in \Phi$, guaranteeing that $\mathbf{J}$ is a representation at least under the aspect P.

[25] For a further discussion of this point, see the end of Sect. 6.4.

In order for $\mathbf{J}$ to represent $\mathbf{I}$ under the aspect P, conditions (i), (ii), and (iii) do not suffice. The following requirements must be met, as well:

(iv) The intentional object $\mathbf{J}$ is a representation of an existing object: $\mathbf{J}$ is realized throughout W. In other words, $W \subseteq marg(\mathbf{J})$.
(v) The set W is suited for the physical object $\mathbf{I}$: the physically individuated world line $\mathbf{I}$ is realized throughout W. In other words, $W \subseteq marg(\mathbf{I})$.
(vi) For every world $w \in W$, there is a world $w' \in W$ internally indistinguishable from w such that $\mathbf{J}(w') = \mathbf{I}(w')$.

Clause (iv) guarantees that the internal modal margin of $\mathbf{J}$ equals W. On the basis of clauses (i)—(iii), we could only infer that the set $W \cap marg(\mathbf{J})$ is non-empty. Clause (v) admits as compatible with the agent's intentional state only worlds in which the physical object $\mathbf{I}$ is realized. Even so, because world representations are indeterminate, the set W contains always a large number of pairwise non-isomorphic worlds. Therefore, although $\mathbf{J}$ is supposed to represent $\mathbf{I}$ as it is in the actual world, clause (v) forces the representation to take into account the counterfactual behavior of $\mathbf{I}$. Moreover, for every world $w \in W$, the set W contains always a large number of worlds isomorphic to w. Clause (vi) states that $\mathbf{J}$ must coincide with $\mathbf{I}$ at least in one world of every world type in W. That is, for any isomorphism type of counterfactual worlds exemplified in W, there is a counterfactual realization of the physical object $\mathbf{I}$ that coincides with the realization of the intentional object $\mathbf{J}$. In addition, because object representations are indeterminate, any isomorphism type contains a large number of worlds u such that $\mathbf{I}(u) \neq \mathbf{J}(u)$. It must be noted that condition (v) does *not* concern all physical objects available in w_0; it only concerns the physical object $\mathbf{I}$ whose representability in terms of $\mathbf{J}$ is being discussed.

Clauses (iii), (v), and (vi) entail that the physical object $\mathbf{I}$ satisfies P and, in fact, all predicates belonging to Φ, throughout the set W. Namely, let $w \in W$ and $Q \in \Phi$ be arbitrary. By (v), $\mathbf{I}$ is realized in w. By (vi), there is a world w' internally indistinguishable from w such that $\mathbf{J}(w') = \mathbf{I}(w')$. Since $w' \in W \cap marg(\mathbf{J})$, by the definition of Φ, the local object $\mathbf{J}(w')$ is Q. Thus, $\mathbf{I}(w')$ is Q, because $\mathbf{J}(w') = \mathbf{I}(w')$. By hypothesis H5 of Sect. 4.5, $\mathbf{I}(w)$ satisfies exactly the same atomic predicates as $\mathbf{I}(w')$. Since Q is atomic, we may conclude that $\mathbf{I}(w)$ is Q. Note that if S is any atomic predicate from outside Φ, it *can* happen that $\mathbf{I}$ satisfies S in w_0 but not throughout W. Further, $\mathbf{I}$ may lack an atomic predicate in w_0 while satisfying it in some world belonging to W. However, if $\neg S(x)$ is intentionally ascribed to $\mathbf{J}$ in w_0, then $\mathbf{I}$ lacks S throughout W: if there was $w \in W$ such that $\mathbf{I}(w)$ is S, then there would be $w' \in W$ internally indistinguishable from w such that $\mathbf{J}(w') = \mathbf{I}(w')$, whence $\mathbf{J}$ would be S in w', contrary to the assumption that it satisfies $\neg S(x)$ in all worlds belonging to W.

It is possible to provide a logical characterization of the condition that must be satisfied in order for an intentional object to represent a physical object under the aspect P, if we use $\Box$ to represent compatibility with the agent's intentional state and another modal operator $\blacksquare$ to speak of mutually isomorphic worlds: $\blacksquare\chi$ holds in w iff χ holds in all worlds isomorphic to w. The condition under which Alice's intentional object represents at w_0 a physical object as being P is characterizable by the formula

22. $\mathsf{E_a}x\, \exists y\, (x = y \;\wedge\; \Box\, P(x) \;\wedge\; \Box\, y = y \;\wedge\; \Box\blacklozenge\, x = y)$,

assuming that the accessibility relation R correlated with $\Box$ satisfies $R(w_0, w_0)$, so that (i) is in force. (This assumption is of course not needed if the state considered is factive.) Suppose, first, that (22) is true in a model M at w_0, where $W = R(w_0) \subseteq dom(M)$. If the witnesses of $\mathsf{E_a}x$ and $\exists y$ in M at w_0 are, respectively, $\mathbf{J}$ and $\mathbf{I}$, clauses (ii) and (vi) hold because $M, w_0, x := \mathbf{J}, y := \mathbf{I} \models x = y \wedge \Box\blacklozenge\, x = y$. Since $M, w_0, x := \mathbf{J} \models \Box\, P(x)$ and P is existence-entailing, it follows that $W \subseteq marg(\mathbf{J})$, whence (iv) holds. Because $\Box\, P(x)$ trivially entails $\Box\,(x = x \rightarrow P(x))$, clause (iii) holds, as well. The condition $M, w_0, y := \mathbf{I} \models \Box\, y = y$ entails clause (v). Conversely, suppose that $\langle W, \mathbf{J}, w_0 \rangle$ is an R-situated content over a model M. Suppose the world lines $\mathbf{I}$ and $\mathbf{J}$ satisfy clauses (ii) through (vi), $\mathbf{I}$ being physically and $\mathbf{J}$ intentionally available in w_0. Clearly, (22) is true in M at w_0 under these assumptions.

Clauses (i)—(vi) yield a reasonable approximative analysis of what it is for an intentional object to represent a specific physical object under a fixed aspect. In order to convert this analysis into an answer to Question (c) of Sect. 4.6, a general notion of physical-object representation must be formulated. In accordance with what was noted in the beginning of this section, an intentional object $\mathbf{J}$ cannot represent a physical object $\mathbf{I}$ unless there is a suitable cluster of aspects Φ such that $\mathbf{J}$ represents $\mathbf{I}$ under all aspects $P \in \Phi$ and unless the set of aspects under which $\mathbf{J}$ fails to represent $\mathbf{I}$ is suitably limited. It is not to be expected that anything very precise could be said about how to delineate the set Φ.

Regarding the accuracy of the analysis provided by clauses (i)—(vi), at the very least, this analysis can be said to show that my framework allows formulating different non-trivial conditions on the interaction of intentional and physical objects. It can surely be suggested that clauses (v) and (vi) are *needlessly strong*: conceivably, the status of $\mathbf{J}$ as a representation of $\mathbf{I}$ tolerates the presence of worlds in which $\mathbf{I}$ is not realized, and it might suffice to formulate the notion of internal indistinguishability using an equivalence relation somewhat weaker than isomorphism. One could even claim that clauses (i)—(vi) are jointly *too weak* by insisting that the set W must be triggered by an intentional state of a specific kind (e.g., knowledge or perception rather than mere belief). Namely, the smaller the set W is, the easier it becomes for it to satisfy conditions (i)—(vi). Someone's beliefs might, without any justification, rule out a huge number of possible scenarios so that W would be very small indeed, for which reason it would be relatively easy for this person's intentional state to satisfy the six clauses. This is why it might be necessary to stipulate that the set W cannot be generated by an arbitrary intentional state but must be subject to some form of justification so that the agent can only exclude a world from W if she can justify the exclusion. Depending on how strong one's intended notion of physical-object representation is, the task of characterizing what it takes for an intentional object to represent a physical object can face major difficulties comparable to those encountered when attempting to characterize knowledge in terms of belief. I will not discuss these problems within the confines of this book.[26]

It is useful to note that I have, in effect, distinguished two senses in which an intentional state can be *de re*. In a weak sense, this happens when the state merely has a material object. A strong sense requires that the intentional object of the state be a representation of a physical object. A necessary condition for this latter requirement is that suitable variants of clauses (i)—(vi) are in force.

[26]In this book, I have taken it to be sufficient for the *veridicality* of perceptual experience that it is a factive intentional state (Sects. 2.3, 4.4). There is, then, no guarantee that object-directed perceptual states that are veridical in this sense actually represent a physical object. If one wishes to qualify only such object-directed perceptual experiences 'veridical' that indeed represent a physical object, then it should be noted that the condition under which propositional experiences are veridical is much weaker and much easier to formulate than the condition under which an object-directed experience is veridical.

Chapter 5
Logical Repercussions of World Line Semantics

5.1 Introduction

In the modal language L, the quantifiers $\exists x$ take as their values physically individuated world lines, while the quantifiers $\mathsf{E}_{\mathsf{a}}x$ range over world lines intentionally individuated by an agent. In Chap. 4, I employed world lines of the latter type in accounting for features of intentional objects and their relation to physical objects. Having thereby acquired a better idea of the contrast between the two modes of individuation of world lines, it becomes natural to explore the consequences of my semantic framework to questions of a logical character.

I phrase two versions of the principle of substitutivity of identicals—one in terms of bound variables and the other in terms of constants symbols. I indicate under what conditions these principles hold according to world line semantics (Sects. 5.2 and 5.3). Recalling the Barcan formula (BF) and its converse (CBF) encountered in Sect. 2.3, I formulate two variants of both formulas, differing in the types of quantifiers used (intentional or physical). I compare the model-theoretic behavior of the resulting formulas with the properties that BF and CBF have under Kripke semantics (Sect. 5.4). In Sect. 5.5, I discern two notions of validity (model-theoretic vs. schematic validity) that are equivalent in first-order logic (FOL) and in first-order modal logic when interpreted according to Kripke semantics (FOML) but not equivalent under world line semantics.[1] I point out in Sect. 5.6 that the logic L lacks a well-behaved notion of logical form. I explain, on the one hand, that this is no reason to dismiss L as a logical language worthy of study and on the other hand that L has a natural extension S_L without this defect—i.e., an extension in which the notion of logical form behaves in the expected way. Further, this extended language S_L can in effect be used to talk about logical forms in L.

In Sect. 5.7, I show that L is translatable into first-order logic. This result is somewhat surprising, given the apparent higher-order character of L: values of quantifiers are world lines, which are partial functions over worlds. Since Lewis's counterpart

[1] For FOL and FOML, see Sect. B.1 of the Appendix B.

T. Tulenheimo, *Objects and Modalities*, Logic, Epistemology, and the Unity of Science 41, DOI 10.1007/978-3-319-53119-9_5

theory is standardly formulated in terms of first-order logic [75], the possibility of such a translation makes it particularly evident that counterpart theory is a special case of world line semantics; cf. the discussion in Sect. 2.7.2. World line semantics is not, conversely, a special case of counterpart theory, because the latter is based on the specific assumption that counterpart relations are triggered by relations of qualitative similarity among world-bound objects instead of being independent of worlds and because counterpart theory cannot simulate the possibility for a world line to be available without being realized—this being essential for the analysis of non-existent intentional objects in connection with world line semantics.

I explain in the concluding section (Sect. 5.8) that anomalous semantic properties of L stem from features of the subject matter discussed, in particular from the fact that the simplest properties considered are existence-entailing. The anomalies of the language do not tell against my framework, because they stem from the nature of the subject matter. The language used for talking about a subject matter must, of course, be designed so as to make the relevant features of the subject matter expressible. Whatever the formal properties of the resulting language are, they must be tolerated as long as the language serves its purpose.

5.2 Quantifiers and the Substitutivity of Identicals

I discern two versions of the principle of *substitutivity of identicals* (SI), according to whether the quantifiers considered are intentional or physical:

SI-V-I $\mathsf{A}\mu\mathsf{A}\nu\big((\mu = \nu \wedge \xi\big) \to \xi[\mu/\nu])$
SI-V-P $\forall\mu\forall\nu\big((\mu = \nu \wedge \xi\big) \to \xi[\mu/\nu])$.

The principles above are to be understood as schemata. Their *instances* are obtained as follows: if x and y are variables, θ is an L-formula in which y is free for x, and θ' is the result of replacing at least one free occurrence of x in θ (not necessarily all its free occurrences) by an occurrence of y, then the formula $\mathsf{A}x\mathsf{A}y([x = y \wedge \theta] \to \theta')$ is an instance of SI-V-I, and the formula $\forall x\forall y([x = y \wedge \theta] \to \theta')$ is an instance of SI-V-P. There are no other ways of producing instances of these schemata.[2]

World lines $\mathbf{I}$ and $\mathbf{J}$ *split* if there are worlds $w \in W$ and $w' \in R(w)$ such that both world lines $\mathbf{I}$ and $\mathbf{J}$ are realized in both worlds w and w', and $\mathbf{I}(w) = \mathbf{J}(w)$ but $\mathbf{I}(w') \neq \mathbf{J}(w')$: these world lines have a common realization in w but distinct realizations in a world w' accessible from w. The *modal depth* of an L-formula ϕ, denoted $md(\phi)$, is the maximum number of nested modal operators occurring in ϕ. The *degree* of ϕ is the number indices of modal operators occurring in ϕ. For

[2] Hintikka [44, pp. 130–1] formulates SI similarly using arbitrary substitutions rather than uniform substitution. Sometimes, SI is referred to as *indiscernibility of identicals*. Such terminology puts the focus on objects in the world instead of linguistic expressions. In reality, the principle concerns the interchangeability of certain expressions: according to it, if we are given the premise $x = y$ and a suitable formula in which x occurs free, then the variables x and y are interchangeable in the formula in a satisfaction-preserving way.

example, if $\mathsf{i} \neq \mathsf{j}$, the degree of $\Box_{\mathsf{i}}\Box_{\mathsf{j}}Q(x)$ is 2 and the degree of $\Box_{\mathsf{i}}\Diamond_{\mathsf{i}}Q(x)$ is 1. The modal depth of both formulas is 2. The modal depth of $\Diamond_{\mathsf{i}}P(x) \vee \Box_{\mathsf{j}}Q(x)$ is 1, its degree being 2. If the degree of ϕ is k and $md(\phi) = n \geq 1$, the maximum number of distinct (non-empty) sequences of nested modal operators in ϕ is $\Sigma_{m=1}^{n} k^m$.

Hintikka remarks that the validity of SI requires two things: (a) world lines never split *and* (b) if world lines **I** and **J** coincide locally, then either can be continued as far as the other along the accessibility relations of the model.[3] I proceed to show that suitable versions of (a) and (b) are necessary and jointly sufficient for SI-V-I. The argument can be readily adapted so as to make it applicable to SI-V-P. In Sect. 5.3, I discuss a version of SI formulated using *constant symbols*. The conditions under which the constant-symbol version of SI is valid are altogether different from those validating the principles SI-V-I and SI-V-P.

Henceforth, I assume that $\mu := x$ and $\nu := y$. The schema SI-V-I has exactly one instance with $\xi := \Box P(x)$—namely, $\mathsf{A}x\mathsf{A}y([x = y \wedge \Box P(x)] \rightarrow \Box P(y))$. This instance is clearly true in a model M at a world w if the following sentence ψ^1 is true in M at w:

$$\mathsf{A}x\mathsf{A}y(x = y \rightarrow \Box((x = x \vee y = y) \rightarrow x = y)).$$

The sentence ψ^1 expresses not only that no two locally coincident intentionally available world lines split but also that if one of them is realized in an accessible world, so is the other. More generally, having ψ^1 as an extra premise suffices for inferring all instances of SI-V-I obtained by letting ξ be a formula of modal depth at most 1. By contrast, ψ^1 is not enough to warrant, for example, the instance of SI-V-I with $\xi := \Box\Diamond P(x)$. For this purpose, the premise ψ^2 suffices:

$$\mathsf{A}x\mathsf{A}y(x = y \rightarrow \Box\Box((x = x \vee y = y) \rightarrow x = y)).$$

In the formula $\Box(Q(x) \wedge \Diamond P(x))$, the variable x appears both in the scope of exactly one modal operator and in the scope of exactly two modal operators. The *conjunction* of ψ^1 and ψ^2 allows us to infer all three instances of SI-V-I with $\xi := \Box(Q(x) \wedge \Diamond P(x))$. (There are three ways of replacing at least one of the two occurrences of x by y.) The general situation is further complicated by the possible presence of modal operators of several types: the degree of the formula replacing ξ need not be 1. However, for any instance of SI-V-I, a finite number of extra premises suffice. We need the following definition to discuss the general case.

Definition 5.1 (*Modal character, modal profile*) If $\phi \in L$, let $\mathsf{i}_1, \ldots, \mathsf{i}_k$ be the indices of modal operators occurring in ϕ. If $m \in \mathbb{N}$, let $\langle \mathsf{j}_1, \ldots, \mathsf{j}_m \rangle$ be a tuple whose members are among the elements of the set $\{\mathsf{i}_1, \ldots, \mathsf{i}_k\}$. (The tuple may contain several occurrences of one and the same index: if $k \geq 1$, we may have $m > k$.) The tuple $\langle \mathsf{j}_1, \ldots, \mathsf{j}_m \rangle$ is a *modal character in* ϕ if $m \geq 1$ and it satisfies the following: there are in ϕ modal operator tokens $\bigcirc^1, \ldots, \bigcirc^m$ with respective indices $\mathsf{j}_1, \ldots, \mathsf{j}_m$ such that for all $1 \leq r < m$, $\bigcirc^{r+1}$ is the immediate successor of $\bigcirc^r$ along the

[3]See [44, pp. 100, 130], [46, pp. 121, 136]. A schema is *valid* over a class $\mathcal{K}$ of models if all its instances are satisfied in all models $M \in \mathcal{K}$ at all worlds $w \in dom(M)$ under all assignments in M.

relation of syntactic subordination among modal operator tokens in ϕ, and $\bigcirc^1$ is not subordinate to any modal operator in ϕ. The *modal profile* of ϕ (denoted P_ϕ) is the set of all modal characters in ϕ. □

If n is the modal depth of ϕ and k is its degree, there are at most $\Sigma_{m=1}^{n} k^m$ different modal characters of ϕ.[4] This is, then, an upper bound to the size of the modal profile P_ϕ of ϕ. Note that if $\langle \mathsf{j}_1, \ldots, \mathsf{j}_m \rangle \in P_\phi$, then $1 \leq m \leq n$. If $n = k = 0$, the set P_ϕ is empty. Now, if $m \geq 1$ and $\mathbf{i} = \mathsf{i}_1 \ldots \mathsf{i}_m$, write $\psi_\mathbf{i}$ for the sentence

$$\mathsf{A}x\mathsf{A}y(x = y \rightarrow \Box_{\mathsf{i}_1} \cdots \Box_{\mathsf{i}_m}((x = x \vee y = y) \rightarrow x = y)).$$

The premise $\bigwedge_{\mathbf{i} \in P_\phi} \psi_\mathbf{i}$ suffices for justifying the instances of SI-V-I with $\xi := \phi$. Actually, we only *need* those conjuncts of the premise $\bigwedge_{\mathbf{i} \in P_\phi} \psi_\mathbf{i}$ that correspond to sequences of modal operators in the scope of which the variable x occurs free in ϕ.[5]

Fact 5.1 *Let ϕ be an L-formula in which y is free for x. Let ϕ' be the result of replacing at least one free occurrence of x in ϕ by y. The following is a valid formula:*

$$\bigwedge_{\mathbf{i} \in P_\phi} \psi_\mathbf{i} \rightarrow \mathsf{A}x\mathsf{A}y([x = y \wedge \phi] \rightarrow \phi').$$

Proof See Sect. A.2 of the Appendix A. □

Conversely, it can be shown that having all instances of SI-V-I as premises suffices for inferring all formulas $\bigwedge_{\mathbf{i} \in P_\phi} \psi_\mathbf{i}$ with $\phi \in L$. Let us take an example. Letting $\xi := \Box_\mathsf{i}[x = x \rightarrow x = x]$, suitable substitutions of y for x allow us to obtain both of the following sentences as instances of schema SI-V-I:

$$\mathsf{A}x\mathsf{A}y(x = y \rightarrow \Box_\mathsf{i}[x = x \rightarrow x = y]) \text{ and } \mathsf{A}x\mathsf{A}y(x = y \rightarrow \Box_\mathsf{i}[y = y \rightarrow x = y]).$$

The schema yields instances of the form $\mathsf{A}x\mathsf{A}y([x = y \wedge \phi] \rightarrow \phi')$, but whenever ϕ is a valid formula—as is the case with $\Box_\mathsf{i}[x = x \rightarrow x = x]$—the conjunct ϕ can be eliminated from the antecedent of the implication *salva veritate*. Jointly, the two sentences yield the sentence ψ_i: $\mathsf{A}x\mathsf{A}y(x = y \rightarrow \Box_\mathsf{i}[(x = x \vee y = y) \rightarrow x = y])$. By similar reasoning, for any finite non-empty tuple $\mathbf{i}$ of indices, we can derive the sentence $\psi_\mathbf{i}$ from schema SI-V-I. Let SI-V-I-n be the schema whose instances are the instances of SI-V-I with values of ξ restricted to formulas of modal depth at most n. Write I_n for the set of non-empty tuples of indices of size at most n—i.e., $I_n = \cup_{1 \leq m \leq n} \mathbb{I}^m$. This set is finite, since $\mathbb{I}$ is a finite set; the set I_0 is empty. I say that a

[4] I stipulate that if $n = 0$ so that the sequence $k^1, \ldots, k^n$ is empty, the sum $\Sigma_{m=1}^{n} k^m$ equals 0.

[5] Hintikka [44, p. 126] takes this fact into account in his definition. The terms 'character' and 'profile' are adapted from Hintikka's definition. Concerning the notation $\bigwedge_{\mathbf{i} \in P_\phi} \psi_\mathbf{i}$, note that if $md(\phi) = 0$ and consequently the set P_ϕ is empty, the conjunction $\bigwedge_{\mathbf{i} \in P_\phi} \psi_\mathbf{i}$ with zero conjuncts is a valid sentence: trivially, all its conjuncts are true in any model at any world. We may stipulate that any conjunction with zero conjuncts equals, for example, $\mathsf{A}x(x = x \rightarrow x = x)$.

formula ϕ *characterizes* a schema ζ (in symbols $\phi \leftrightsquigarrow \zeta$) if for all models M, worlds $w \in dom(M)$, and assignments g in M, we have:

$$M, w, g \models \phi \text{ iff } (M, w, g \models \theta \text{ for all instances } \theta \text{ of } \zeta).$$

Fact 5.2 *For all $n \in \mathbb{N}$, we have $\bigwedge_{\mathbf{i} \in I_n} \psi_{\mathbf{i}} \leftrightsquigarrow$ SI-V-I-n.*

Proof Trivially, for every $n \in \mathbb{N}$ and $\phi \in L$ with $md(\phi) \leq n$, the formula $\bigwedge_{\mathbf{i} \in I_n} \psi_{\mathbf{i}}$ entails $\bigwedge_{\mathbf{i} \in P_\phi} \psi_{\mathbf{i}}$, which, by Fact 5.1, entails $\mathsf{A}x\mathsf{A}y([x = y \wedge \phi] \rightarrow \phi')$ for every ϕ' obtained from ϕ by a relevant substitution of y for x. (In the special case of $n = 0$, all formulas $\mathsf{A}x\mathsf{A}y([x = y \wedge \phi] \rightarrow \phi')$ are simply valid.) Thus, the left-to-right entailment holds. For the right-to-left direction, let $\mathsf{i}_1 \ldots \mathsf{i}_m \in I_n$ be arbitrary. If $n = 0$, the formula $\bigwedge_{\mathbf{i} \in I_n} \psi_{\mathbf{i}}$ is valid, and there is nothing to prove. Suppose, then, that $n > 0$ so that $\mathsf{i}_1 \ldots \mathsf{i}_m$ is a non-empty tuple of indices. Let M, w, and g be any model, world, and assignment such that $M, w, g \models \theta$ for all instances θ of SI-V-I-n. Among these θ, there are $2^4 - 1 = 15$ instances corresponding to the value $\xi := \Box_{\mathsf{i}_1} \ldots \Box_{\mathsf{i}_m}[x = x \rightarrow x = x]$. (They all have the modal depth $m \leq n$.) By the same reasoning as above in the case of modal depth 1, two of these 15 instances are jointly sufficient for entailing $\psi_{\mathsf{i}_1 \ldots \mathsf{i}_m}$. It ensues that $M, w, g \models \psi_{\mathsf{i}_1 \ldots \mathsf{i}_m}$. □

Let us write $\boxplus\phi$ for 'ϕ is true in all logically possible worlds'. As the character portrayed as Quine in Hintikka's dialogue notes [46, p. 123],[6] one would expect *any* world to be logically possible. If so, Fact 5.2 can be reformulated as the statement $\psi_0 \leftrightsquigarrow$ SI-V-I, where ψ_0 is the following formula:

$$\mathsf{A}x\mathsf{A}y(x = y \rightarrow \boxplus((x = x \vee y = y) \rightarrow x = y)).$$

Namely, ψ_0 entails $\bigwedge_{\mathbf{i} \in I_n} \psi_{\mathbf{i}}$ for all $n \in \mathbb{N}$. What is more, ψ_0 is entailed by the schema SI-V-I-n with $n := 1$, provided that $\boxplus$ is among the modal operators of L.[7] Hintikka is uncomfortable with this outcome, as he takes the possibility of splitting world lines to depend on the type of modality considered: as he sees it, this phenomenon cannot occur in relation to alethic modality, though it can occur in connection with intentional states, such as belief and perceptual experience. By his standards, ψ_0 is valid, but many of the formulas $\psi_{\mathbf{i}}$ are refutable, so that ψ_0 does not entail all formulas $\psi_{\mathbf{i}}$. He ends up suggesting that some possible worlds can violate analytical truths [46, p. 123] and that some doxastically possible worlds are not conceptually possible [49, pp. 559–60]. This type of defense is not convincing in the present setting; it would require a major deviation from the semantics as formulated thus far. In fact, Hintikka runs into a dilemma, because he attempts to tie the behavior of world lines to a type of modality—rather than to a mode of individuation of world lines.

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[6]The essay 'Quine on Quantifying In: A Dialogue' [46, pp. 102–36] is presented in the form of a dialogue between two parties: Q and C, or Quine and his critics.

[7]Actually, instead of $\boxplus$, it would be sufficient to use the global modality $\ominus$ ranging over all worlds of the model of evaluation: $M, w, g \models \ominus\phi$ iff $M, w', g \models \phi$ for all worlds $w' \in dom(M)$.

Let us compare the formula ψ_0 with the following formula ψ_0^{phys}, obtained by replacing the intentional quantifiers of ψ_0 by physical quantifiers:

$$\forall x \forall y (x = y \to \boxplus((x = x \vee y = y) \to x = y)).$$

The formula ψ_0^{phys} characterizes the schema SI-V-P in the same way that ψ_0 characterizes SI-V-I. That is, $\psi_0^{\text{phys}} \leftrightsquigarrow$ SI-V-P. Since I have assumed that no two *physically* individuated world lines overlap (H1), the formula ψ_0^{phys} is valid and consequently so are all instances of SI-V-P. However, this does not provide the slightest reason to think that the formula ψ_0 is valid or that no instance of SI-V-I is refutable. If ψ_0 were valid, Example 4.3 about Alice, a spy, and a parliament member would be contradictory, provided that in fact the spy and the parliament member happen to be one and the same person. That is, the following formula could not be true:

$$\mathsf{E_a} x \mathsf{E_a} y \big(x = y \ \wedge \ \Box[S(x) \ \wedge \ P(y)] \ \wedge \ \Diamond \neg x = y\big).$$

Namely, ψ_0 entails $\mathsf{A_a} x \mathsf{A_a} y (x = y \to \Box((x = x \vee y = y) \to x = y))$, which means that if Alice's intentional objects coincide in the actual world, her beliefs represent these objects as being identical. Rather than seeing this as a reason to revise the whole semantic framework by postulating impossible worlds while holding on to ψ_0, this is a good reason not to accept the principle ψ_0 unconditionally. In fact, any temptation to accept the validity of ψ_0 is explicable in terms of the contrast between physical and intentional quantifiers. If one fails to make the distinction, reasons for accepting the principle ψ_0^{phys} are erroneously transferred so as to appear as reasons for accepting the principle ψ_0. Given that ψ_0^{phys} is valid, the formula

$$\exists x \exists y \big(x = y \ \wedge \ \Box[S(x) \ \wedge \ P(y)] \ \wedge \ \Diamond \neg x = y\big)$$

is indeed contradictory.

5.3 The Logical Behavior of Constant Symbols

In the previous section, I discerned two versions of the principle of substitutivity of identicals. Both were formulated in terms of bound variables. One of the variants was seen to be refutable (SI-V-I), the other being valid (SI-V-P). A further version of the substitutivity principle can be formulated in terms of constant symbols:[8]

SI-C $\quad (\gamma = \delta \wedge \xi) \to \xi[\gamma/\delta].$

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[8] Recall that as the expression 'constant symbol' is used in this book, the referent of a constant symbol is required to be constant only relative to a fixed world (cf. Sect. 3.4). It is, actually, impossible that the referent of a constant symbol in one world be extensionally identical to its referent in another world (because such referents are local objects of distinct worlds). Further, while allowed interpretations of constant symbols could be limited by imposing conditions formulated in terms of world lines, in this book no such limitations are imposed.

The principle SI-C is a schema. If c and d are constant symbols, θ is an L-formula, and θ' is the result of replacing at least one occurrence of c in θ (not necessarily all its occurrences) by an occurrence of d, then the formula $([c = d \wedge \theta] \rightarrow \theta')$ is an *instance* of SI-C. There are no other ways of producing instances of this schema.

The schema SI-C is not valid: it has refutable instances. One way of showing this is by observing that the instance $(c = d \wedge \Box c = c) \rightarrow \Box c = d$ of SI-C is refutable. This is because interpretations of constant symbols in distinct worlds can be chosen independently of each other. Let M be a model with two worlds w and w' and an interpretation Int satisfying: $R(w, w')$ and $Int(c, w) = a_w = Int(d, w)$ and $Int(c, w') = a^1_{w'} \neq a^2_{w'} = Int(d, w')$. Then, $M, w \models (c = d \wedge \Box c = c)$ but $M, w \not\models \Box c = d$. In passing, we may note that this example shows that in L, identity statements formulated in terms of constant symbols can be true in one world without being true in all worlds: the model M refutes the formula $c = d \rightarrow \Box c = d$. That is, identity statements that are true factually need not be true necessarily. Indeed, the truth-condition of the identity statement $c = d$ is purely local. The truth of the sentence $c = d$ in a world w depends exclusively on the world w. From the fact that $c = d$ is true in w, nothing can be inferred concerning its truth in worlds other than w.[9]

Incidentally, the quantifier principle $\forall x \forall y(x = y \rightarrow \Box x = y)$ is also refutable (cf. Example 3.5), but it is important to note that the reason for this is entirely unrelated to the reason why the principle $c = d \rightarrow \Box c = d$ is refutable. The former fails because a physical object may be realized in a world without also being realized in all accessible worlds; the latter fails because interrelations of interpretations of fixed constant symbols may vary according to the world considered.

In order to refute SI-C, it is not essential to consider as values of ξ formulas that use the identity symbol. For example, the instance $[c = d \wedge \Box P(c)] \rightarrow \Box P(d)$ of SI-C is refutable, as well. Let $\Box$ range over the set $R(w_0)$ of Bob's doxastic alternatives in w_0. Let 'c' and 'd' be abbreviations of the names 'Molière' and 'J. B. Poquelin', respectively. In w_0, these names are coextensive: $Int(c, w_0) = Int(d, w_0)$. Suppose we have $\Box P(c)$ at w_0: Bob believes that Molière is the author of *Le Bourgeois gentilhomme*. That is, $Int(c, w) \in Int(P, w)$ for all $w \in R(w_0)$. Bob's belief need not be stronger: he might have acquired his belief by having asked who authored the named piece, without having any idea as to who Molière is, so that he has only learned that someone named 'Molière' wrote the piece. The fact that in every $w \in R(w_0)$, the author of the play in w is the object $Int(c, w)$ does not entail that these various interpretations of the constant symbol c are correlated by a *physically* individuated world line. There might be no two worlds w and w' in $R(w_0)$ such that their corresponding interpretations $Int(c, w)$ and $Int(c, w')$ are thus correlated. From the assumption $\Box P(c)$, it does not even follow that these interpretations are correlated by an *intentionally* individuated world line, though it may be a plausible

[9] As is well known, according to Kripke [68, 71], such identity statements as 'Hesperus = Phosphorus' are necessarily true, if true at all. He arrives at this conclusion because he takes proper names to be rigid designators. Conceptual problems with the notion of rigid designator were discussed in Sect. 2.7.1. In the present section, I indicate how the notion of rigid designator can be simulated in my framework, cf. Definition 5.2.

hypothesis about the use of proper names in doxastic contexts that these local objects must be so correlated.[10] Now, even if there was such an intentional object, the name 'J. B. Poquelin' need not go together with the name 'Molière' in Bob's doxastic alternatives. It may well be compatible with Bob's beliefs that 'J. B. Poquelin' names a Belgian painter or a Swiss fisherman, neither of whom is active as a playwright. That is, there may be a doxastic alternative for Bob in which such a person carries the name 'J. B. Poquelin'. If so, there are doxastic alternatives w in which $Int(c, w) \neq Int(d, w)$. Thus, $[c = d \wedge \Box P(c)] \rightarrow \Box P(d)$ is false at w_0.

In addition to the constant-symbol version of SI, another much-discussed principle involving constant symbols is *existential generalization* (EG). My framework motivates discerning a weak and a strong version of EG:[11]

EG-W $\quad \xi \rightarrow \mathsf{E}\mu(\mu = \mu \wedge \xi[\gamma/\mu])$
EG-S $\quad \xi \rightarrow \exists\mu\xi[\gamma/\mu]$.

If x is a variable, c is a constant symbol, θ is an L-formula containing at least one occurrence of c, and θ' is the result of replacing at least one occurrence of c in θ (not necessarily all its occurrences) by an occurrence of x, then the formula $\theta \rightarrow \mathsf{E}x(x = x \wedge \theta')$ is an *instance* of EG-W and $\theta \rightarrow \exists x\theta'$ is an *instance* of EG-S. There are no other ways of producing instances of these schemata. The consequent of any instance of EG-W states the availability of an actually realized *intentionally* individuated world line. The consequent of any instance of EG-S affirms the availability of a *physically* individuated world line. Observe that both formulas $\mathsf{E}x(x = x \wedge \theta')$ and $\exists x\theta'$ are ontologically committing: each of them can only be true in a world w if the relevant existential quantifier is witnessed by a world line *existing* in w—i.e., a world line realized in w. An ontologically non-committing version of EG can be formulated, as well:

EG-I $\quad \xi \rightarrow \mathsf{E}\mu\xi[\gamma/\mu]$.

The instances of EG-I are of the form $\theta \rightarrow \mathsf{E}x\theta'$. Here, the formula $\mathsf{E}x\theta'$ is ontologically non-committing, since its truth in a world w merely requires that the witness of the existential quantifier $\mathsf{E}x$ be an intentionally individuated world line available in w—it is not required that the world line be realized in w.

In order to compare the schemata EG-I, EG-W, and EG-S, let us begin by considering the formulas $\mathsf{E}x\,\Box P(x)$, $\mathsf{E}x\,[x = x \wedge \Box P(x)]$, and $\exists x\,\Box P(x)$. First, trivially $\mathsf{E}x\,[x = x \wedge \Box P(x)]$ entails $\mathsf{E}x\,\Box P(x)$. Second, $\mathsf{E}x\,\Box P(x)$ does not entail $\mathsf{E}x\,[x = x \wedge \Box P(x)]$. A value of x witnessing the quantifier $\mathsf{E}x$ in a world need not be realized in that world. For example, suppose $\mathbf{J}$ is an intentionally individuated world line available in w_1. Suppose $\mathbf{J}$ is realized in w_2 but not in w_1. Finally, suppose the local object $\mathbf{J}(w_2)$ is P. If w_2 is the only world accessible from w_1, then the assignment $x := \mathbf{J}$ satisfies the formula $\Box P(x)$ in w_1, but this

[10] That is, it is reasonable to think that if Bob has a belief he would express by using the proper name 'Molière', then 'Molière' marks an intentional object of Bob's belief by naming each realization of a certain intentionally individuated world line available to Bob at w_0. If so, the inference from $\Box P(c)$ to $\mathsf{E}_{\mathsf{Bob}}x\,\Box P(x)$ is valid, though the inference from $\Box P(c)$ to $\exists x\,\Box P(x)$ is not.

[11] These versions of the principle EG can also be formulated in terms of variables, cf. Sect. 6.6.

assignment does *not* satisfy the formula $x = x \wedge \Box P(x)$ in w_1, as **J** is not realized in w_1. Since **J** is available but not realized in w_1, it follows that $\mathsf{E}x \Box P(x)$ is true in w_1 but $\mathsf{E}x[x = x \wedge \Box P(x)]$ is false therein. Third, $\exists x \Box P(x)$ entails neither $\mathsf{E}x\,[x = x \wedge \Box P(x)]$ nor $\mathsf{E}x \Box P(x)$. This is because a physical object may be available in w_0 without any intentional object being available in w_0. Fourth, the formula $\mathsf{E}x\,[x = x \wedge \Box P(x)]$ is compatible with the formula $\mathsf{A}x\,\forall y\,(x = y \rightarrow \Diamond \neg x = y)$, so it does not entail $\exists x \Box P(x)$.[12] Fifth, the formula $\mathsf{E}x \Box P(x)$ is compatible with the formula $\mathsf{A}x\,\forall y \Box \neg x = y$, whence $\mathsf{E}x \Box P(x)$ does not entail $\exists x \Box P(x)$. These observations allow us to conclude, in particular, that the two schemata EG-W and EG-S are mutually independent in the sense of having mutually independent instances. Their instances $\Box P(c) \rightarrow \mathsf{E}x\,[x = x \wedge \Box P(x)]$ and $\Box P(c) \rightarrow \exists x \Box P(x)$ are indeed mutually independent, since neither of the formulas $\mathsf{E}x\,[x = x \wedge \Box P(x)]$ and $\exists x \Box P(x)$ entails the other. Similarly, it follows that EG-I and EG-S are mutually independent. As for EG-I and EG-W, trivially the latter is a stronger principle than the former. Any instance of EG-W entails the corresponding instance of EG-I, but not vice versa. For example, $\Box P(c) \rightarrow \mathsf{E}x\,[x = x \wedge \Box P(x)]$ entails $\Box P(c) \rightarrow \mathsf{E}x \Box P(x)$. Yet, the latter can be true while the former is false.

In extensional settings, certain instances of EG-S are valid. A case in point is $c = c \rightarrow \exists x\; x = x$. If $M, w \models c = c$, then 'c' stands for an element of $dom(w)$. As every local object is the realization of a physical object (H4), we have $M, w \models \exists x\; x = x$. Generally, $\phi^c \rightarrow \exists x\; \phi(x)$ is valid if $\phi(x)$ is existence-entailing and has modal depth 0, given that ϕ^c is the result of replacing all free occurrences of x in $\phi(x)$ by c. By contrast, $\neg P(c) \rightarrow \exists x\; \neg P(x)$ is refutable: if $Int(c, w) = *$, then $M, w \models \neg P(c)$, but still, all physically individuated world lines available in w may satisfy P in w. Similarly, $\Diamond P(c) \rightarrow \exists x \Diamond P(x)$ is refutable. The local object that 'c' denotes in an accessible world need not be the realization of any actually available physical object. For comparison, if we let the value of ξ be any of the formulas $c = c$, $P(c)$, $\neg P(c)$, or $\Diamond P(c)$, the corresponding instances of EG-W and EG-I are refutable. In fact, $\phi^c \rightarrow \mathsf{E}x[x = x \wedge \phi(x)]$ and $\phi^c \rightarrow \mathsf{E}x\; \phi(x)$ are refutable whenever $\phi(x)$ contains no intentional quantifiers. Examples of valid instances of EG-W are $[P(c) \wedge \mathsf{E}y\; y = c] \rightarrow \mathsf{E}x(x = x \wedge [P(x) \wedge \mathsf{E}y\; y = x])$ and $[\Box P(c) \wedge \mathsf{E}y \Box y = c] \rightarrow \mathsf{E}x(x = x \wedge [\Box P(x) \wedge \mathsf{E}y \Box y = x])$ if $\Box$ stands for a factive modality. Finally, $[P(c) \wedge \mathsf{E}y\; y = c] \rightarrow \mathsf{E}x[P(x) \wedge \mathsf{E}y\; y = x]$ and $[\Box P(c) \wedge \mathsf{E}y \Box y = c] \rightarrow \mathsf{E}x[\Box P(x) \wedge \mathsf{E}y \Box y = x]$ are valid instances of EG-I.

As has already been noted in Sect. 2.7.1, the notion of rigid designator is misguided according to the analysis adopted in this book: it conceals important conceptual distinctions instead of clarifying them. Language users may manage to associate the same name with realizations of one and the same world line over a set of worlds, but this is a derivative use of names. Primarily, a name is a name of a *realization* of a physical object (physically individuated world line), not a name of a physical object itself. The notion of rigid designator can, however, be simulated as follows.

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[12] There is *no* formula ϕ such that $\exists x\, \phi$ entails $\mathsf{E}x[x = x \wedge \phi]$. By contrast, $\mathsf{E}x[x = x \wedge \phi]$ entails $\exists x\, \phi$ if $md(\phi) = 0$: any realization of an intentional object is a realization of a physical object (H3, H4).

Definition 5.2 (*Relative rigid designator*) If $\mathsf{i}_1 \dots \mathsf{i}_m$ is a finite string of indices and c is a constant symbol, let us write $\theta^c_{\mathsf{i}_1 \dots \mathsf{i}_m} := \exists x(x = c \wedge \Box_{\mathsf{i}_1} \cdots \Box_{\mathsf{i}_m} x = c)$. We say that c is a *relative rigid designator of type* $\mathsf{i}_1 \dots \mathsf{i}_m$ in M at w iff $M, w \models \theta^c_{\mathsf{i}_1 \dots \mathsf{i}_m}$. □

Observe that if the tuple $\mathsf{i}_1 \dots \mathsf{i}_m$ is empty, the formula $\theta^c_{\mathsf{i}_1 \dots \mathsf{i}_m}$ is equivalent to $\exists x\ x = c$. Note also that the same constant symbol can be a relative rigid designator of several types. When $\theta^c_{\mathsf{i}_1 \dots \mathsf{i}_m}$ is true at w, there is a physical object $\mathbf{I} \in \mathcal{P}_w$ whose realization is named by 'c' in w and in all worlds w' such that $(R_{\mathsf{i}_1} \circ \cdots \circ R_{\mathsf{i}_m})(w, w')$ but possibly not in any further world.[13] This does *not* mean that 'c' behaves as a rigid designator in Kripke's sense: there is no entity functioning as the referent of 'c' over all those worlds w' (the referent of 'c' in each world is a local object and does not belong to the domain of any other world), though there is a fixed physical object $\mathbf{I}$ such that 'c' denotes the *realization* of $\mathbf{I}$ in each w'. This notion of relative rigid designator is typically 'unrealistic', at least when attention is confined to modalities corresponding to intentional states (instead of metaphysical, alethic, or physical modalities): in connection with experiences and propositional attitudes of an agent, names are reasonably assumed to go together with realizations of intentional objects and to be related to physical objects only if those intentional objects happen to represent specific physical objects. My goal is not to argue for the intrinsic interest of relative rigid designators but to show how the Kripkean notion of rigid designator can be mimicked in the present framework.

Relative rigid designators behave differently from arbitrary constant symbols. First, any instance of EG-S becomes valid if it is required that the value of γ be a relative rigid designator with a suitable type profile. (The requisite types depend on the syntax of the value of ξ.) Here is a simple example. If c is a relative rigid designator of type i, we can validly infer by EG-S from the assumption $\Diamond_{\mathsf{i}} P(c)$ to the conclusion $\exists x \Diamond_{\mathsf{i}} P(x)$. Namely, if $\exists x(x = c \wedge \Box_{\mathsf{i}} x = c)$ and $\Diamond_{\mathsf{i}} P(c)$ are true at w, there is $\mathbf{I} \in \mathcal{P}_w$ such that $w, x := \mathbf{I} \models \Diamond_{\mathsf{i}} (x = c \wedge P(c))$. It follows that $w \models \exists x \Diamond_{\mathsf{i}} P(x)$. More generally, let $\psi(x)$ be an arbitrary formula whose sole free variable is x. Suppose that at w, the constant symbol c is a relative rigid designator of all types $\mathbf{i} \in P_\psi$, where P_ψ is the modal profile of ψ. This means that there is a physical object whose realizations are uniformly labeled by 'c' in w and in all worlds to which the evaluation of ψ can lead when ψ is evaluated at w. This is why 'c' can be seen as standing proxy for this individual relative to those worlds, which is what allows inferring the truth of $\exists x\, \psi(x)$ at w from the truth of ψ^c at w. That is, the formula $([\bigwedge_{\mathbf{i} \in P_\psi} \theta^c_{\mathbf{i}}] \wedge \psi^c) \to \exists x\, \psi(x)$ is valid.

Second, if c and d are relative rigid designators of type i, we can infer $c = d \to \Box_{\mathsf{i}}\, c = d$. Namely, if $\exists x(x = c \wedge \Box_{\mathsf{i}}\, x = c)$ and $\exists y(y = d \wedge \Box_{\mathsf{i}}\, y = d)$ are both true at w, the respective witnesses $\mathbf{I}$ and $\mathbf{J}$ of the quantifiers $\exists x$ and $\exists y$ satisfy: $\mathbf{I}(v) = \mathit{Int}(c, v)$ and $\mathbf{J}(v) = \mathit{Int}(d, v)$ for all $v \in \{w\} \cup R_{\mathsf{i}}(w)$. If, furthermore, $c = d$ is true at w, we have $\mathbf{I}(w) = \mathit{Int}(c, w) = \mathit{Int}(d, w) = \mathbf{J}(w)$. As $\mathbf{I}$ and $\mathbf{J}$ are physically individuated

[13] The composition of two binary relations is defined as follows: $R_1 \circ R_2 = \{\langle x, y\rangle :$ there is z such that $R_1(x, z)$ and $R_2(z, y)\}$. Generally, $R_1 \circ \cdots \circ R_{n+1} = (R_1 \circ \cdots \circ R_n) \circ R_{n+1}$.

world lines, it ensues by hypothesis H1 of Sect. 3.4 that $\mathbf{I}$ equals $\mathbf{J}$. We may conclude that $\Box_{\mathsf{i}}\, c = d$ is true at w, for otherwise there would be an R_{i}-accessible world w' such that $\mathbf{I}(w') = Int(c, w') \neq Int(d, w') = \mathbf{J}(w')$, which is impossible, because $\mathbf{I} = \mathbf{J}$.

Third, given that SI-C has refutable instances, it may be asked whether suitable extra premises could be formulated relative to which its instances become valid. Actually, it suffices to require that the values of γ and δ be relative rigid designators. For example, the instance $[c = d \wedge \Box_{\mathsf{i}}\, P(c)] \rightarrow \Box_{\mathsf{i}}\, P(d)$ of SI-C holds conditionally to the premises $\exists x(x = c \wedge \Box_{\mathsf{i}}\, x = c)$ and $\exists y(y = d \wedge \Box_{\mathsf{i}}\, y = d)$. This is because these premises, together with the assumption $c = d$, allow inferring $\Box_{\mathsf{i}}\, c = d$, as just explained. Together, the formulas $\Box_{\mathsf{i}}\, c = d$ and $\Box_{\mathsf{i}}\, P(c)$ entail, then, $\Box_{\mathsf{i}}\, P(d)$.

We may note that the premise $\bigwedge_{\mathbf{i} \in P_\phi} (\theta^c_{\mathbf{i}} \wedge \theta^d_{\mathbf{i}})$ suffices for justifying the instances of SI-C with $\xi := \phi$, $\gamma := c$, and $\delta := d$.

Fact 5.3 *Let ϕ be an L-formula. Let ϕ' be the result of replacing at least one occurrence of c in ϕ by d. The following is a valid formula:*

$$\bigwedge_{\mathbf{i} \in P_\phi} (\theta^c_{\mathbf{i}} \wedge \theta^d_{\mathbf{i}}) \rightarrow ((c = d \wedge \phi) \rightarrow \phi').$$

Proof See Sect. A.3 of the Appendix A. □

5.4 The Barcan Formula and Its Converse

In Sect. 2.3, I showed that the Barcan formula and its converse are both refutable in logic L_0. It was stressed that the refutability of these two formulas has nothing to do with local objects being world-bound. Instead, they are refutable, because according to the semantics of L_0, world lines that are available as values of quantifiers in one world need not be available as values of quantifiers in another world and because realization of a world line in a world does not entail its availability therein. With two types of quantifiers, two versions of these formulas can be formulated in logic L:

BF-P	$\Diamond \exists x\, Q(x) \rightarrow \exists x \Diamond Q(x)$
CBF-P	$\exists x \Diamond Q(x) \rightarrow \Diamond \exists x\, Q(x)$
BF-I	$\Diamond \mathsf{E}_{\mathsf{a}} x\, Q(x) \rightarrow \mathsf{E}_{\mathsf{a}} x \Diamond Q(x)$
CBF-I	$\mathsf{E}_{\mathsf{a}} x \Diamond Q(x) \rightarrow \Diamond \mathsf{E}_{\mathsf{a}} x\, Q(x)$,

where Q is a unary predicate. The arguments of Example 2.2 are directly applicable for showing that BF-I, CBF-I, and BF-P are refutable according to the semantics of L. By contrast, the counter-model constructed for CBF in that example does not help us to refute CBF-P. In the counter-model, there was a world in which a world line is realized but not available, while for physical objects, availability coincides with realizability. In fact, CBF-P is valid.

Fact 5.4 *The physical-quantifier version of the converse Barcan formula is valid.*

Proof Suppose $M, w \models \exists x \Diamond Q(x)$. That is, there is $\mathbf{I} \in \mathcal{P}_w$ such that $M, w, x := \mathbf{I} \models \Diamond Q(x)$. Thus, there is w' with $R(w, w')$ such that $\mathbf{I}$ is realized in w' and $\mathbf{I}(w') \in Int(Q, w')$. Since for physical world lines realization implies availability, it follows that $\mathbf{I} \in \mathcal{P}_{w'}$, and so, $M, w' \models \exists x Q(x)$. As $R(w, w')$, we have $M, w \models \Diamond \exists x Q(x)$. □

I take the syntax of 'first-order modal logic', or FOML, to be that of L_0. For the notion of Kripke frame, see Sect. B.1 of the Appendix B. A Kripke frame $F = \langle W, R, (D_w)_{w \in W} \rangle$ has *anti-monotonic* (or *decreasing*) domains if $D_w \supseteq D_{w'}$ whenever $R(w, w')$ and *monotonic* (or *increasing*) domains if $D_w \subseteq D_{w'}$ whenever $R(w, w')$.[14] A sentence ϕ is *valid in the frame F* (in the sense of Kripke semantics), denoted $F \models_K \phi$, if ϕ is true at all worlds in all Kripke models based on F.[15] (Here, I restrict attention to formulas without free variables.) It is well known that if F is any frame, we have:

- $F \models_K$ BF iff F has anti-monotonic domains
- $F \models_K$ CBF iff F has monotonic domains.

The notions of monotonicity and anti-monotonicity can be transferred into my framework as follows. Recall the definition of a frame $\langle W, \mathcal{R}, \mathcal{P}, \mathcal{I} \rangle$ from Sect. 3.4. Here, it suffices to consider frames with $\mathcal{R} = \{R\}$—i.e., frames with a single accessibility relation. Such a frame is *physically anti-monotonic* if all $w, w' \in W$ with $R(w, w')$ satisfy $\mathcal{P}_w \supseteq \mathcal{P}_{w'}$ and *physically monotonic* if all $w, w' \in W$ with $R(w, w')$ satisfy $\mathcal{P}_w \subseteq \mathcal{P}_{w'}$. The notions of *intentional monotonicity* and *intentional anti-monotonicity* relative to an agent α can be defined similarly, in terms of the sets $\mathcal{I}^\alpha_w$ and $\mathcal{I}^\alpha_{w'}$. An L-formula ϕ is *valid in a frame F* (in the sense of my semantics), denoted $F \models \phi$, if for all suitable interpretations *Int* and worlds $w \in W$, we have $\langle F, Int \rangle, w \models \phi$.

Kripke semantics can be compared with the semantics of L by asking how the versions of BF and CBF relate to the variants of the properties of monotonicity and anti-monotonicity. In analogy with the case of Kripke semantics, the frame validity of BF-P characterizes physical anti-monotonicity. In all other cases, the analogy fails.

Fact 5.5 *Let F be any frame.*

(a) $F \models$ BF-P $\quad \Leftrightarrow \quad$ *F is physically anti-monotonic.*
(b) $F \models$ CBF-P $\quad \not\Rightarrow \Leftarrow \quad$ *F is physically monotonic.*
(c) $F \models$ BF-I $\quad \not\Rightarrow \Leftarrow \quad$ *F is intentionally anti-monotonic relative to agent α.*
(d) $F \models$ CBF-I $\quad \not\Rightarrow \Leftarrow \quad$ *F is intentionally monotonic relative to agent α.*

Proof See Sect. A.4.1 of the Appendix A. □

[14] Fitting and Mendelsohn [27, p. 101] speak of *augmented frames* where I speak of Kripke frames. For the terminology used for frame properties, see, e.g., [27, pp. 110, 112] (monotonicity, anti-monotonicity) and [5, p. 557] (increasing, decreasing).

[15] The notions 'model based on a frame' and 'valid in a frame' are relative to a vocabulary. I assume we employ a fixed vocabulary that includes all predicate symbols I explicitly mention.

By now, we know, then, that BF-P characterizes the frame property of physical anti-monotonicity and that CBF-P is valid. Given the negative nature of items (b), (c), and (d) of Fact 5.5, it is of interest to ask, first, which frame properties *are* characterized by BF-I and CBF-I and, second, whether physical and intentional monotonicity and intentional anti-monotonicity *can* be characterized in terms of L.

A frame $F = \langle W, \{R\}, \mathcal{P}, \mathcal{I}\rangle$ is *intentionally weakly monotonic* relative to agent α if for all $w, w' \in W$ with $R(w, w')$ and all $\mathbf{I} \in \mathcal{I}^\alpha_w$ realized in w', there is $\mathbf{J} \in \mathcal{I}^\alpha_{w'}$ such that $\mathbf{J}$, too, is realized in w' and $\mathbf{I}(w') = \mathbf{J}(w')$. This condition is much weaker than intentional monotonicity: we consider only those $\mathbf{I}$ available in w that are realized in w', and furthermore, such an $\mathbf{I}$ itself need not be available in w'; it is merely required that at least one intentionally individuated world line that coincides with $\mathbf{I}$ in w' be available in w'. Further, F is *intentionally weakly anti-monotonic* relative to α if for all $w, w' \in W$ with $R(w, w')$ and all $\mathbf{I} \in \mathcal{I}^\alpha_{w'}$ realized in w', there is $\mathbf{J} \in \mathcal{I}^\alpha_w$ such that $\mathbf{J}$, too, is realized in w' and $\mathbf{I}(w') = \mathbf{J}(w')$. This condition is much weaker than intentional anti-monotonicity. Unlike the concepts of intentional monotonicity and anti-monotonicity themselves, the weakened notions are *not* entirely symmetric: in both cases, we are interested in two world lines, both of which are realized in w'.

Fact 5.6 *Let F be any frame.*

(a) $F \models$ BF-I $\;\Leftrightarrow\;$ *F is weakly intentionally anti-monotonic relative to α.*
(b) $F \models$ CBF-I $\;\Leftrightarrow\;$ *F is weakly intentionally monotonic relative to α.*

Proof The right–left directions of both claims hold, obviously. For the converse direction of claim (a), let $F = \langle W, \{R\}, \mathcal{P}, \mathcal{I}\rangle$ and assume $F \models$ BF-I. Suppose for contradiction that there are $w, w' \in W$ with $R(w, w')$ and $\mathbf{I} \in \mathcal{I}^\alpha_{w'}$ realized in w' such that there is no $\mathbf{J} \in \mathcal{I}^\alpha_w$ realized in w' and satisfying $\mathbf{I}(w') = \mathbf{J}(w')$. Consider a model $M = \langle F, Int\rangle$, where $Int(Q, w') = \{\mathbf{I}(w')\}$ and $Int(Q, w'') = \varnothing$ for all $w'' \in W \setminus \{w'\}$. Since $F \models$ BF-I and $M, w \models \Diamond \mathsf{E}_{\mathsf{a}} x\, Q(x)$, it follows that $M, w \models \mathsf{E}_{\mathsf{a}} x \Diamond Q(x)$. We may conclude that there is $\mathbf{J} \in \mathcal{I}^\alpha_w$ and w'' with $R(w, w'')$ such that $\mathbf{J}(w'') \in Int(Q, w'')$. It ensues that $w'' = w'$ and $\mathbf{J}(w') = \mathbf{I}(w')$. This is a contradiction. The left–right direction of claim (b) can be proven by a similar argument, supposing for contradiction that CBF-I is valid in a frame that is not weakly intentionally monotonic. □

To complete the comparative discussion, I show that physical monotonicity is characterizable in L, but intentional monotonicity and anti-monotonicity are not.

Fact 5.7 (Characterizing physical monotonicity) *Let F be any frame. We have:*

$$F \models \forall x \Box x = x \;\; \textit{iff} \;\; F \textit{ is physically monotonic}.$$

Proof Because $\forall x \Box x = x$ contains no non-logical predicates, it is valid in a frame F iff it is true at every world of the unique model M_F that has empty vocabulary and is based on F. Suppose first that F is physically monotonic and $w \in dom(M_F)$. If w is R-maximal, trivially, $M_F, w \models \forall x \Box x = x$. Else, let $\mathbf{I} \in \mathcal{P}_w$ and w' with $R(w, w')$

be arbitrary. By physical monotonicity, we have $\mathbf{I} \in \mathcal{P}_{w'}$. Since physical availability entails realization, we have $M, w', x := \mathbf{I} \models x = x$ and, therefore, $M, w \models \forall x \Box x = x$. Conversely, assume $F \models \forall x \Box x = x$. If F was not physically monotonic, there would be $w, w' \in dom(M_F)$ with $R(w, w')$ and $\mathbf{I} \in \mathcal{P}_w$ such that $\mathbf{I} \notin \mathcal{P}_{w'}$. However, $F \models \forall x \Box x = x$ entails $M_F, w \models \forall x \Box x = x$, whence $M_F, w', x := \mathbf{I} \models x = x$ and $\mathbf{I}$ is realized in w'. Thus, $\mathbf{I} \in \mathcal{P}_{w'}$. This is a contradiction. $\Box$

The intentional versions of monotonicity and anti-monotonicity *cannot* be characterized in L. Due to the semantics of atomic formulas, L-formulas can express conditions on world lines only in terms of their realizations. Yet, an intentionally individuated world line can be available in a world w without being realized in w or in any world reachable from w in a finite number of steps along the relevant accessibility relations. The following negative result is proven in Sect. A.4.2 of the Appendix A:

Theorem 5.1 *(a) There is no L-formula that is valid in a frame iff the frame is intentionally monotonic. (b) Neither is there an L-formula that is valid in a frame iff the frame is intentionally anti-monotonic.*

In connection with Kripke semantics, the Barcan formula and its converse are typically not considered as single formulas. They are viewed as schemata with an infinity of formulas as instances. If BF or $\Diamond \exists x Q(x) \rightarrow \exists x \Diamond Q(x)$ is viewed as a schema of FOML, the symbol '$Q(x)$' is understood as a variable, and the *instances* of this schema are those FOML-formulas that are either themselves obtained by substituting for $Q(x)$ an FOML-formula with x as its sole free variable or else logically equivalent to formulas obtained from BF by such a substitution.[16] In this way $\Diamond \exists x \neg Q(x) \rightarrow \exists x \Diamond \neg Q(x)$ and its equivalents $\neg \exists x \Diamond \neg Q(x) \rightarrow \neg \Diamond \exists x \neg Q(x)$ and $\forall x \Box Q(x) \rightarrow \Box \forall x Q(x)$ are instances of the schema BF.

Sometimes, the notions of Barcan formula and converse Barcan formula are introduced—remaining at the object-language level, without recourse to schemata—by saying that all FOML-formulas of the forms BF and BF′ are Barcan formulas and all formulas of the forms CBF and CBF′ are converse Barcan formulas:[17]

[16] Williamson [127, pp. 33, 123] draws a strict line between instantiation and substitution, expressing doubts as to whether it makes sense to say that a formula like $\forall x \Box \neg Q(x)$ is obtained by substitution from $\forall x \Box Q(x)$. He insists that only variables of specified grammatical categories can be substituted by expressions of the same grammatical category and that '$Q(x)$' is not a variable of any category, though 'Q' would be. (He qualifies only first-order variables and predicates of the non-logical vocabulary as expressions belonging to a grammatical category.) This is an unmotivated self-imposed restriction on the notion of substitution. Generally, substitution is a syntactic operation that consists of replacing a syntactic constituent of a well-formed formula by an expression so that the result is still a well-formed formula. There is no conceptual reason why the first-mentioned expression could not be of any syntactic complexity and no reason not to call any such expression a 'variable' in connection with a corresponding notion of substitution.

[17] Cf., e.g., Fitting and Mendelsohn [27, pp. 108–9].

BF $\Diamond \exists x\, Q(x) \to \exists x \Diamond Q(x)$
BF′ $\forall x \Box Q(x) \to \Box \forall x\, Q(x)$
CBF $\exists x \Diamond Q(x) \to \Diamond \exists x\, Q(x)$
CBF′ $\Box \forall x\, Q(x) \to \forall x \Box Q(x)$.

While BF and BF′ are not equivalent, according to the Kripke semantics they are valid in the same frames. The same goes for CBF and CBF′. These interrelationships break down in L.[18]

Fact 5.8 *(a) There are frames F such that $F \models$ BF′ but $F \not\models$ BF. (b) There are frames F such that $F \models$ CBF but $F \not\models$ CBF′.*

Proof For (a), it suffices to find a frame that is not physically anti-monotonic but in which BF′ is valid, cf. Fact 5.5(a). Let w_1 and w_2 be two worlds with $dom(w_1) = \{a_1\}$ and $dom(w_2) = \{a_2\}$. Let $\mathcal{P}_{w_1} = \{\mathbf{I}\}$ and $\mathcal{P}_{w_2} = \{\mathbf{J}\}$ with $\mathbf{I} = \{\langle w_1, a_1\rangle\}$ and $\mathbf{J} = \{\langle w_2, a_2\rangle\}$. Let us consider the frame $F = \langle W, \{R\}, \mathcal{P}, \mathcal{I}\rangle$ such that $W = \{w_1, w_2\}$, $R = \{\langle w_1, w_2\rangle, \langle w_2, w_1\rangle\}$, $\mathcal{P} = \{\mathcal{P}_{w_1}, \mathcal{P}_{w_2}\}$, and $\mathcal{I}$ is empty. Now, the frame F is not anti-monotonic, since $R(w_1, w_2)$ but $\{\mathbf{I}\} = \mathcal{P}_{w_1} \not\supseteq \mathcal{P}_{w_2} = \{\mathbf{J}\}$. However, the formula BF′ or $\forall x \Box Q(x) \to \Box \forall x\, Q(x)$ is trivially valid in F: its antecedent is false at both worlds w_1 and w_2 in any model based on F. Indeed, let M be any such model. We have $M, w_1 \not\models \forall x \Box Q(x)$ because $\mathbf{I} \in \mathcal{P}_{w_1}$ and $R(w_1, w_2)$ but $w_2 \notin marg(\mathbf{I})$. Symmetrically, we have $M, w_2 \not\models \forall x \Box Q(x)$ because $\mathbf{J} \in \mathcal{P}_{w_2}$ and $R(w_2, w_1)$ but $w_1 \notin marg(\mathbf{J})$. That is, BF′ is true at both worlds in M.

As for (b), since CBF is valid (Fact 5.4), the claim follows if we show that CBF′ is refutable. I move on to describe a counter-model of CBF′. Let us consider the equivalent CBF″ of CBF′, obtained from CBF by replacing $Q(x)$ by $\neg Q(x)$:

CBF″ $\exists x \Diamond \neg Q(x) \to \Diamond \exists x \neg Q(x)$.

Let M be a model with distinct worlds w and w' such that $R(w, w')$. Let $\mathcal{P}_w = \{\mathbf{I}\}$ and $\mathcal{P}_{w'} = \{\mathbf{J}\}$, where $w' \notin marg(\mathbf{I})$, $w' \in marg(\mathbf{J})$, and $\mathbf{J}(w') \in Int(Q, w')$. We have $M, w \models \exists x \Diamond \neg Q(x)$ since trivially, $M, w', x := \mathbf{I} \not\models Q(x)$. Yet, $M, w \not\models \Diamond \exists x \neg Q(x)$, because $R(w) = \{w'\}$ and $\mathbf{J}$ is the only element of $\mathcal{P}_{w'}$ and M, w', $x := \mathbf{J} \models Q(x)$. □

5.5 Validity and Its Preservation Under Substitution

By Fact 5.8(b), we cannot use CBF and CBF′ as alternative formulations of the same frame condition. Further, since CBF is valid but CBF″ is not, validity is not preserved under arbitrary substitutions in L: replacing $Q(x)$ by $\neg Q(x)$ in CBF turns a validity into a refutable formula. For another example, consider $\exists x \Diamond\Diamond Q(x) \to \Diamond\exists x \Diamond Q(x)$, obtained from CBF by substituting $\Diamond Q(x)$ for $Q(x)$. The truth of the antecedent at w entails that there are a world line $\mathbf{I} \in \mathcal{P}_w$ and worlds w' and w'' such

[18] Note that BF, BF′, CBF, and CBF′ all are L-formulas employing physical quantifiers.

that $R(w, w')$, $R(w', w'')$, and $w'' \in marg(\mathbf{I})$. This does not guarantee that $\mathbf{I} \in \mathcal{P}_{w'}$. Using this observation, a counter-model for $\exists x \Diamond \Diamond Q(x) \rightarrow \Diamond \exists x \Diamond Q(x)$ can be constructed. It follows that CBF cannot be understood as a schema and that the notions of validity, schema, and logical form do not behave in the expected way in L.

Recall from Sect. 2.4 the notation $\phi[x_1 /\!\!/ y_1, \ldots, x_n /\!\!/ y_n]$ for uniform substitution of y_i for x_i. We need a precise notion of substituting a formula for an atomic formula.

Definition 5.3 (*Substitution, base of substitution*) Let τ_1 and τ_2 be disjoint vocabularies. Let V_1 and V_2 be disjoint subsets of *Var*, with $V_2 = \{v_i : i \geq 1\}$. A *base of substitution* is a map $\upsilon : \tau_1 \rightarrow L[\tau_1 \cup \tau_2, V_2]$ that assigns to every n-ary predicate P of τ_1 an $L[\{P\} \cup \tau_2, V_2]$-formula $\upsilon(P)$ whose free variables are $v_1, \ldots, v_n$. A map $\sigma : L[\tau_1, V_1] \rightarrow L[\tau_1 \cup \tau_2, V_1 \cup V_2]$ is an υ-*substitution* (or *substitution based on* υ) if it satisfies the following:

- $\sigma[P(x_1, \ldots, x_n)] := \upsilon(P)[v_1 /\!\!/ x_1, \ldots, v_n /\!\!/ x_n]$
- $\sigma[x = y] := x = y$
- $\sigma[\neg\psi] := \neg\sigma[\psi]$
- $\sigma[(\psi_1 \wedge \psi_2)] := (\sigma[\psi_1] \wedge \sigma[\psi_2])$
- $\sigma[\Box_{\mathsf{i}}\psi] := \Box_{\mathsf{i}}\sigma[\psi]$
- $\sigma[\mathsf{Q}x\psi] := \mathsf{Q}x\sigma[\psi]$ for $\mathsf{Q} \in \{\exists\} \cup \{\mathsf{E}_{\mathsf{a}} : \mathsf{a} \in \mathbb{A}\}$. □

It is useful to note the following features of the υ-substitution σ. First, there is exactly one υ-substitution σ based on a given map υ. In particular, υ uniquely determines the formula $\sigma[P(x_1, \ldots, x_n)]$, because the free variables of $P(x_1, \ldots, x_n)$ correspond to the fixed free variables $v_1, \ldots, v_n$ of $\upsilon(P)$ in a unique fashion: the variable v_i of $\upsilon(P)$ is replaced by the variable of $P(x_1, \ldots, x_n)$ that has the order position i in the left–right order determined by the syntax of this atomic formula. Second, the only predicate symbol of the vocabulary τ_1 that may occur in $\sigma[P(x_1, \ldots, x_n)]$ is P. I introduce this restriction to avoid needless clashes between predicates in ϕ and those in $\sigma[\phi]$: otherwise, the formula $\upsilon(P)[v_1 /\!\!/ x_1, \ldots, v_n /\!\!/ x_n]$ replacing the atomic subformula $P(x_1, \ldots, x_n)$ of ϕ could involve predicate symbols Q distinct from P such that Q already appears in ϕ. Third, since all variables occurring in $\upsilon(P)$ come from V_2, each x_i is trivially free for v_i in $\upsilon(P)$. Fourth, the formula $\sigma[P(x_1, \ldots, x_n)]$ employs the variables $x_1, \ldots, x_n \in V_1$—these are uniformly substituted for the free variables $v_1, \ldots, v_n \in V_2$ of $\upsilon(P)$—but any further variables it may contain come from V_2. Fifth, observe that $Free(\phi) = Free(\sigma[\phi])$ for all $\phi \in L$.[19]

Henceforth, I will say that an L-formula ϕ is *model-theoretically valid* if it is satisfied in all models M in all worlds $w \in dom(M)$ under all assignments in M. This has been my definition of the unqualified notion of 'validity' from Sect. 2.3 on. However, as will soon be explicated, this notion does not behave in all respects as we might expect—hence, the qualification *model-theoretically* valid. Let us say that ϕ is *schematically valid* if for all substitutions σ, the formula $\sigma[\phi]$ is model-theoretically

[19]Unsurprisingly, I write $Free(\psi)$ for the set of free variables occurring in ψ.

valid. Trivially, any schematically valid formula is model-theoretically valid.[20] In first-order logic, the converse also holds. For example, suppose we replace in a sentence $\phi \in \mathrm{FOL}[\tau_1]$ an atomic formula $P(x)$ by a formula $\psi[v/\!\!/x] \in \mathrm{FOL}[\{P\} \cup \tau_2]$, mapping all other atomic formulas to themselves. This yields a sentence $\sigma[\phi] \in \mathrm{FOL}[\tau_1 \cup \tau_2]$. Now, for any model $\mathcal{M} = \langle D, \mathbf{Int} \rangle$ of vocabulary $\tau_1 \cup \tau_2$, there is a model $\mathcal{M}' = \langle D', \mathbf{Int}' \rangle$ of vocabulary τ_1 with $D' = D$ such that the set of values of x satisfying $\psi[v/\!\!/x]$ in $\mathcal{M}$ equals the set of values of x satisfying $P(x)$ in $\mathcal{M}'$: we can simulate the semantic value of an arbitrary formula by a suitable interpretation of an atomic formula. If ϕ is model-theoretically valid, it is true in $\mathcal{M}'$. By the compositionality of the semantics of FOL, $\sigma[\phi]$ is true in $\mathcal{M}$.[21] In FOL, model-theoretic validity suffices for schematic validity. This means that in FOL, *model-theoretic validity is preserved under uniform substitution*: if ϕ is model-theoretically valid and σ is any substitution, $\sigma[\phi]$ is also model-theoretically valid.

The examples discussed in the beginning of this section show that in L, not all model-theoretically valid formulas are schematically valid. In L, model-theoretic validity is *not* preserved under uniform substitution. There are formulas whose semantic values *cannot* be simulated by interpretations of atomic formulas. Atomic formulas can only simulate existence-entailing formulas, but not all formulas are existence-entailing (Fact 2.1). In order for a formula to be existence-entailing, it suffices that it has the form $\chi(x_1, \ldots, x_n) \wedge \bigwedge_{1 \le i \le n} x_i = x_i$. This is not a necessary condition: for example, the formulas $Q(x)$ and $Q(x) \wedge \exists y P(y)$ and $Q(x) \vee P(x)$ and $\exists z S(x, y, z) \vee (P(x) \wedge Q(y))$ are existence-entailing without being of the mentioned form. (For the problem of deciding whether a formula is existence-entailing, see Sect. 5.6.) However, if $\psi(x_1, \ldots, x_n)$ is any existence-entailing formula, it is *equivalent* to the formula $\psi(x_1, \ldots, x_n) \wedge \bigwedge_{1 \le i \le n} x_i = x_i$. This observation motivates the following modified notion of substitution.

Definition 5.4 (*Strong substitution, strong base of substitution*) Let the sets τ_1, τ_2, V_1, and V_2 be as in Definition 5.3. A *strong base of substitution* is a base of substitution $\rho : \tau_1 \to L[\tau_1 \cup \tau_2, V_2]$ such that for all $P \in \tau_1$, there is $\chi_P \in L[\{P\} \cup \tau_2, V_2]$ satisfying $\rho(P) = \chi_P(v_1, \ldots, v_n) \wedge \bigwedge_{1 \le i \le n} v_i = v_i$. A *strong ρ-substitution* is a ρ-substitution, where ρ is a strong base of substitution. □

An arbitrary base of substitution may but a strong base of substitution must assign to every predicate symbol a conjunction of the form $\chi(v_1, \ldots, v_n) \wedge \bigwedge_{1 \le i \le n} v_i = v_i$. In L, the conjunct $\bigwedge_{1 \le i \le n} v_i = v_i$ is semantically non-trivial: it forces the values of the variables v_i to be realized in the context of evaluation. In Kripke semantics, formulas $v_i = v_i$ are trivially satisfied, wherefore the two formulations of the notion

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[20] The base of substitution υ can be chosen so that the υ-substitution σ is an identity map—i.e., satisfies $\sigma[\phi] = \phi$ for all $\phi \in L$: we may choose $\upsilon(P) = P(v_1, \ldots, v_n)$ for all n and n-ary $P \in \tau_1$.

[21] The semantic value $|\phi|^{\mathcal{M}}$ of an FOL-formula $\phi(x_1, \ldots, x_n)$ in a model $\mathcal{M} = \langle D, \mathbf{Int} \rangle$ is the set $\{\langle a_1, \ldots, a_n \rangle : \mathcal{M}, x_1 := a_1, \ldots, x_n := a_n \models \phi(x_1, \ldots, x_n)\}$. The semantics of FOL is compositional: the semantic value of a formula depends only on the semantic values of its syntactic components and the way they are put together. It does not depend on those components themselves: if $|\psi|^{\mathcal{M}} = |\psi'|^{\mathcal{M}}$ and $\mathit{Free}(\psi) = \mathit{Free}(\psi')$ and ϕ' is obtained by replacing the syntactic component ψ of ϕ by ψ', then $|\phi|^{\mathcal{M}} = |\phi'|^{\mathcal{M}}$.

of substitution are, for all relevant purposes, equivalent when Kripke semantics is applied.

In L, model-theoretic validity is preserved under *strong* substitutions. In order to prove this, let us observe first that the semantics of L is compositional.

Fact 5.9 (Compositionality) *In any given model, the semantic value of a syntactically complex L-formula depends only on the semantic values of its immediate syntactic constituents and the way in which they are syntactically combined.*

Proof The following relations hold directly on the basis of the semantics:

- If $(\phi \wedge \psi)(x_1, \ldots, x_n) = \phi(x_{i_1}, \ldots, x_{i_m}) \wedge \psi(x_{j_1}, \ldots, x_{j_k})$, then $|(\phi \wedge \psi)(x_1, \ldots, x_n)|^M = \{\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle :$ $\langle w, \mathbf{I}_{i_1}, \ldots, \mathbf{I}_{i_m}\rangle \in |\phi(x_{i_1}, \ldots, x_{i_m})|^M$ and $\langle w, \mathbf{I}_{j_1}, \ldots, \mathbf{I}_{j_k}\rangle \in |\psi(x_{j_1}, \ldots, x_{j_k})|^M\}$
- $|\neg\phi(x_1, \ldots, x_n)|^M = [dom(M) \times \mathrm{WL}(M)^n] \setminus |\phi(x_1, \ldots, x_n)|^M$
- $|\Box_{\mathsf{i}}\phi(x_1, \ldots, x_n)|^M = \{\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle : \{\langle w', \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle : R_{\mathsf{i}}(w, w')\} \subseteq |\phi(x_1, \ldots, x_n)|^M\}$
- $|(\exists x\phi)(x_1, \ldots, x_n)| = \{\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle :$ there is $\mathbf{I} \in \mathcal{P}_w$ such that $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n, \mathbf{I}\rangle \in |\phi(x_1, \ldots, x_n, x)|^M\}$
- $|(\mathsf{E}x_{\mathsf{a}}\phi)(x_1, \ldots, x_n)| = \{\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle :$ there is $\mathbf{I} \in \mathcal{I}_w^\alpha$ such that $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n, \mathbf{I}\rangle \in |\phi(x_1, \ldots, x_n, x)|^M\}$.

For a fixed frame F, write $|\phi|^{\langle F, Int\rangle}$ for the semantic value of ϕ in the model $\langle F, Int\rangle$. Consequently, if Int and Int' are interpretations on F and $\phi_1, \phi_2, \phi'_1, \phi'_2$ are formulas such that $Free(\phi_1) = Free(\phi'_1)$ and $Free(\phi_2) = Free(\phi'_2)$ and $|\phi_1|^{\langle F, Int\rangle} = |\phi'_1|^{\langle F, Int'\rangle}$ and $|\phi_2|^{\langle F, Int\rangle} = |\phi'_2|^{\langle F, Int'\rangle}$, then $|\phi_1 \wedge \phi_2|^{\langle F, Int\rangle} = |\phi'_1 \wedge \phi'_2|^{\langle F, Int'\rangle}$ and $|\star\, \phi_1|^{\langle F, Int\rangle} = |\star\, \phi'_1|^{\langle F, Int'\rangle}$ for all $\star \in \{\neg, \Box, \exists x, \mathsf{E}_{\mathsf{a}}x\}$. $\Box$

Because the relation of functional dependence of the semantic value of a formula on the semantic values of its immediate subformulas is transitive, the semantic value of a formula in a model is determined by the semantic values of its *atomic* subformulas.

Fact 5.10 *If $\phi \in L$ is model-theoretically valid and σ is a strong substitution, then $\sigma[\phi]$ is model-theoretically valid, as well.*

Proof Suppose $\phi(x_1, \ldots, x_m) \in L[\tau_1]$ is model-theoretically valid. Let $\rho : \tau_1 \to L[\tau_1 \cup \tau_2]$ be a strong base of substitution, and let $\sigma : L[\tau_1] \to L[\tau_1 \cup \tau_2]$ be the strong substitution based on ρ. Let $F = \langle W, \mathcal{R}, \mathcal{P}, \mathcal{I}\rangle$ be any frame and $M = \langle F, Int\rangle$ any model of vocabulary $\tau_1 \cup \tau_2$. Let $w \in dom(M)$ be arbitrary and $g : \{x_1, \ldots, x_m\} \to \mathrm{WL}(M)$ any assignment. We must show that $M, w, g \models \sigma[\phi]$. Define a model $M' = \langle F, Int'\rangle$ of vocabulary τ_1 as follows. For all $n \geq 1$, n-ary $P \in \tau_1$, and $w \in W$, let

$$Int'(P, w) = \{\langle \mathbf{I}_1(w), \ldots, \mathbf{I}_n(w)\rangle : M, w, v_1 := \mathbf{I}_1, \ldots, v_n := \mathbf{I}_n \models \rho(P)(v_1, \ldots, v_n)\}.$$

The interpretation $Int'(P, w)$ is well-defined: if $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle$ is an n-tuple of world lines satisfying $\rho(P) = \chi_P(v_1, \ldots, v_n) \wedge \bigwedge_{1 \leq i \leq n} v_i = v_i$, each $\mathbf{I}_i$ is realized in w. Now, syntactically, $\sigma[\phi]$ is built from the formulas $\rho(P)[v_1 /\!\!/ x_1, \ldots, v_n /\!\!/ x_n]$ in exactly the same way that ϕ is built from the atomic formulas $P(x_1, \ldots, x_n)$.

By the construction of M', we have $|P(x_1, \ldots, x_n)|^{M'} = |\rho(P)|^M$ for all predicate symbols P appearing in ϕ. Since ϕ is model-theoretically valid, we have $M', w, g \models \phi$. By the compositionality of the semantics of L (Fact 5.9), it ensues that $M, w, g \models \sigma[\phi]$. $\square$

Working with standard logical languages, such as FOL, may lead one to become suspicious of any notion of validity not preserved under uniform substitution. Williamson [127, p. 76] affirms that the set of valid formulas being closed under uniform substitution is normally considered mandatory for a logical system and suggests that deviations from this normality count as symptoms of philosophical confusion.[22] He discusses Carnap's account of the semantics of propositional modal logic. If p is atomic, $\neg\Box p$ is true in every model in Carnap's sense. Yet, substituting $(p \to p)$ for p in $\neg\Box p$ results in $\neg\Box(p \to p)$, which is not true in every model in Carnap's sense but actually false in every model. In Carnap's case, the fact that validity is not preserved under uniform substitution indeed suggests that the semantics he employs is somehow anomalous. In his semantics, the failure of this preservation property *cannot* be traced back to a semantic difference between atomic and complex formulas—the phenomenon stems from the way in which modal operators are construed (as ranging invariably over all models). In his setting, any set of models and a fortiori the semantic value of any complex formula can be the semantic value of an atomic formula. By contrast, in L, there are formulas whose semantic values *cannot* coincide with semantic values of atomic formulas: atomicity has, in L, a stronger semantic import than it does in many other logics.

Once one understands how the semantics of L functions, it would be downright misguided to expect that arbitrary substitutions preserve model-theoretic validity in L. It would also be out of place to give a *normative* status to semantic features of certain familiar languages, so that L would be discredited just because it does not obey all the semantic regularities that one's favorite languages do. It is easy to devise respectable languages in which atomic formulas cannot simulate arbitrary formulas. For example, think of a first-order language L^* in which all (unary) predicates are *typed*, while variables are always allowed to range over all elements of the domain. Thus, models of L^* are structures $\mathcal{M} = \langle D, T_1, \ldots, T_n, \mathbf{Int}\rangle$, where the T_i are subsets of D referred to as *types*. Each unary predicate P has its associated type $t(P) \in \{T_1, \ldots, T_n\}$. It is required that the interpretation $\mathbf{Int}(P)$ of the predicate P be a subset of the type $t(P)$. In L^*, all entities in the domain of a model are nevertheless viewed indiscriminately as things on an equal footing. This is why syntactically, there are in

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[22] For a discussion, Williamson refers to Burgess [8, 9], who understands 'laws of logic' logic-externally, as logical representations of natural-language expressions having a privileged semantic property (such as truth independently of the circumstances of evaluation). The idea is that such statements are in a precise sense 'instantiations' of these logical laws; cf. [9, p. 147]. Laws of logic can also be discussed logic-internally, as schematically construed formulas of a given logical language—taken to stand for a whole set of valid formulas of the same language. The question is, then, what options there are as to how a representative relation suitable for such schematism may emerge. The intuitive connection between logic-external and logic-internal views on logical laws comes from the fact that the notion of schematic validity is a way of mimicking—internally to a logic—the relation of instantiation between the logic and a set of natural-language statements.

L^* variables of only one sort, ranging over all elements of the domain considered. Now, suppose $\mathcal{M}$ is a model whose types are non-empty and pairwise disjoint. Let P and Q be unary predicates of distinct types, and suppose the interpretations of these predicates in $\mathcal{M}$ are non-empty sets. Consider the formula $P(x) \vee Q(x)$. Its semantic value in $\mathcal{M}$ is the set $\mathbf{Int}(P) \cup \mathbf{Int}(Q)$. Because $\varnothing \neq \mathbf{Int}(P) \subseteq t(P)$, $\varnothing \neq \mathbf{Int}(Q) \subseteq t(Q)$, and $t(P) \cap t(Q) = \varnothing$, it follows that there is no type T_i such that $\mathbf{Int}(P) \cup \mathbf{Int}(Q) \subseteq T_i$. Consequently, the semantic value of $P(x) \vee Q(x)$ in $\mathcal{M}$ cannot be represented by a unary atomic formula and therefore not by any atomic formula.

5.6 Schemata, Logical Forms, and Schematic Formulas

In connection with languages in which model-theoretic validity is not preserved under uniform substitution, the very notion of *logical form* must be rethought. Typically, we can discuss ‘logical forms’ of certain natural-language sentences in terms of a given logical language, and logic-internally, we can speak of formulas having such-and-such ‘form’. Insofar as a formula of a given logical language is able to represent logical forms, it must be implicitly clear in what way the syntax of the formula gives rise to a representational relation. Normally, a formula B is taken to have the form represented by the formula A if B can be obtained from A by uniformly replacing atomic subformulas of A by suitable formulas of the logical language considered. From the fact that B has the form A, it does not follow that, conversely, A has the form B. Further, B can have several forms in this sense. For example, in propositional logic, it can be said that the formula $([p_1 \vee p_2] \wedge [p_3 \rightarrow p_4])$ has both forms $(p \wedge q)$ and $([r \vee s] \wedge q)$, though neither of the latter two formulas has the form $([p_1 \vee p_2] \wedge [p_3 \rightarrow p_4])$. It is certainly a part of any reasonable definition of ‘logical form’ that if B has the form A and A is model-theoretically valid, then so is B. If a proposed notion of logical form allows B to have the form A while A is model-theoretically valid but B is not, the proposed notion is unacceptable. Since not all model-theoretically valid L-formulas are schematically valid, the way in which a formula represents a logical form in L cannot be based on the idea of allowing arbitrary formulas to be uniformly substituted for its atomic subformulas.

Can an L-formula be used to represent a logical form in the first place? Which aspects of the syntax of a formula are relevant for its form? In L, predicate letters contribute more to the form than they do in many other languages: the behavior of atomic formulas is semantically limited in a way the behavior of arbitrary formulas is not. In a model-theoretically valid formula, not just any formula can be substituted for an atomic formula while preserving model-theoretic validity. Generally, we can be sure that a formula $\psi(x_1, \ldots, x_n)$ meets this criterion just in case its semantic value can be simulated by the semantic value of an atomic formula, which is the case if and only if ψ is existence-entailing. An L-formula is *safe for substitution* if it satisfies this semantic condition. All formulas $\chi(x_1, \ldots, x_n) \wedge \bigwedge_{1 \leq i \leq n} x_i = x_i$ with $\chi \in L$

are trivially safe for substitution. While there are further formulas likewise safe for substitution, in fact, this notion escapes all attempts at syntactic implementation.

Theorem 5.2 *The problem of determining whether an L-formula is existence-entailing (safe for substitution) is undecidable.*

Proof See Sect. A.5 of the Appendix A. □

By Theorem 5.2, the notion of being safe for substitution is 'syntactically ineffable': there is no absolute or natural or even algorithmically definable unqualified notion of schema or logical form for L. In FOL and FOML, such notions are trivially available: in them, semantic values of arbitrary formulas can be simulated by interpretations of atomic formulas.[23] An L-formula can be construed as a schema only *relative* to some substitutions for its atomic predicates. Reasonable candidates are the formulas safe for substitution, because they and only they preserve model-theoretic validity when substituted for atomic subformulas within an arbitrary model-theoretically valid formula.[24] Similarly, L-formulas can be used for representing a logical form only *relative* to safe substitutions. With such notions of schema and logical form, we are *very far* from the situation we have in standardly studied languages, such as FOL or FOML.

Since there is no hope of construing L-formulas themselves as representing logical forms in a syntactically manageable way, we can—if we so wish—introduce expressions that are *not* L-formulas but whose very task is to be schematic syntactic representatives of logical forms. These expressions will look exactly like L-formulas, except that in place of predicate symbols, they contain *schematic symbols*, such as X, Y, Z, X_1, X_2—each symbol with its specified arity. The set $S_L[\Sigma]$ of *schematic formulas* of vocabulary Σ is generated from a set Σ of schematic symbols in the same way that the set $L[\tau]$ is generated from the vocabulary τ.

Schematic formulas are evaluable relative to *schematic models* in the same way as formulas are evaluated relative to models. If F is a frame and Σ is a set of schematic symbols, a schematic model is a structure $\langle F, INT\rangle$, where INT is a function associating a subset $INT(X, w)$ of $\mathrm{WL}(F)^n$ for every $n > 0$, n-ary $X \in \Sigma$, and $w \in dom(F)$. Schematic symbols of positive arity are predicates applied to tuples of world lines, instead of being predicates applied to tuples of local objects. Bressan [6] advocates this sort of 'intensional predication' in his account of quantified modal logic.[25] By definition, $M, w, g \models X(x_1, \ldots, x_n)$ iff $\langle g(x_1), \ldots, g(x_n)\rangle \in INT(X, w)$, and $M, w, g \models x = y$ iff $g(x)$ equals $g(y)$. The clauses for complex formulas are

[23] In connection with FOML, this is because the domain constraint is not assumed: interpretations of predicates in a world w can involve objects lying outside the domain of w.

[24] If $\phi(x)$ is not safe for substitution, then there are M, w, and g such that $M, w, g \models \phi(x)$ and $g(x)$ is not realized in w. This fact allows us to construct a model M' containing worlds w and w' with $R = \{\langle w', w\rangle\}$ such that $g(x) \in \mathcal{P}_{w'}$ and $g(x)$ satisfies $\Diamond\phi(x)$ in w', but no world line in $\mathcal{P}_w$ satisfies $\phi(x)$ in w. It follows that $\exists x \Diamond \phi(x) \to \Diamond \exists x\, \phi(x)$ is false in M' at w'. That is, substituting $\phi(x)$ for $Q(x)$ in the model-theoretically valid formula $\exists x \Diamond Q(x) \to \Diamond \exists x\, Q(x)$ yields a refutable formula.

[25] We could also consider nullary schematic symbols standing for world lines instead of local objects, but for simplicity, I ignore them here. (See, however, footnote 29 in Sect. 6.6.) It should be noted

exactly as in L. A schematic formula ζ is said to be *model-theoretically valid* if it is satisfied in every schematic model M at every world $w \in dom(M)$ under all assignments. Letting Σ_1 and Σ_2 be disjoint sets of schematic symbols, if υ is a map of type $\Sigma_1 \to S_L[\Sigma_1 \cup \Sigma_2, V_2]$ assigning to every n-ary schematic symbol X of Σ_1 an $S_L[\{X\} \cup \Sigma_2, V_2]$-formula $\upsilon(X)$ with free variables $v_1, \ldots, v_n$, the *schematic substitution* based on υ is the υ-substitution $\sigma : S_L[\Sigma_1, V_1] \to S_L[\Sigma_1 \cup \Sigma_2, V_1 \cup V_2]$. A schematic formula ζ is *schematically valid* if for all schematic substitutions σ, the schematic formula $\sigma[\zeta]$ is model-theoretically valid. Model-theoretic validity and schematic validity coincide in S_L. In particular, the former entails the latter, since *any* semantic value of an S_L-formula can be simulated by a semantic value of a schematic symbol: any subset of $\mathrm{WL}(F)^n$ can be a value of an interpretation function *INT*.

A schematic formula ζ is *valid in a frame* F (denoted $F \models_{\mathrm{sch}} \zeta$) if for all schematic models $\langle F, INT\rangle$ based on F, all $w \in dom(F)$, and all suitable assignments g, we have $M, w, g \models \zeta$. If ζ is valid in a frame F and σ is a schematic substitution, then clearly, the schematic formula $\sigma[\zeta]$ is valid in the frame F, as well. Let us consider schematic versions of the Barcan formula and its converse:

- BF-P-S $\Diamond \exists x\, X(x) \to \exists x \Diamond X(x)$
- CBF-P-S $\exists x \Diamond X(x) \to \Diamond \exists x\, X(x)$
- BF-I-S $\Diamond \mathsf{E}_{\mathsf{a}} x\, X(x) \to \mathsf{E}_{\mathsf{a}} x \Diamond X(x)$
- CBF-I-S $\mathsf{E}_{\mathsf{a}} x \Diamond X(x) \to \Diamond \mathsf{E}_{\mathsf{a}} x\, X(x)$,

where X is a unary schematic symbol. We have seen that the L-formula $\exists x \Diamond Q(x) \to \Diamond \exists x Q(x)$, or CBF-P, is model-theoretically valid and that it therefore does not characterize monotonicity. The schematic version of this formula, or CFP-P-S, is *not* model-theoretically valid. This can be seen by noting that the L-formula $\exists x \Diamond \neg Q(x) \to \Diamond \exists x \neg Q(x)$, or CBF$''$, is refutable; cf. the proof of Fact 5.8(b). In terms of schematic formulas, we recover the familiar connection between the converse Barcan formula and monotonicity. The requirement $F \models_{\mathrm{sch}}$ CFP-P-S is much stronger than the requirement that $F \models$ CBF-P: in any world w, the schematic symbol X ranges over *all* subsets of $\mathrm{WL}(F)$, whereas in a world w, the predicate symbol Q in effect ranges only over those subsets of $\mathrm{WL}(F)$ whose elements are realized in w.

(Footnote 25 continued)
that adopting intensional predication would not automatically increase the expressive power of our modal language. It does not, if both of the following assumptions (a) and (b) are made: (a) no two world lines overlap, whence we get a unique grip of a world line by speaking of any of its realizations, and (b) intensional atomic predicates are quasi-extensional (cf. Definition 2.2). Giving up (a), two world lines may share their realizations in w while only one satisfies a given intensional predicate. If (b) is given up, a world line may satisfy in w an intensional atomic predicate without being realized in w. It should be noted that in the absence of (b), atomic formulas lose their capacity to express conditions concerning their context of evaluation (for a discussion, see Sect. 5.8).

Fact 5.11 (Schematic CBFs and monotonicity) *Let F be an arbitrary frame.*

(a) $F \models_{\text{sch}}$ CBF-P-S $\quad\Leftrightarrow\quad$ *F is physically monotonic.*
(b) $F \models_{\text{sch}}$ CBF-I-S $\quad\Leftrightarrow\quad$ *F is intentionally monotonic relative to agent α.*

Proof For (a), let $F = \langle W, \{R\}, \mathcal{P}, \mathcal{I}\rangle$ be a frame. If F is physically monotonic, CFP-P-S is trivially valid in F. Conversely, assume $F \models_{\text{sch}}$ CBF-P-S. Suppose for contradiction that there are w, w' with $R(w, w')$ and $\mathbf{I} \in \mathcal{P}_w$ such that $\mathbf{I} \notin \mathcal{P}_{w'}$. Let $INT(w', X) = \{\mathbf{I}\}$ and $INT(w'', X) = \varnothing$ for all $w'' \neq w'$. Write $M = \langle F, INT\rangle$. Since $M, w \models \exists x \Diamond X(x)$ and CFP-P-S is valid in F, we have $M, w \models \Diamond \exists x X(x)$ and thus $M, u, x := \mathbf{J} \models X(x)$ with $R(w, u)$ and $\mathbf{J} \in \mathcal{P}_u$. By the definition of INT, it follows that $u = w'$ and $\mathbf{J} = \mathbf{I}$. This is impossible, as $\mathbf{I} \notin \mathcal{P}_{w'}$. Claim (b) can be proven similarly: the reasoning above did not make use of the fact that $\mathbf{I}$ is physically individuated. □

Like BF-P, BF-P-S also characterizes physical anti-monotonicity. What is more, BF-I-S characterizes intentional anti-monotonicity.

Fact 5.12 (Schematic BFs and anti-monotonicity) *Let F be an arbitrary frame.*

(a) $F \models_{\text{sch}}$ BF-P-S $\quad\Leftrightarrow\quad$ *F is physically anti-monotonic.*
(b) $F \models_{\text{sch}}$ BF-I-S $\quad\Leftrightarrow\quad$ *F is intentionally anti-monotonic relative to agent α.*

Proof The right–left directions are trivial. The converse directions can be proven by arguments similar to the one used in the proof of Fact 5.11. □

Schematic formulas are better behaved than L-formulas. Any schematic formula ζ represents a logical form in S_L: all schematic formulas obtained from ζ by schematic substitutions have the form ζ. If ζ is model-theoretically valid in the sense of S_L, so are all schematic formulas having the logical form that ζ represents. Now, while L-formulas themselves cannot be used for representing logical forms of L-formulas, we can use the language S_L for that purpose. An S_L-formula ζ can be taken to represent any L-formula that could be obtained by uniformly substituting suitable L-formulas for the schematic symbols of ζ. If the S_L-formula ζ is model-theoretically valid in the sense of S_L, then all L-formulas it represents are model-theoretically valid in the sense of L. It should be noted that among L-formulas obtainable by substituting L-formulas for schematic symbols in a refutable schematic formula, there can still of course be formulas, some of which are model-theoretically valid while others are not. A case in point is the schematic formula $\exists x \Diamond X(x) \rightarrow \Diamond \exists x X(x)$ with the substitution instances $\exists x \Diamond Q(x) \rightarrow \Diamond \exists x Q(x)$ and $\exists x \Diamond \neg Q(x) \rightarrow \Diamond \exists x \neg Q(x)$, the former being valid and the latter refutable.

5.7 Relation to First-Order Logic

Let us compare FOL and L. For simplicity, I restrict attention to relational vocabularies—i.e., I concentrate on formulas without constant symbols. Syntactically, FOL is a fragment of L. What is more, FOL is translatable into L. For any first-order model $\mathcal{M} = \langle D, \mathbf{Int} \rangle$ of vocabulary τ, let $M = \langle W, \mathcal{R}, \mathcal{P}, \mathcal{I}, Int \rangle$ be a model of L of vocabulary τ, satisfying: $W = \{w_0\}$ with $dom(w_0) = D$; $\mathcal{R} = \mathcal{I} = \varnothing$; $Int(Q, w_0) = \mathbf{Int}(Q)$ for all $Q \in \tau$; and $\mathcal{P}_{w_0} = \{\mathbf{I}_d : d \in D\}$, where each $\mathbf{I}_d = \{\langle w_0, d \rangle\}$. Consequently, $\mathbf{I}_d \neq \mathbf{I}_{d'}$ whenever $d \neq d'$. The following fact holds trivially.

Fact 5.13 *For all FOL-formulas $\phi(x_1, \dots, x_n)$, first-order models $\mathcal{M}$, and assignments $\Gamma : Var \to D$, we have:*

$$\mathcal{M}, \Gamma \models_{\mathrm{FOL}} \phi(x_1, \dots, x_n) \textit{ iff } M, w_0, g \models \phi(x_1, \dots, x_n),$$

where g is a map of type $Var \to \mathcal{P}_{w_0}$ such that $g(x) = \mathbf{I}_{\Gamma(x)}$ for all $x \in Var$. □

Semantically, the logic L has an unmistakable higher-order flavor: its quantifiers range over partial functions. Now, the simplest entities the semantics employs are local objects. We can think of worlds as (pairwise disjoint) sets of local objects. If world lines are regarded as functions with worlds as arguments and local objects as values, this amounts to viewing worlds as second- and world lines as third-order entities. Alternatively, as explained in Sect. 2.6, we may view world lines as sets of local objects: any world line corresponds to such a set—the set of its realizations. The latter way of construing the two types of modal unities could be conveniently utilized to show that L admits a translation into monadic second-order logic. Somewhat surprisingly, perhaps, we can actually do much better: L can be translated into FOL, if the first-order structures used in the comparison of these two languages are suitably chosen. As noted in Sect. 5.1, the possibility of such a translation serves to highlight the fact that Lewis's counterpart theory is a special case of my world line semantics. (Cf. the discussion in Sect. 2.7.2.)

Both worlds and world lines can be treated as values of first-order variables: we need not quantify over objects lower in the type hierarchy.[26] If we adopt this viewpoint, we may recover realizations of world lines as *pairs* of worlds and world lines. Thus, realizations are treated as entities *more complex* than worlds and world lines. Yet, we do not need second-order quantifiers: it suffices that we can ascribe predicates to such pairs. The described maneuver entails one complication. We must group together pairs $\langle w, \mathbf{I} \rangle$ corresponding to one and the same local object: distinct (intentionally individuated) world lines may have a common realization. Let E be a relation that holds of a triple $\langle w, \mathbf{I}, \mathbf{J} \rangle$ iff $\mathbf{I}$ and $\mathbf{J}$ are both realized in w and satisfy $\mathbf{I}(w) = \mathbf{J}(w)$. Thus, $\langle w, \mathbf{I}, \mathbf{J} \rangle \in E$ iff $Im(\mathbf{I}) \cap Im(\mathbf{J}) \cap dom(w) \neq \varnothing$.[27] Note that $\langle w, \mathbf{I}, \mathbf{I} \rangle \in E$ iff $\mathbf{I}$ is realized in w. In the models relative to which the translation of L into FOL is effected, we also need, in addition to counterparts of the relations R_i and counterparts of the sets $\mathcal{P}_w$ and $\mathcal{I}^{\alpha}_w$, the relation E to handle the use of the identity symbol of L.

[26] There is nothing extravagant in using first-order quantifiers to range over entities that, substantially speaking, are sets: this is precisely what is done in axiomatic set theory. It is one thing for an entity to be of first order for the purposes of a specific logical analysis and another thing to be of first order metaphysically speaking.

[27] If $f : A \to B$ is a function, its *image* $Im(f)$ is the subset $\{f(a) : a \in A\}$ of B.

I will describe a translation of L into FOL. I assume that FOL uses variables of two *sorts*. Variables of *Var* are of *sort 1*. Their values play the role of world lines. Variables in the syntactically disjoint set $\{t, s, t_1, t_2, \ldots\}$ are of *sort 2*. Their values play the role of worlds. For any model $M = \langle W, \mathcal{R}, \mathcal{P}, \mathcal{I}, Int\rangle$ of vocabulary τ, let P, I_{a} (with $\mathsf{a} \in \mathbb{A}$), and R_{i} (with $\mathsf{i} \in \mathbb{I}$) be binary predicate symbols, and let E be a ternary predicate symbol. Finally, for every n-ary $Q \in \tau$, let Q be an $(n+1)$-ary predicate symbol. For specifying a first-order model $\mathcal{M} = \langle D, \mathbf{Int}\rangle$ of vocabulary $\tau^* = \{\mathsf{Q} : Q \in \tau\} \cup \{\mathsf{E}, \mathsf{P}\} \cup \{\mathsf{I}_{\mathsf{a}} : \mathsf{a} \in \mathbb{A}\} \cup \{\mathsf{R}_{\mathsf{i}} : \mathsf{i} \in \mathbb{I}\}$, let us define sets U_1 and U_2 of modal unities and relations R_i^*, $\mathcal{P}^*$, $\mathcal{I}_\alpha^*$, and E among these modal unities as follows:

- $U_1 := \{dom(w) : w \in W\}$
- $U_2 := \{Im(\mathbf{I}) : \mathbf{I} \in \mathcal{P}_w \text{ and } w \in W\} \cup \{Im(\mathbf{I}) : \mathbf{I} \in \mathcal{I}_w^\alpha \text{ and } w \in W \text{ and } \alpha \in A\}$
- $R_{\mathsf{i}}^* := \{\langle dom(w), dom(w')\rangle : R_{\mathsf{i}}(w, w')\}$
- $\mathcal{P}^* := \{\langle dom(w), Im(\mathbf{I})\rangle : \mathbf{I} \in \mathcal{P}_w\}$
- $\mathcal{I}_\alpha^* := \{\langle dom(w), Im(\mathbf{I})\rangle : \mathbf{I} \in \mathcal{I}_w^\alpha\}$
- $E = \{\langle m, i, j\rangle \in U_1 \times U_2 \times U_2 : m \cap i \cap j \neq \varnothing\}$.

For any $m \in U_1$, there is a unique world $w \in W$ such that $m = dom(w)$; I denote this world by $|m|$. The structure $\mathcal{M} = \langle D, \mathbf{Int}\rangle$ is defined by setting: $D = U_1 \cup U_2$, $\mathbf{Int}(\mathsf{R}_{\mathsf{i}}) = R_{\mathsf{i}}^*$, $\mathbf{Int}(\mathsf{P}) = \mathcal{P}^*$, $\mathbf{Int}(\mathsf{I}_{\mathsf{a}}) = \mathcal{I}_\alpha^*$, $\mathbf{Int}(\mathsf{E}) = E$, and

> $\langle m, i_1, \ldots, i_n\rangle \in \mathbf{Int}(\mathsf{Q})$ iff $m \in U_1$ and $i_1, \ldots, i_n \in U_2$ and there are elements $b_1, \ldots, b_n$ of m such that $b_1 \in i_1$ and...and $b_n \in i_n$ and $\langle b_1, \ldots, b_n\rangle \in Int(Q, |m|)$.

Elements of $D = U_1 \cup U_2$ are *sets* of local objects of M. Elements of U_1 derive from worlds of M, while those of U_2 are induced by world lines of M. Whatever internal structure worlds w and world lines $\mathbf{I}$ of M may have is immaterial here. This is why I simply consider the sets of local objects $dom(w)$ and $Im(\mathbf{I})$ they determine. For every $i \in U_2$ there is a unique world line $\mathbf{I} \in \mathrm{WL}(M)$ such that $i = Im(\mathbf{I})$; I denote this world line by $|i|$. (It may belong to many sets $\mathcal{P}_w$ and/or to many sets $\mathcal{I}_w^\alpha$.)

An FOL-formula $\theta(t, x_1, \ldots, x_n)$ is a *translation* of an L-formula $\phi(x_1, \ldots, x_n)$ if for all models M, worlds w, and assignments $g : \{x_1, \ldots, x_n\} \to \mathrm{WL}(M)$, we have:

$$M, w, g \models \phi(x_1, \ldots, x_n) \textit{ iff } \mathcal{M}, \Gamma_{t,w,g} \models \theta(t, x_1, \ldots, x_n),$$

where $\Gamma_{t,w,g}$ is an assignment on the set of variables $\{t, x_1, \ldots, x_n\}$ satisfying $\Gamma_{t,w,g}(t) = dom(w)$ and $\Gamma_{t,w,g}(x_i) = Im(g(x_i))$. Let us take examples.

Example 5.1 The following table displays four formulas with their translations:

L-formula	its FOL translation
$Q(x)$	$\mathsf{Q}(t, x)$
$x = y$	$\mathsf{E}(t, x, y)$
$\Box Q(x)$	$\forall s(\mathsf{R}_{\mathsf{i}}(t, s) \to \mathsf{Q}(s, x))$
$\exists x Q(x)$	$\exists x(\mathsf{P}(t, x) \wedge \mathsf{Q}(t, x))$

Let us check that each of these L-formulas is indeed translated by the corresponding FOL-formula. Let us begin with $Q(x)$. If $M, w, g \models Q(x)$, then $g(x)$ is realized in w, and $g(x)(w) \in Int(Q, w)$. Writing $m := dom(w)$ and $i := Im(g(x))$, it follows that $\Gamma_{t,w,g}(t) = m \in U_1$ and $\Gamma_{t,w,g}(x) = i \in U_2$. Further, $\langle m, i\rangle \in \mathbf{Int}(\mathsf{Q})$, because $g(x)(w) \in m \cap i \cap Int(Q, w)$. Consequently, $\mathcal{M}, \Gamma_{t,w,g} \models \mathsf{Q}(t, x)$. Conversely, if $m = \Gamma_{t,w,g}(t)$ and $i = \Gamma_{t,w,g}(x)$ and $\langle m, i\rangle \in \mathbf{Int}(\mathsf{Q})$, the set $m \cap i \cap Int(Q, |m|)$ is non-empty. Since $m = dom(w) \in U_1$ and $i = Im(g(x)) \in U_2$, this means that $g(x)$ is realized in w and $g(x)(w) \in Int(Q, |m|)$, where $|m| = w$. Therefore, $M, w, g \models Q(x)$.

As for the formula $x = y$, if $M, w, g \models x = y$, then $g(x)$ and $g(y)$ are both realized in w, and $g(x)(w)$ equals $g(y)(w)$. Letting $m := dom(w)$, $i := Im(g(x))$ and $j := Im(g(y))$, we have $m \cap i \cap j = \{g(x)(w)\} = \{g(y)(w)\}$, whence $\langle m, i, j\rangle \in E$. Since $\Gamma_{t,w,g}(t) = m$, $\Gamma_{t,w,g}(x) = i$ and $\Gamma_{t,w,g}(y) = j$, it ensues that $\mathcal{M}, \Gamma_{t,w,g} \models \mathsf{E}(w, x, y)$. Conversely, suppose $\mathcal{M}, \Gamma_{t,w,g} \models \mathsf{E}(t, x, y)$. This means that there is b such that $\Gamma_{t,w,g}(t) \cap \Gamma_{t,w,g}(x) \cap \Gamma_{t,w,g}(y) = \{b\}$, whence $g(x)$ and $g(y)$ are realized in w and $g(x)(w) = b = g(y)(w)$. It follows that $M, w, g \models x = y$.

Let us move on to take a look at the formula $\Box Q(x)$. Suppose $M, w, g \models \Box Q(x)$. Write $m := dom(w)$. Let m' be any element of U_1 such that $\mathcal{M}, \Gamma_{t,w,g}[s := m'] \models \mathsf{R_i}(t, s)$. It ensues that $\langle m, m'\rangle \in \mathbf{Int}(\mathsf{R_i}) = R^*_{\mathsf{i}}$. This means that $R_{\mathsf{i}}(w, w')$, where $w' = |m'|$. Thus, $M, w', g \models Q(x)$. Given that $Q(x)$ is translated by $\mathsf{Q}(t, x)$, we have $\mathcal{M}, \Gamma_{t,w',g} \models \mathsf{Q}(t, x)$—and therefore, by replacing t by s, further $\mathcal{M}, \Gamma_{s,w',g} \models \mathsf{Q}(s, x)$. Here, $\Gamma_{s,w',g}[t := m] = \Gamma_{t,w,g}[s := m']$, and we may conclude that $\mathcal{M}, \Gamma_{t,w,g}[s := m'] \models \mathsf{Q}(s, x)$. Conversely, suppose $\mathcal{M}, \Gamma_{t,w,g} \models \forall s(\mathsf{R_i}(t, s) \to \mathsf{Q}(s, x))$. Let w' with $R_{\mathsf{i}}(w, w')$ be arbitrary, and write $m' := dom(w')$. Now, $\mathcal{M}, \Gamma_{t,w,g}[s := m'] \models \mathsf{R_i}(t, s)$, where $\Gamma_{t,w,g}[s := m'] = \Gamma_{s,w',g}[t := m]$. It follows that $\mathcal{M}, \Gamma_{s,w',g} \models \mathsf{Q}(s, x)$. Consequently, $M, w', g \models Q(x)$, which allows us to conclude that $M, w, g \models \Box Q(x)$.

Finally, let us consider the formula $\exists x Q(x)$. Suppose that $M, w, g \models \exists x Q(x)$. Thus, $M, w, g[x := \mathbf{I}] \models Q(x)$ for some $\mathbf{I} \in \mathcal{P}_w$. Let $m := dom(w)$ and $i := Im(\mathbf{I})$. On the one hand, we have $\langle m, i\rangle \in \mathcal{P}^*$, entailing that $\mathcal{M}, \Gamma_{t,w,g}[x := i] \models \mathsf{P}(t, x)$. On the other hand, we have $\mathcal{M}, \Gamma_{t,w,g[x:=\mathbf{I}]} \models \mathsf{Q}(t, x)$. Since $\Gamma_{t,w,g[x:=\mathbf{I}]} = \Gamma_{t,w,g}[x := i]$, it follows that $\mathcal{M}, \Gamma_{t,w,g}[x := i] \models \mathsf{P}(t, x) \wedge \mathsf{Q}(t, x)$, whence $\mathcal{M}, \Gamma_{t,w,g} \models \exists x(\mathsf{P}(t, x) \wedge \mathsf{Q}(t, x))$. Conversely, suppose there is $i \in U_2$ such that $\mathcal{M}, \Gamma_{t,w,g}[x := i] \models \mathsf{P}(t, x) \wedge \mathsf{Q}(t, x)$. Thus, $M, w, g[x := \mathbf{I}] \models Q(x)$, where $\mathbf{I} = |i| \in \mathcal{P}_w$. Therefore $M, w, g \models \exists x Q(x)$. $\square$

In order to translate L into FOL, let us associate a map $T_t : L[\tau] \to \text{FOL}[\tau^*]$ for every variable t of sort 2 as follows:

- $T_t[Q(x_1, \ldots, x_n)] := \mathsf{Q}(t, x_1, \ldots, x_n)$
- $T_t[x_1 = x_2] := \mathsf{E}(t, x_1, x_2)$
- $T_t[\neg\phi] := \neg T_t[\phi]$
- $T_t[(\phi \wedge \psi)] := (T_t[\phi] \wedge T_t[\psi])$
- $T_t[\exists x\phi] := \exists x(\mathsf{P}(t, x) \wedge T_t[\phi])$
- $T_t[\mathsf{E_a} x\phi] := \exists x(\mathsf{I_a}(t, x) \wedge T_t[\phi])$
- $T_t[\Box_{\mathsf{i}}\phi] := \forall s(\mathsf{R_i}(t, s) \to T_s[\phi])$,

where s is a variable of sort 2 syntactically distinct from t. The sole free variable of the FOL-formula $T_t[\phi]$ is t. Note that all quantifiers in $T_t[\phi]$ are 'bounded' by a relational condition expressed in terms of one of the binary predicates P, $\mathsf{I_a}$, or $\mathsf{R_i}$. Observe also that the FOL-formula $T_t[\phi]$ contains no occurrences of the identity symbol.

Theorem 5.3 (First-order translation of L) *For all $\phi(x_1, \ldots, x_n) \in L$, models M, worlds $w \in dom(M)$, and assignments $g : \{x_1, \ldots, x_n\} \to \mathrm{WL}(M)$, we have:*

$$M, w, g \models \phi \textit{ iff } \mathcal{M}, \Gamma_{t,w,g} \models T_t[\phi].$$

Proof See Sect. A.6 of the Appendix A. □

5.8 Negative Properties Put into Perspective

We have seen that two notions of validity must be distinguished in L and that L lacks notions of scheme and logical form—notions that are entirely unproblematic in such languages as FOL and FOML. Some philosophers tend to give the semantic behavior of familiar logical languages a *normative* status. A good example is Quine, who took certain metatheoretic properties of FOL to constitute criteria that a putative language must satisfy in order to qualify as a logic (see, e.g., [101]). In particular, he gave axiomatizability (the existence of a sound and complete proof procedure, recursive enumerability of valid formulas) a normative status in this connection. Already on this ground, he was prepared to disqualify second-order logic as a logic proper. Williamson's comments on 'mandatory properties of logical systems' suggest that he shares, in this respect, Quine's way of thinking about logic.[28]

Surely the property of admitting a notion of logical form has at least as good a claim for being a necessary condition of logicality than a technical meta-property such as axiomatizability has. Should L, then, be dismissed as a logic? Should we conclude that despite having perhaps looked initially as worthy of logical study, we have shown that in reality L lacks the relevant mandatory properties? When our interest is in philosophical considerations, the grounds for studying a language are not to be searched within the language itself. I am interested in the language L because it is a linguistic tool for talking about certain non-linguistic phenomena, notably the behavior of physical and intentional objects in modal settings. It may be a pity that L lacks certain seemingly innocent and apparently obvious representational properties,

[28]Some logicians and logic-oriented philosophers may think that the question of the logicality of a language is a nonissue. For them, anything anyone might wish to call a 'logic' just is a logic, period. For my part, I think that even though one can hardly lay down rigid logicality criteria of the sort Quine had in mind, one gains a better understanding of a language when reflecting on its formal properties, positive or negative. The philosophical interest of the language is dependent on how one can situate these properties in a larger setting. Therefore, not just anything is qualifiable as a logic in an interesting sense.

but this is hardly a good reason for changing the subject matter of our investigations. The language that is used to talk about a subject matter must reflect the subject matter, not the other way around. It would be absurd to conclude that just because our means of expression fail to comply with some predetermined standards, we must throw those means of expression overboard—and with them the subject matter they in fact serve to describe. Methodologically, such a maneuver would be as laudable as it would be for a physicist to renounce resorting to quantum mechanics because of an independently acquired aesthetic judgment according to which it must be possible to measure simultaneously the momentum and the location of a particle. Incidentally, Williamson [127, p. 27] declares that his thinking is guided by 'a conception of theories in logic and metaphysics as scientific theories, to be assessed by the same overall standards as theories in other branches of science'. In science, preconceptions about the requisite means of describing phenomena are never among the relevant standards, so in Williamson's lights, the fact that L lacks a reasonable notion of logical form should not be found disturbing—even if Williamson himself is suspicious of some languages due to their lacking certain 'mandatory' properties.

I take it, then, that negative semantic properties of L cannot provide sufficient grounds for giving up the study of worlds and world lines in my framework. Still, weird properties of a language designed for talking about a subject matter might provide indirect evidence that there is something weird about one's conceptualization of the subject matter itself. The main features of my framework are the fact that physical and intentional objects are understood as world lines and the fact that atomic formulas serve to ascribe quasi-extensional predicates that world lines satisfy in a world, depending on how their *realizations* are in that world. It is this latter feature that leads to the semantic anomalies that have been discussed in this chapter.

We have seen that the anomalies disappear when attention is turned to schematic formulas. The languages L and S_L have no semantic differences except those that are induced by their diverging interpretation of atomic formulas. In S_L, the simplest n-ary expressions are n-ary schematic symbols. Their semantic values in a model M at a world w are subsets of $\mathrm{WL}(M)^n$. By contrast, in L, the simplest n-ary expressions are n-ary predicates, whose interpretations are subsets of $dom(w)^n$. Given an assignment $x_1 := \mathbf{I}_1, \ldots, x_n := \mathbf{I}_n$, a schematic atomic predication $X(x_1, \ldots, x_n)$ is satisfied in a world w iff $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in INT(X, w)$—a condition that by no means requires that the world lines $\mathbf{I}_j$ be realized in w. By contrast, an atomic L-formula $Q(x_1, \ldots, x_n)$ is satisfied in w iff each $\mathbf{I}_j$ is realized in w and $\langle \mathbf{I}_1(w), \ldots, \mathbf{I}_n(w) \rangle \in Int(Q, w)$. The two languages differ similarly in their interpretation of identities $x_1 = x_2$. Unlike in L, in S_L, the formula $x = x$ is valid. If we look at S_L in its own right so that schematic symbols are viewed as predicates, then S_L is a formulation of a language in which quasi-extensional predication exemplified by atomic formulas in L is replaced by full-fledged 'intensional predication' already at the atomic level. If one should make a choice between L and S_L, depending on which language is better behaved metatheoretically, the choice should fall on S_L.

While a fully general formulation of my semantic framework must allow intensional predication, the language S_L, as it stands, is too weak for my purposes. In this language, we lose grip of the distinction between world lines and their realizations.

In S_L, a world line need not be realized in w in order to satisfy a formula $X(x)$. If the value of x is physically individuated, there is, admittedly, a roundabout way of expressing that this value is realized in w: by requiring that $\exists y(X(x) \wedge x = y)$ be satisfied at w. Namely, the physical availability of the value of y in w entails that this value is realized in w. Therefore, if the value of y is identical to that of x, the latter is realized in w. (Recall that in S_L, the identity symbol is interpreted in terms of world lines, not in terms of their realizations.) However, if the value of x is intentionally individuated, binding y in $X(x) \wedge x = y$ by an intentional quantifier evaluated in w does not suffice for guaranteeing that the value of y is realized in w, and neither is there any other way of expressing this condition. Yet, my analysis of intentional objects requires that we are able to speak of their realizations in specific contexts.[29] Certainly, S_L can be generalized so as to allow expressing such features—for example, by incorporating into the syntax two types of predicates and identity symbols: extensional and intensional. The point is, however, that world-internal ascriptions of characteristics and therefore local evaluation of atomic formulas are an integral part of the framework I am discussing—not an artifact of a badly formulated language designed to be used for talking about this framework. Therefore, the semantic anomalies of the language L do not tell against my framework, and my decision to concentrate at the linguistic level on L instead of a suitable generalization of S_L (which would be both syntactically and semantically essentially more involved than L) has merely the status of a theoretical choice allowing us to concentrate on certain aspects of my framework without blurring the big picture by needless complications.

[29]While being too weak for my purposes, S_L can also express conditions that L cannot express. We could have two world lines $\mathbf{I}$ and $\mathbf{J}$ and a world w with $\mathbf{I}(w) = \mathbf{J}(w)$, such that in a model at w, the schematic formula $X(x)$ is satisfied by the assignment $x := \mathbf{I}$ but not by the assignment $y := \mathbf{J}$. Namely, we could have $\mathbf{J} \notin INT(X, w) \ni \mathbf{I}$. By contrast, in L the formula $Q(x)$ would be satisfied either by both assignments or by neither: if $\mathbf{I}(w) = \mathbf{J}(w)$, then $\mathbf{I}(w) \in Int(Q, w)$ iff $\mathbf{J}(w) \in Int(Q, w)$.

Chapter 6
General Consequences

6.1 Introduction

My framework offers a novel semantic analysis of sentences that ascribe to an agent a mental state having an intentional object. Making use of the analysis of intentional states presented in Sect. 4.7, I discern in Sect. 6.2 four senses of the notion of intentional object and indicate how mental states involving different types of objects of thought can be uniformly represented in my semantic framework: propositional thoughts, plural thoughts, thoughts with an indeterminate object, singular thoughts, and thoughts representing specific physical objects. In Sect. 6.3, I define singular contents as situated contents involving a single intentional object. I compare my account of singular contents to the theory of singular thought that François Recanati formulates in his 2012 book, *Mental Files*.

In the recent philosophical literature, Tim Crane is one of the most important proponents of intentionalism—the thesis that all mental phenomena are intentional.[1] At the heart of his analysis is the notion of mental representation. Of specific interest is Crane's discussion of thoughts about the non-existent in his 2013 book, *The Objects of Thought*. His theory is formulated in the context of the philosophy of mind, while my analysis is largely driven by semantic considerations. His goal is to develop a philosophical account of psychological phenomena, whereas my aim is to develop a semantic account of linguistic expressions describing some such phenomena. In Sect. 6.4, I spell out the differences and similarities between Crane's approach and mine.

I point out in Sect. 6.5 that in the context of L, *variables* can be viewed as *formulas*. Syntactically, variables are singular terms, but semantically, they have satisfaction conditions. The formula x is satisfied in w iff the value of x (a certain world line) is realized in w. This double role of variables opens up a way of representing certain intensional transitive verbs in my semantic framework. I discern a semantic criterion that an intensional transitive verb must satisfy to be thus analyzable and refer to the

[1] Further important proponents include Alex Byrne [10] and Michael Tye [121].

T. Tulenheimo, *Objects and Modalities*, Logic, Epistemology,
and the Unity of Science 41, DOI 10.1007/978-3-319-53119-9_6

relevant class of verbs as *robust intensional verbs* (Sect. 6.6). I compare my logical analysis of these verbs with Friederike Moltmann's linguistically driven account of what she calls intentional—as opposed to intensional—verbs (Sect. 6.7).

6.2 Objects of Thought: A Uniform Analysis

As Prior stressed and as noted in Sect. 3.2, the term 'object of thought' is ambiguous between two readings. It can mean either a proposition or an intentional object. In my analysis, both propositions and intentional objects come out as objects of intentional states of the general form $\langle W, \mathfrak{J}, w_0 \rangle$ discerned in Sect. 4.7. I have referred to such states $\langle W, \mathfrak{J}, w_0 \rangle$ alternatively as 'situated contents' (Sect. 2.5). If $\mathfrak{J} = \emptyset$, the state $\langle W, \mathfrak{J}, w_0 \rangle$ has as its object a proposition—namely, the set W of worlds. A propositional thought may but need not be general. General thoughts have propositions as their objects. A possible object of a general thought would be *that there are winged horses*. Not all propositions give rise to general thoughts—e.g., *that it is raining* does not. In addition to propositional thoughts, there are object-directed thoughts. If $\mathfrak{J}$ consists of a single world line, $\mathbf{J}$, the state $\langle W, \mathfrak{J}, w_0 \rangle$ has $\mathbf{J}$ as its single intentional object. If, again, the list $\mathfrak{J}$ has several members, the state $\langle W, \mathfrak{J}, w_0 \rangle$ has a plurality of intentional objects. The intentional objects involved in object-directed thoughts may be *specific* or *unspecific*. The two cases differ in the amount of indeterminacy tolerated. I take it that an intentional object may be specific without representing a (past or present) physical object. In order to be a representation of a physical object, an intentional object must satisfy the criteria discussed in Sect. 4.8. When these criteria are met, the intentional object is relatively well behaved: it cannot manifest indeterminacy in an arbitrary way, since it must reflect the behavior of a physical object over the corresponding set of worlds $R(w_0)$. It cannot be excluded that an intentional object is sufficiently well behaved even in the absence of a corresponding physical object. This is why we cannot exclude the possibility of specific intentional objects that do not represent any specific physical object.

How are contents $\langle W, \mathbf{J}, w_0 \rangle$ with a single intentional object related to *singular thoughts*? According to a popular view, a singular thought is a specific thought about one particular existing object. It is taken that agents cannot have singular thoughts unless the relevant object exists (see, e.g., McDowell [85, Chap. 9]). As the popular view has it, singular thoughts can be described using *singular propositions* (or Russellian propositions)—structured propositions literally containing a particular object as a constituent.[2] These particular objects are typically supposed to be ordinary material objects, not, for example, indeterminate intentional objects. (Russell himself, however, took singular propositions to pertain to sense data, not to material objects.) It appears reasonable to think that a singular proposition cannot be used

[2]For Russell's theory of singular propositions, see [104, 105]. For neo-Russellian theories of structured propositions, see, e.g., Soames [114, 115] and Salmon [108].

for describing anything unless its constituent particular object exists. Consequently, the connection between singular thoughts and singular propositions breaks down if singular thoughts are not assumed to be existence-dependent—if there can be singular thoughts about non-existent objects. Now, manifestly, contents $\langle W, \mathbf{J}, w_0\rangle$ are structured. If the intentional object $\mathbf{J}$ counts as specific, the content $\langle W, \mathbf{J}, w_0\rangle$ looks much like a singular proposition. I refer to such contents as *singular contents*. A sufficient condition for an intentional object to be specific is that it represents a physical object. Crane [21, Sect. 6] argues—against the above-mentioned popular view—that not all specific thoughts are existence-dependent: that a specific thought may have a non-existent object. Similarly, as noted in the previous paragraph, I do not assume that the object $\mathbf{J}$ of a singular content $\langle W, \mathbf{J}, w_0\rangle$ must represent a physical object. In particular, I do not assume that it must exist in the sense of being realized in w_0.

It should be noted that generally, we need entire structures $\langle W, \mathfrak{J}, w_0\rangle$ to represent intentional objects; it is not always sufficient to confine attention to world line components of such states. Actually, the notion of intentional object is potentially four-ways ambiguous. First, we may adopt an *internal* or *external* viewpoint on an intentionally individuated world line according to whether we limit attention to its internal modal margin or consider it in its entirety. Second, we may opt for an *intrinsic* or *contextualized* viewpoint: we can consider an intentionally individuated world line in its own right or in relation to the world representation of an intentional state. Combined, the two distinctions yield four ways in which we may view object representations of intentional states. Intentional states semantically modeled as structures $\langle W, \mathfrak{J}, w_0\rangle$ comprise enough information to allow discerning the four senses of 'intentional object'. If $\mathbf{J} \in \mathfrak{J}$, it may but need not happen that $marg(\mathbf{J}) \subseteq W \subseteq marg(\mathbf{J})$.[3] The choice between an internal and external viewpoint on $\mathbf{J}$ presents itself when the first inclusion fails. This is what happens in qualitatively illusory experiences (the bear seen as a stone) and in those cases of intentional identity in which two agents ascribe mutually incompatible predicates to a fixed intentional object. The choice between an intrinsic and contextualized viewpoint on $\mathbf{J}$ forces itself upon us when the second inclusion fails. It fails when the agent does not believe that her intentional object exists (beliefs about a winged horse without commitment to its existence). An external but intrinsic viewpoint on $\mathbf{J}$ yields the world line $\mathbf{J}$ itself, whereas an external but contextualized point of view leads us to consider the pair $\langle \mathbf{J}, W\rangle$. Similarly, an internal and intrinsic standpoint yields the restriction $\mathbf{J}\restriction_W$ of $\mathbf{J}$ to the set W, while from the internal but contextualized point of view, it is the pair $\langle \mathbf{J}\restriction_W, W\rangle$ that is considered. The internal viewpoint is appropriate when we consider an intentional state as it appears to the agent. We may then disregard any worlds *not* belonging to the set $W = R(w_0)$, as by definition the agent excludes the possibility that the world in which she finds herself lies outside $R(w_0)$. Even when adopting the internal viewpoint on an intentional object, we cannot generally confine attention to its internal modal margin in our account of the agent's intentional state. The agent can represent the object $\mathbf{J}$ as non-existent only *in relation to* the set W: such a representation must

[3] Cf. footnote 13 in Sect. 3.5 and formula (11) in Sect. 4.4 (bear seen as a stone).

encode the information that $W \not\subseteq marg(\mathbf{J})$. This is not a matter intrinsic to the world line $\mathbf{J}$; what is at stake is the relation between the set W and the (internal) modal margin of $\mathbf{J}$.

6.3 Singular Contents and Singular Propositions

How are singular contents related to singular propositions? Consider the presumed singular propositions [Socrates is sitting] and [Socrates does not exist] with the individual Socrates as their constituent. Let the corresponding singular contents be $[\![$Socrates is sitting$]\!] := \langle W_1, \mathbf{I}_1, w_0\rangle$ and $[\![$Socrates does not exist$]\!] := \langle W_2, \mathbf{I}_2, w_0\rangle$. In the former content, $marg(\mathbf{I}_1) = W_1$ and '— is sitting' is intentionally predicated of $\mathbf{I}_1$ at w_0. In the latter content, $W_2 \not\subseteq marg(\mathbf{I}_2)$. Physical objects are physically individuated world lines. Socrates the physical object is one such world line. Writing $\mathbf{J}$ for Socrates, we may note that there is no reason to think that either $\mathbf{I}_1$ or $\mathbf{I}_2$ equals $\mathbf{J}$; cf. Sect. 4.8. However, both world lines $\mathbf{I}_1$ and $\mathbf{I}_2$ coincide with $\mathbf{J}$ on sufficiently many worlds belonging to the respective sets W_1 and W_2 so as to allow the contents to qualify as representations of Socrates. That is, there are non-empty sets $S_1 \subseteq W_1$ and $S_2 \subseteq W_2$ such that $\mathbf{I}_1(w_1) = \mathbf{J}(w_1)$ and $\mathbf{I}_2(w_2) = \mathbf{J}(w_2)$ for all $w_1 \in S_1$ and $w_2 \in S_2$.

Doubts have been raised on the notion of singular proposition on metaphysical grounds.[4] If propositions are abstract objects and exist necessarily, how could they have as their constituents contingent particulars such as Socrates? In my analysis, an agent's thought about Socrates may pertain to the real Socrates (a certain physically individuated world line), but even so, the corresponding intentional object *is not* Socrates: the intentional object merely coincides with the real Socrates over a suitable span of worlds, as just explained. Since there are no intentionally individuated world lines without those agents whose intentional objects they are, it should not occur to anyone that a singular content, such as $[\![$Socrates is sitting$]\!]$ or $[\![$Socrates does not exist$]\!]$, might 'exist' independently of an agent. An intentional object vanishes as soon as the agents thinking about it do—quite possibly much sooner! (Agents are forgetful: an intentional object available to an agent at t need not be available to her at a later time t'.)

Given that the presumed singular propositions [Socrates is sitting] and [Socrates does not exist] have the individual Socrates as their constituent, one might be led to wonder how this is compatible with the fact that the truth-values of these singular propositions are context-dependent. Can constituents of propositions be things that undergo changes? It might seem counterintuitive to say that a proposition has a component that *was* standing but *is* currently sitting or that it has a component that once existed but no longer does. Yet, if propositions literally have individuals as constituents, such conclusions appear hard to avoid. On the other hand, if constituents of propositions are immutable, then apparently those constituents cannot be

[4] For a discussion, see, e.g., Fitch and Nelson [26, Sect. 6].

individuals but rather world-bound time-slices of individuals. However, in that case, singular propositions must have context-independent truth-values: if Socrates is sitting in w at t, then it holds true in every context that the world-bound time-slice of Socrates corresponding to w and t is sitting. No similar dilemma occurs in connection with singular contents in my sense.

Take the singular content $[\![$Socrates is sitting$]\!]$. In the following comments, I make use of the relations of uniform and local support (see Definition 2.5). We recall that a situated content $C = \langle V, \mathbf{I}, w_0\rangle$ uniformly supports a formula ϕ (denoted $C \Vdash_{uni} \phi$) iff ϕ is true at every world $v \in V$, and that C locally supports ϕ (denoted $C \Vdash_{loc} \phi$) iff ϕ is true at w_0. The intentional object $\mathbf{I}_1$ of the singular content $[\![$Socrates is sitting$]\!]$ indeed satisfies the predicates it is represented as satisfying—namely, the predicates $\phi(x)$ such that $\langle W_1, \mathbf{I}_1, w_0\rangle \Vdash_{uni} \phi(x)$. More specifically, this intentional object satisfies these predicates necessarily, not contingently (cf. the discussion in Sect. 4.2). Among the predicates in question, there is the predicate '— is sitting'. That is, the intentional object has as its essential, characterizing feature the property expressed by this predicate. Then again, the intentional object $\mathbf{I}_1$ must not be mistaken for $\mathbf{J}$, Socrates the physical object. It is not in the least paradoxical that the *intentional* object satisfies, necessarily, the predicate '— is sitting'. To think otherwise is to confuse the intentional object $\mathbf{I}_1$ representing Socrates (typically within some more or less narrow temporal and spatial bounds) with Socrates himself. The *physical* object Socrates satisfies the predicate '— is sitting' contingently: he sits in some contexts and fails to sit in others. The same content $\langle W_1, \mathbf{I}_1\rangle$ can be associated with several worlds (by one agent or by several agents). If two situated contents differ only in their world-component, they have the same world representation and the same object representations, and thus, they *uniformly* support the same formulas. Yet, they need not *locally* support the same formulas. The question of whether a situated content locally supports a formula depends on the world in which the agent is located. For all situated contents $\langle W, \mathbf{I}, w\rangle$ with $W = W_1$ and $\mathbf{I} = \mathbf{I}_1$, we have $\langle W, \mathbf{I}, w\rangle \Vdash_{uni}$ (x is sitting), but $\langle W, \mathbf{I}, w\rangle \Vdash_{loc}$ (x is sitting) holds only if $\mathbf{I}$ is realized in w and $\mathbf{I}(w)$ is sitting. Similarly, all situated contents $\langle W', \mathbf{I}', w\rangle$ with $W' = W_2$ and $\mathbf{I}' = \mathbf{I}_2$ satisfy $\langle W', \mathbf{I}', w\rangle \nVdash_{uni}$ (x exists). By contrast, we have $\langle W', \mathbf{I}', w\rangle \nVdash_{loc}$ (x exists) only if $\mathbf{I}'$ is not realized in w.

What about cases in which an agent appears to have contradictory beliefs about one and the same physical object? Suppose Alice, unbeknownst to herself, finds herself looking at Mont Blanc. She believes that Mont Blanc is 4000 meters high, but at the same time, she believes that the mountain she is looking at is less than 4000 meters high.[5] Let us consider the singular contents $[\![$The mountain I am looking at is less than 4000 meters high$]\!] := \langle W, \mathbf{I}, w_0\rangle$ and $[\![$Mont Blanc is 4000 meters high$]\!] := \langle W', \mathbf{I}', w_0\rangle$, where W is the set of scenarios compatible with what Alice perceives in w_0; W' is the set of scenarios compatible with what Alice believes in w_0; $marg(\mathbf{I}) = W$; $marg(\mathbf{I}') = W'$; and $w_0 \in W \cap W'$. Generally, it might well happen that the sets W and W' are distinct or even disjoint. First, examples like the

[5] For the correspondence between Russell and Frege about the question of whether Mont Blanc is a component of the proposition that Mont Blanc has such-and-such height, see [30].

Müller-Lyer illusion show that an agent can perceive p and believe not-p.[6] However, let us suppose for simplicity that Alice's perceptual beliefs (beliefs triggered by her perceptual situation) are determined by her perception in the straightforward sense that the set of scenarios compatible with Alice's perceptual beliefs is the same as the set W of scenarios compatible with her perception. Second, Alice's perceptual beliefs might not exclude scenarios that are indeed excluded by her overall body of beliefs: we can have $W' \subsetneq W$. If Alice is watching Mont Blanc at t, her perceptual information leaves open the possibility that the world began five minutes before t and that there are opossums in South Africa. Yet, unless Alice is prone to serious skepticism, the set W' contains only worlds with a considerably longer history and may well exclude South African opossum scenarios, as well, provided that Alice has interest in zoology. Now, if we adopt an internal and intrinsic viewpoint on objects of Alice's thought, we may take $\langle W, \mathbf{I}, w_0\rangle$ and $\langle W', \mathbf{I}', w_0\rangle$ to be the singular contents of Alice's beliefs. If $\mathbf{J}$ is the physical entity Mont Blanc, a necessary condition for these contents being about Mont Blanc is that $\mathbf{I}(w_0) = \mathbf{J}(w_0) = \mathbf{I}'(w_0)$. Mont Blanc is the material object of both contents, although one or both intentional objects $\mathbf{I}$ and $\mathbf{I}'$ might fail to represent Mont Blanc in the sense of Sect. 4.8. In practice, neither $\mathbf{I}$ nor $\mathbf{I}'$ can be identical to $\mathbf{J}$. It is also unlikely that $\mathbf{I}$ and $\mathbf{I}'$ are identical. This follows automatically if $W' \subsetneq W$, since in that case there is a world in which $\mathbf{I}$ is realized but $\mathbf{I}'$ is not. Further, even if the sets W and W' were equal and both $\mathbf{I}$ and $\mathbf{I}'$ were representations of $\mathbf{J}$, there could be worlds $w \in W$ in which $\mathbf{I}(w) \neq \mathbf{I}'(w)$. By the clause (vi) of Sect. 4.8, both $\mathbf{I}$ and $\mathbf{I}'$ must coincide with $\mathbf{J}$ in some world of every world type in W. However, this condition is satisfied if for every $w \in W$ there are $u, u' \in W$ with $u \cong w \cong u'$ such that $\mathbf{I}(u) = \mathbf{J}(u)$ and $\mathbf{I}'(u') = \mathbf{J}(u')$. We can still have $\mathbf{I}(u) \neq \mathbf{I}'(u)$, since it is not required that $u = u'$. If so, Alice does not think of $\mathbf{I}$ and $\mathbf{I}'$ as identical, but neither does she think that they are distinct: she has no definite opinion about their identity.

The analysis above can be related to Recanati's theory of singular thought. Recanati distinguishes *modes of presentation* of two kinds: *non-descriptive* modes are ways an object is given to an agent in experience, whereas *descriptive* modes are ways an object is indirectly given via properties it uniquely instantiates [103, p. 16]. He takes the proposition [The mountain I am looking at is less than 4000 meters high] to be singular and to utilize a non-descriptive mode of presentation based on the agent's perceptual relation to Mont Blanc, while the proposition [Mont Blanc is 4000 meters high] is supposed to employ a descriptive mode of presentation and not to be singular. According to Recanati's view, an agent can grasp the latter but not the former proposition without being acquainted with Mont Blanc. In his analysis, non-descriptive modes of presentation are *mental files*. These are understood as being, typically, results of storing information about objects of acquaintance.[7] Mental files are 'about objects'. They refer to an object—but the reference is not accomplished descriptively, by virtue of properties the agent considers the object as having, but

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[6] For the Müller-Lyer illusion, see, e.g., [17, p. 150].

[7] Acquaintance in this sense is not triggered only through perception but, more generally, via causal chains allowing information transmission. See Lewis [79, pp. 380–1], Recanati [103, p. 34].

through ‘acquaintance relations’ on which the file is based. These relations are taken to be *epistemically rewarding*: by being thus related to an object, the agent gains information about the object [103, p. 35]. Yet, a file may misrepresent its object: the object need not satisfy the information stored in the file. Reference is accomplished by the fact that the file is generated in a certain way. Recanati is even prepared to accept acquaintanceless mental files. As he puts it [ibid. p. 164], to open a mental file, actual acquaintance is not necessary; expected acquaintance suffices. However, he takes it that unless a mental file achieves reference to an object, it does not allow us to grasp a singular thought-content [ibid. pp. 164, 170]. For him, singular thoughts are existence-dependent, although mental files are not.

By Recanati’s standards, a proposition is singular and pertains to a physical object o if it employs a mental file that refers to o. Referring to o is not an intrinsic feature of the mental file: the file does not determine o as its referent. For me, a content $\langle W, \mathbf{J}, w_0\rangle$ is singular and pertains to a physical object $\mathbf{I}$ if the intentional object $\mathbf{J}$ represents $\mathbf{I}$. Merely by inspecting the situated content $\langle W, \mathbf{J}, w_0\rangle$, we cannot tell which attitude or experience has generated the set of worlds W. Neither can we tell on that basis whether $\mathbf{J}$ represents a physical object or whether it just accidentally behaves in a sufficiently orderly way, thereby giving the impression that it does. Even if there is such a physical object, $\mathbf{I}$, the intentional object need not (and in practice cannot) represent $\mathbf{I}$ as it is; some residue of indeterminacy is unavoidable.[8]

In my analysis, a singular content need not be triggered by an acquaintance relation—it need not be based on an agent’s direct cognitive relation to her environment. By contrast, it is essential to singular thoughts in Recanati’s sense to have as their component a mental file thus triggered, or at least this is how singular thoughts typically behave in his analysis.[9] Also, the intentional object of a singular content need not represent a physical object. My framework does not motivate any distinction among intentional objects corresponding to Recanati’s distinction between descriptive and non-descriptive modes of presentation. All contents involving at least one world line are object-directed. Some contents with a sole intentional object are singular—those whose intentional object happens to be specific (indeterminate to a sufficiently small degree). Finally, the intentional object of some singular contents represents a physical object. Undoubtedly, in many important cases, singular contents involve an acquaintance-based intentional state, and they are indeed directed on a physical object. However, the structure of our intentional states is not altered when they are not based on acquaintance, nor when they are not singular. They can always be represented as situated contents $\langle W, \mathfrak{J}, w_0\rangle$, consisting of a set of worlds structured by a number of intentionally individuated world lines relative to a fixed world in which the agent is located.

[8]Given the analysis presented in Sect. 4.8, $\mathbf{J}$ cannot represent $\mathbf{I}$ as satisfying a predicate that $\mathbf{I}$ does not in fact satisfy—but there will be many predicates P such that $\mathbf{J}$ fails to represent $\mathbf{I}$ as satisfying P, although, in fact, $\mathbf{I}$ satisfies P.

[9]For exceptions, see Recanati’s discussion on the possibility of singular thought without acquaintance [103, Chap. 13, esp. pp. 171–2].

6.4 Comparison with Crane's Account

Crane distinguishes three general characteristics of intentional states [21, Chap. 4]: they have an *object*, a *content*, and a *mode*. He maintains that every intentional state is about something (its object), represents its object in a certain way (its content), and has its characteristic psychological type (its mode) in accordance with which it is directed on its object, via its content. As examples of states that have an object, Crane lists thoughts, hopes, desires, wishes, and fears; sense experiences and perceptions; intentions, decisions, and actions; emotions like love, hate, and disgust; and bodily sensations and moods [21, p. 90]; cf. [19]. He distinguishes between three categories of intentional modes: propositional attitudes, relational intentional states, and object-directed non-propositional non-relational intentional states reported by intensional transitive verbs [21, p. 103]. Examples of relational intentional states are seeing, noticing, and loving. They are states directed upon objects under aspects, and they imply the existence of their relata. They do not have propositional contents. The following intentional transitive verbs serve to express object-directed non-propositional non-relational intentional states: *imagine*, *write (about)*, and *believe (in)*; *anticipate*, *expect*, *fear*, and *plan*; *prefer*, *want*, and *hope (for)*; *worship* and *respect*; and *need*, *require*, and *deserve*. All intentional states have a content, but insofar as the second and third categories cannot be reduced to the first, there are intentional states whose content is not propositional. The states of the third category are important for my purposes for the same reason they are important for Crane: they provide 'cases of superficially relational structures which cannot really be so' [ibid. p. 116]. In Sect. 6.7, I take up the semantics of intensional transitive verbs in order to illustrate how my framework deals with intentional states that have a non-existent object.

In my logical analysis, modes are syntactically represented by modal operators, semantically interpreted in terms of accessibility relations. Construing them in this way does not mean that I subscribe to *propositionalism*, the position according to which contents of intentional states are propositional.[10] The content of a state is propositional only in special cases; generally, it involves not only a world representation but also object representations. My analysis does *not* predict that all intentional states are propositional. It allows intentional states representing a specific physical object and ones with a non-existent or indeterminate intentional object.

Crane takes the subject's point of view—subject's representation of the world—to be a basic psychological notion. According to him, we must accept representation as a basic, indefinable feature of our psychological makeup [21, pp. 115–6]. Now, even if representation was a basic notion, it does not follow that so is the notion of two representations having the same object. I have gone further than just saying

[10] For an argument against propositionalism, see M. Montague [90]. Her strategy is to consider different variants of propositionalism and to show that there are irreducibly 'objectual' attitudes—i.e., attitudes that have a non-propositional object and that cannot be accounted for in terms of any variant of propositionalism. She does not develop a positive account of objectual attitudes and in particular does not formulate a systematic analysis of the difference between physical and intentional objects.

that representation is something basic. At the abstract level of semantic contents, I have attempted to articulate what it is for an intentional state to represent.[11] I take such intentional states to be representational in an intrinsic sense, and I take them to involve, generally, both a world representation and a number of object representations. The former need not accurately represent the actual world, and the latter need not represent specific physical objects or even be actually realized. When Crane speaks of representations, his comments are meant to apply to concrete phenomenological contents. (For the distinction between semantic and phenomenological contents, cf. Sect. 3.2.) Representations, in my sense, are the result of a theoretical analysis of types of representations in Crane's sense. Unlike the latter representations, the former are abstract, and they are not psychologically primary. Such abstractions are unavoidable: as a matter of fact, already the notion of two Cranean representations having the same object requires moving away from the concrete reality of an experiential episode. For me, the notion of modal unity is a basic notion. Worlds are one variety of modal unities, world lines another. Not all world lines correspond to representations in the sense in which Crane uses this word. Intentionally individuated world lines do; physically individuated world lines do not.[12] A content, in my semantic sense, is always relative to an agent and to a fixed current world, though not for this reason private or unrepeatable. Its world representation depends on the mode of the state in Crane's sense. A world representation is never alone capable of representing intentional objects. To this end, object representations are needed, as well.

Crane distinguishes three features of representations in terms of which we may usefully discuss intentional contents: *aspect*, *accuracy*, and *absence*. He takes it that any representation has at least one of these characteristics. In my framework, these features are manifested as follows. Where Crane speaks of representing an intentional object under an aspect, I have spoken of ascribing predicates to an intentionally individuated world line in the sense of intentional predication. From my viewpoint, it makes sense to speak of accuracy of an object representation in connection with intentional states having a material object. The accuracy of a representation must be judged by comparing the set X_I of predicates intentionally ascribed to the intentional object of the state with the set X_P of predicates that the material object of the state

[11] Precisely because my goal is merely to discuss the issue of representation at the level of semantic contents, what I say is *not* meant as any sort of replacement of Crane's theory. I am expressly not proposing an analysis of the representational nature of phenomenological contents. What I say is not intended as a psychological account of representational mental states.

[12] Actually, according to the *idealist* variant of the transcendental interpretation of world lines discussed in Sect. 2.2, physically individuated world lines *are* also representations, though not representations in Crane's sense: on this view, they are appearances. For Kant, those representations that conform to empirical causal laws yield knowledge of actual things. Representations we encounter in dreams or in illusory experiences do not meet this criterion. (Cf. A218/B266, A225–6/B272–4, B278–9, A376–7; see also Pereboom [93].) One way of trying to find a place for the distinction between physical and merely intentional objects in Kant's philosophy, staying within the realm of representations, would be to identify it with the distinction between representations congruent with empirical laws and representations that are products merely of imagination.

satisfies in the sense of physical predication. The more predicates of X_P are covered by X_I, the more accurate is the representation. Another factor affecting the accuracy is the behavior of the intentional object in worlds in which it fails to satisfy one or more of the predicates belonging to X_P: the more the relevant realizations of the intentional object diverge from the material object of the state, the less accurate and the more indeterminate is the representation. Where Crane speaks of representations manifesting absence, I speak of intentionally individuated world lines not being actually realized.

Crane takes it to be a fundamental feature of intentionality that the *same* object of thought can be represented in *different ways* [21, p. 97]: different intentional contents can be associated with the same intentional object. That is, according to him, two representations can have the same object while differing in the aspects under which they represent the objects. This means, positively, that the object of a representation can be represented in many ways and, negatively, that any particular way of representing the object excludes other ways of representing it. As examples, Crane mentions imagining his mother listening to the radio and imagining her frying onions. It follows that aspect-boundedness entails indeterminacy: if an intentional object is represented as P but neither as Q nor as not-Q, it is indeterminate with respect to the predicate Q. In my framework, two intentional states can well involve different object representations and still have the same material object. Different selections of predicates of a fixed physical object give rise to different characterizations of an intentional object. My analysis is, then, in conformity with what Crane says of intentionality—as long as we are dealing with existent intentional objects. By his own standards, Crane should be reluctant to extend his claim to non-existent intentional objects. After all, as I remarked in Sect. 3.5, he maintains that the idea of 'same non-existent object' cannot be taken at face value, arguing that mere intentional identity amounts to similarity of representation [ibid. p. 164]. If so, we cannot frame the idea of the same *non-existent* intentional object being represented in different ways without suitably paraphrasing the associated idea of sameness. By contrast, my framework allows formulating a notion of 'same non-existent object': all world lines correspond to partial functions with worlds as arguments and local objects as values, the notion of 'same partial function' being perfectly well defined. Since there is no physical object to which a non-existent intentional object could be compared, its status as an object is very strongly tied to the way in which it is characterized.[13] This fact may render it less likely that we could actually be in a position to say that mutually grossly divergent sets of predicates characterize the same intentional object on two occasions. Even if this was so, it does not follow that we cannot speak of the same non-existent object characterized in some *fixed way* by two agents on a given occasion or by one and the same agent on two occasions, instead of speaking of objects present in the two cases as merely being similar.

[13]The object-status of a non-existent intentional object is, however, not *exclusively* tied to its characterizing properties: as world lines, intentional objects of an agent are dependent on the set of worlds compatible with the agent's intentional state, and this set is not determined by the predicates that an intentional object satisfies in the sense of intentional predication.

Concerning accuracy, Crane's idea is that a representation is (more or less) accurate if the way it represents its object is (more or less) the way the object is. His example of imagining an inexpensive bottle of champagne [21, p. 98] is not a particularly good one, since the imagining does not in this case pertain to any particular bottle. The representation is inaccurate, because *there is no* inexpensive bottle of champagne; this is an inaccurate general representation. A more interesting example would be an existing object represented in a certain way—say, in perceptual experience. Illusions provide a clear-cut example of inaccurate representations of this kind. More generally, the unavoidable indeterminacy of representation automatically induces inaccuracy: even if an intentional state has a material object (which satisfies all predicates characterizing the intentional object of the state), the intentional object is an inaccurate representation due to all predicates relative to which it is indeterminate. Even perceptual experience that represents a specific physical object in the strong sense discussed in Sect. 4.8 need not (and in practice cannot) represent its object with full accuracy, in the sense of characterizing the intentional object of perception as having exactly those properties the material object has—just like propositional knowledge need not (and in practice cannot) represent the world as making true exactly those propositions that are true in the actual world.

Contents in my abstract sense are structures $\langle W, \mathbf{J}_1, \ldots, \mathbf{J}_n, w_0 \rangle$, where $W = R(w_0)$ and the $\mathbf{J}_i$ are the world lines belonging to the set $\mathcal{I}^{\alpha}_{w_0}$ for some agent α and accessibility relation R; see Sect. 4.7. They are representations in an intrinsic sense: they do not describe something as representing something. The set W need not contain the actual world w_0. The intentionally individuated world lines $\mathbf{I}_i$ need not represent specific physical objects; they need not even be realized in w_0. The usefulness of contents for semantic theorizing does not depend on there being correlations between intentional and physical objects. Yet, such contents have *accuracy conditions*. The accuracy of a content depends on its world representation and on those of its object representations that are realized in the actual world. A content may be accurate to a greater or lesser degree, but in order to be at least more or less accurate, a certain minimal requirement must be met. Consider a content $C = \langle W, \mathbf{J}_1, \ldots, \mathbf{J}_{m+k}, w_0 \rangle$, where $\mathbf{J}_i$ is realized in w_0 iff $1 \leq i \leq m$. Thus, the $\mathbf{J}_i$ with $m + 1 \leq i \leq m + k$ are non-existent intentional objects. The content C is 'more or less accurate' if $w_0 \in W$ and there are physical objects $\mathbf{I}_1, \ldots, \mathbf{I}_m$ such that $\mathbf{J}_i$ represents $\mathbf{I}_i$ in the sense of Sect. 4.8, whenever $1 \leq i \leq m$. The degree of accuracy of the content is proportional to the degree of accuracy of the world representation W and to the degree of accuracy of the intentional objects representing physical objects. Note that if $m = k = 0$, the content is propositional and the condition for its being more or less accurate reduces to the mere requirement that $w_0 \in W$. If C is more or less accurate, none of the world lines $\mathbf{J}_1, \ldots, \mathbf{J}_m$ can be a representation of a non-existent object, since each of them represents a physical object and therefore, by clause (iv) of Sect. 4.8, they all are realized throughout W and therefore, in particular, in w_0. Further, if C is more or less accurate, it *accurately* represents the world lines $\mathbf{J}_{m+1}, \ldots, \mathbf{J}_{m+k}$ as being non-existent: the world representation W contains a world (in any event, the world w_0) in which they are not realized. That is, C may be accurate in representing an intentional object as non-existent—as not being realized throughout the set W. It would indeed be too restrictive to require that a content can accurately represent only what exists: an agent can, for example, accurately think of a winged horse as non-existent. Yet, it must be observed that in connection with non-existent intentional objects, it does *not* generally make sense to speak of pleonastic properties being accurately represented. Non-existence is the only exception.

Obviously, there is no neat distinction between a representation having a non-existent object and a representation having an existent but inaccurate object: if we start with a more or less accurate representation of an object present to our senses and imagine gradually modifying it qualitatively, the representation becomes first illusory and eventually ceases to be a representation of any physical object present in the perceptual situation. In Crane's analysis, there are two cases in which a representation manifests absence: not only when the object is non-existent but also when it is indeterminate.[14] I agree that terms of 'real relations' are physical objects and that real relations are existence-entailing: when such a relation prevails, its terms exist. I also agree that no physical object can be indeterminate (in the sense of physical predication) and that quantification over physical objects is ontologically committing. By contrast, according to my analysis, indeterminate intentional objects certainly can manifest absence, but they need not do so. Actually, if no indeterminate intentional object could exist in the sense of being realized (i.e., lack absence in Crane's terminology), then an intentional state could never have a material object, since our object representations are always indeterminate to some degree. It would be a result of erroneous reasoning to conclude from the fact that physical objects are subject to the principle of complete determination that they cannot be represented by indeterminate intentional objects.[15] In my framework, there is no contradiction in saying that an indeterminate intentional object represents a physical object, while according to Crane's account, one could not coherently say so. Namely, the standards of characterization used when talking about intentional objects are different from the standards applied in connection with physical objects. An intentional object being indeterminate in the sense of intentional predication is in no way contradictory with the intentional state's material object being determinate in the sense of physical predication.

6.5 Talking About Objects of Thought

If we ask how different types of objects of thought can be linguistically described, it is evident that descriptions of propositions are sentences, descriptions of intentional objects are formulas of one free variable, and descriptions of pluralities of

[14] Crane holds that indeterminate intentional objects and non-existent intentional objects both are examples of objects of intentional states that do not exist in any sense; see [21, p. 130]. Therefore, one would expect him to classify indeterminate intentional objects as a subvariety of non-existent intentional objects. Yet, Crane systematically treats these two classes of intentional objects independently of each other.

[15] Cf. the comments about formula (21) in Sect. 4.8.

intentional objects are formulas of several free variables. When we are talking about their characterizing properties, intentional objects can be described by ascribing predicates to them under the intentional mode. It was noted in Sect. 4.2 that ascribing a predicate P to a world line $\mathbf{I}$ in w_0 under the intentional mode means affirming that $w_0, x := \mathbf{I} \models \Box[x = x \to P(x)]$. Actually, intentional predication can be characterized in terms of the relations of local and uniform support. It will facilitate discussion to have available a terminology that allows an easy reference to the converses of these relations. I will say that ϕ *is true of* C iff $C \Vdash_{loc} \phi$ and that ϕ *describes* C iff $C \Vdash_{uni} \phi$. (That ϕ describes C does not suggest exhaustiveness: it only means that ϕ is one of the formulas that C uniformly supports.) Recalling the definitions of local and uniform support, it follows, then, that a formula $\phi(x_1, \ldots, x_n)$ *describes* a situated content $C = \langle W, \mathbf{J}_1, \ldots, \mathbf{J}_n, w_0\rangle$ iff $w, \mathbf{J}_1, \ldots, \mathbf{J}_n \models \phi$ for all $w \in W$. That is, in this case, ϕ can be seen as providing a partial description of the (unsituated) content $\langle W, \mathbf{J}_1, \ldots, \mathbf{J}_n\rangle$. Further, the formula $\phi(x_1, \ldots, x_n)$ is *true of* C iff $w_0, \mathbf{J}_1, \ldots, \mathbf{J}_n \models \phi$. Here, the formula serves to describe the specific evaluation context $\langle w_0, \mathbf{J}_1, \ldots, \mathbf{J}_n\rangle$ consisting of a single world and a number of world lines. Since ϕ may involve modal operators, its being true of C may depend on many other worlds besides w_0. Now, by Fact 2.2, we have the following equivalences: P is ascribed to $\mathbf{I}$ in w_0 under the intentional mode iff we have $w_0, x := \mathbf{I} \models \Box[x = x \to P(x)]$ iff the corresponding situated content $\langle R(w_0), \mathbf{I}, w_0\rangle$ *locally* supports the formula $\Box[x = x \to P(x)]$ iff the formula $\Box[x = x \to P(x)]$ is *true of* the situated content $\langle R(w_0), \mathbf{I}, w_0\rangle$ iff the situated content $\langle R(w_0), \mathbf{I}, w_0\rangle$ *uniformly* supports the formula $x = x \to P(x)$ iff the formula $x = x \to P(x)$ *describes* the situated content $\langle R(w_0), \mathbf{I}, w_0\rangle$.

If the content $\langle W, \varnothing, w_0\rangle$ represents the world as rainy (i.e., if it is raining in all worlds that belong to W), the sentence 'It is raining' describes the content $\langle W, \varnothing, w_0\rangle$. If Alice thinks of a pink elephant and her object of thought is $\mathbf{I}$ with $W \subseteq marg(\mathbf{I})$, the formula '$x$ is a pink elephant' describes the content $\langle W, \mathbf{I}, w_0\rangle$. If Alice neither believes the elephant to have blue eyes nor believes it not to have blue eyes, this is a feature of the intentional object we might wish to capture by a linguistic description, but it is not a sort of property we can ascribe to $\mathbf{I}$ by saying that it satisfies a certain formula $\chi(x)$ in all worlds $w \in W$. The format offered by the relation of uniform support (i.e., $\Vdash_{uni}$) is not useful here. This is because in order to talk about an intentional object, we may need to speak of properties relative to which it is indeterminate, in addition to its characterizing properties. In terms of the relation $\Vdash_{loc}$, we have no problem in expressing what we want.[16] The formula

$$\Box(x \text{ is a pink elephant}) \wedge \Diamond(x \text{ has blue eyes}) \wedge \Diamond\neg(x \text{ has blue eyes})$$

is true of the content $\langle W, \mathbf{I}, w_0\rangle$. Further, if Alice does not seriously believe that her intentional object exists, there are worlds in W in which $\mathbf{I}$ is realized and others in which it is not. The relevant feature of the content $\langle W, \mathbf{I}, w_0\rangle$ can be characterized by saying that the following formula is true of $\langle W, \mathbf{I}, w_0\rangle$:

[16] Cf. formulas (3) of Sect. 4.3 and (12) of Sect. 4.5 for talking about indeterminacy and formula (18) of Sect. 4.7 for expressing that an intentional object is represented as non-existent.

$$\Box(x = x \to x \text{ is a pink elephant}) \wedge \Diamond x = x \wedge \Diamond\neg x = x.$$

Note that we have $\langle W, \mathbf{I}, w_0\rangle \Vdash_{loc} \Diamond\neg x = x$ iff $\langle W, \mathbf{I}, w_0\rangle \nVdash_{uni} x = x$. Note further that this negative condition is much weaker than the positive condition $\langle W, \mathbf{I}, w_0\rangle \Vdash_{uni} \neg x = x$, according to which $\mathbf{I}$ is not realized in *any* world $w \in W$.

One can object to the idea of equating contents of thought with contents generated by formulas on many counts—notably because contents generated by formulas are situated contents in the sense of Definition 2.3, and such mathematical structures with all likelihood lack the sufficient psychological reality required for literally *being* contents of thought in the phenomenological sense. Situated contents—formulated as triples consisting of a set of worlds, a tuple of world lines, and a world—are mathematical representations of contents in the *semantic* sense. Semantic contents are not epistemologically primary. However, the epistemologically primary contents—phenomenological contents—encode information that can be rendered explicit. Semantic contents render explicit what several phenomenological contents may have in common: semantic contents are what may be shared by two agents by virtue of having the phenomenological contents they have. However, what I have wanted to stress by the above considerations is that even if contents of thought literally were situated contents, it would not be enough to talk about them in terms of formulas they uniformly support—i.e., as contents generated by certain formulas. As just observed, we may need to complement such descriptions by specifying formulas true of the content.

The formula $x = x$ describes the content $\langle W, \mathbf{I}, w_0\rangle$ iff $\mathbf{I}$ is realized throughout W iff the formula $\Box x = x$ is true of $\langle W, \mathbf{I}, w_0\rangle$. The formula $x = x$ ascribes no predicates to the value of x. In a fixed world, its role is to state that the value exists (i.e., is realized). Semantically, the formula $x = x$ behaves much like a non-empty singular term, with the exception that instead of presupposing that its semantic value exists, it states that its semantic value exists. These observations motivate slightly generalizing the syntax of L: I will allow *variables* x as an additional form of *formulas*. From now on, I will consider not only expressions of the forms $Q(x_1, \ldots, x_n)$ and $x_1 = x_2$ but also expressions of the form x with $x \in Var$ as atomic L-formulas. By stipulation, $M, w, g \models x$ iff $g(x)$ is realized in w, whence the formulas x and $x = x$ are logically equivalent. Just like the formula x describes the content $\langle W, \mathbf{I}, w_0\rangle$ iff $\mathbf{I}$ is realized throughout W, the formula $x_1 \wedge \cdots \wedge x_n$ describes the content $\langle W, \mathbf{I}_1, \ldots, \mathbf{I}_n, w_0\rangle$ iff all intentional objects $\mathbf{I}_1, \ldots, \mathbf{I}_n$ are realized throughout W.

Syntactically, variables are, at the same time, formulas and singular terms. In this generalized setting, variables have satisfaction conditions. Yet, variables stand for world lines. While the maneuver of accepting x as a formula may appear strange at first sight, a similar approach is commonplace in hybrid logic, in which certain syntactic items have semantically a double role as names and formulas. A special class of proposition symbols called *nominals* is distinguished, with the characteristic feature that unlike arbitrary proposition symbols, nominals are true at exactly one world and can therefore be considered as names of worlds. The idea stems from Prior, who proposed to avoid mentioning instants in our language by construing them as propositions of a special kind. Instead of qualifying instants by predicates, we may then qualify propositions by modalities.[17] Prior showed that by changing the informal interpretation of our modal-like language, we can similarly attempt to avoid ontological commitments to worlds (modal logic) and even to selves and other bona fide individuals (egocentric logic). In egocentric logic, the difference between individuals and propositions is not categorical, since individuals are treated as conjunctions of (egocentric) propositions. In my analysis, a proposition symbol true in exactly one world could be understood as standing for a *set of local objects* (namely, a single world), just as a variable can be understood as standing for a *set of local objects* (namely, a world line). Sets of the latter kind are not a special case of sets of the former variety: distinct elements of a world line belong to distinct worlds.

Since x and $x = x$ are logically equivalent, we gain nothing in expressive power by having available formulas of both forms. However, the augmented syntax allows us to syntactically represent in a uniform manner cases in which an agent has a proposition as an object of thought and cases in which the object of thought is an intentional object. Examples of cases of the latter type are sentences using such intensional transitive verbs as *see* and *think of*. These may be compared with extensional verbs taking a direct or prepositional object, such as *shake hands with* or *hit*.[18] While the expressions 'Alice shakes hands with x' and 'Alice hits x' have the form

1. $R(\mathsf{a}, x)$,

where R is a binary predicate symbol, I claim that the expressions 'Alice thinks of x' and 'Alice sees x' have the form

2. $\Box_{\mathsf{a}} x$.

Whereas the expression 'Alice hits x who is a philosopher' has the form

3. $R(\mathsf{a}, x) \wedge P(x)$,

the expression 'Alice thinks of x who is a philosopher' has a reading of the form

4. $\Box_{\mathsf{a}} x \wedge \Box_{\mathsf{a}} P(x)$;

this is the reading according to which Alice's intentional object is characterized as a philosopher, so that the predicate '— is a philosopher' is ascribed to the intentional object under the intentional mode.[19] The two occurrences of $\Box$ in (4) require

[17] See Prior [98, Chaps. 11, 12]. For hybrid logic, see, e.g., [3].

[18] Grammatically, transitive verbs are verbs capable of combining two noun phrases into a sentence without using auxiliary material, such as prepositions. Logically, it is uninteresting in which way an expression of a binary relation is syntactically realized; all that matters is the logical type of the relation. In a logical analysis, then, all extensional verbs are treated on a par (whether they are transitive or intransitive), as long as they serve to produce sentences out of two noun phrases.

[19] It should be noted, first, that the expression 'Alice thinks of x who is a philosopher' admits an alternative reading—namely, a reading of the form $\Box_{\mathsf{a}} x \wedge P(x)$, according to which the predicate '— is a philosopher' is ascribed to the intentional object $\mathbf{I}$ under the *physical* mode. If the predicate P is atomic, this entails that the intentional object is actually realized, this realization $\mathbf{I}(w_0)$ being P. Moreover, if there is a physical object $\mathbf{J}$ such that $\mathbf{I}(w_0) = \mathbf{J}(w_0)$, then $\mathbf{J}$ is the material object of Alice's intentional state, and the formula $\Box_{\mathsf{a}} x \wedge P(x)$ entails that this material object is P in w_0.

some words of explanation. The first occurrence of $\Box$ in (4) represents the relevant intensional transitive verb. Its second occurrence represents an intentional predication (expressed in the relative clause). The formula (4) has the equivalent forms $\Box_{\mathsf{a}}(x \wedge P(x))$ and $\Box_{\mathsf{a}}P(x)$. In fact, we can rarely speak of intentional objects without talking about properties they are represented as having, and as soon as we state a formula $\Box_{\mathsf{a}}\phi(x)$, where ϕ is existence-entailing, it becomes superfluous to state $\Box_{\mathsf{a}}x$. Nevertheless, it is convenient, at least for the purposes of linguistic theorizing, to have available in the syntax a way of forming schematic forms of such intensional constructions as *think of* and *see*, in abstraction from the predicates ascribed to the corresponding grammatical objects.[20]

The proposal to use formulas $\Box_{\mathsf{a}}x$ to represent the form of certain intensional transitive verbs requires comments. To begin with, I do not pretend to provide a *linguistic* analysis of constructions involving intensional verbs. In terms of the semantic conceptualizations developed in this book, it is possible to represent truth-conditions of certain natural-language sentences. On the other hand, there are many linguistically important syntactic and morphological issues that cannot be addressed in terms of my truth-conditional analysis. By itself, this fact does not tell against my semantic framework. First, the goal of my book is not to analyze fragments of natural language but to use logic to address philosophical problems concerning objects in many-world settings. I merely take up certain issues inspired by natural-language semantics to illustrate the semantics of my logical language. Second, it is methodologically reasonable to limit attention to questions of certain well-defined sorts. Even if one can logically analyze phenomena that involve finer distinctions than those that can be truth-conditionally provided (e.g., hyperintensionality and fine-grained modalities), it is preferable to begin by getting clear about more fundamental notions such as cross-world identity. Third, even the most thoroughgoing logical analyses risk being linguistically irrelevant, because linguistic constructions are often conditioned by logically unmotivated but empirically real restrictions—for example, interactions among embedded grammatical tenses in languages such as English are in fact much more limited than what they combinatorially speaking could be.[21] Fourth, when

(Footnote 19 continued)
A further thing to note is that there are other natural-language expressions besides 'Alice thinks of x who is a philosopher' whose form could in suitable circumstances be represented using (4). If it is clear from the context that Alice's intentional state has a certain person as its material object, then (4) can be understood as affirming that Alice has a thought pertaining to this physical object of whom Alice thinks *that* he or she is a philosopher. In such a case, the formula $\Box_{\mathsf{a}}P(x)$ serves to represent a *that*-clause (*thinks of ... that ... is*), instead of modifying the grammatical object of an intensional transitive verb (*thinks of ... who*). This phenomenon is discussed in Sect. 6.6.

[20]The semantics of my modal language renders the two formulas $\Box_{\mathsf{a}}x \wedge \Box_{\mathsf{a}}\phi(x)$ and $\Box_{\mathsf{a}}(x \wedge \phi(x))$ logically equivalent. What is more, if ϕ is existence-entailing, according to my semantics they both are logically equivalent to the formula $\Box_{\mathsf{a}}\phi(x)$. Given that $P(x)$ is atomic and therefore existence-entailing, it would, consequently, be meaningless to suggest that one of the three formulas $\Box_{\mathsf{a}}x \wedge \Box_{\mathsf{a}}P(x)$, $\Box_{\mathsf{a}}(x \wedge P(x))$, and $\Box_{\mathsf{a}}P(x)$ is better than the others as a representation of the logical form of the expression 'Alice thinks of x who is a philosopher'—the criterion of the success of representation being the capacity to capture the relevant satisfaction condition.

[21]For a discussion, see, e.g., Hornstein [53].

a logical analysis of a natural-language construction is possible, the goal of the analysis is not to save the linguistic appearances but to discern the logical form of the construction. The linguist must pay attention to the rules allowing to generate such expressions, but the logician with an interest in natural languages only needs to clarify, say, how the linguistic contexts involving such expressions as ‘the present king of France’ or ‘the book John needs to write’ can be suitably paraphrased.[22] In sum, all I wish to accomplish in the remaining sections of this chapter is to give reasons to believe that the conceptualizations used in my framework, notably the notion of world line, help us to capture truth-conditions of certain statements about intentional states.

A lengthier discussion is needed to clarify further aspects of my logical analysis of intensional verbs. First, I wish to point out that the commonplace idea according to which a modal-logical framework is, by its nature, forced to treat intentional states in terms of *that*-clauses is based on a hasty generalization. Second, I address the question of what is characteristic of those intensional verbs that are analyzable in the way I propose. Third, I relate my analysis to Moltmann’s distinction between intentional and intensional verbs [88], and I comment on her claim that intentional objects cannot be understood as ‘variable objects’ [89]; in Moltmann’s analysis, variable objects are a generalization of what Fine calls variable embodiments. Formally, the notion of variable embodiment is close to the notion of world line, as explained in Sect. 2.7.3. The first of these three points is discussed in the remainder of the present section, while the remaining two points are explored in Sects. 6.6 and 6.7, respectively.

In text book presentations of different variants of modal logic, the semantics of the operator $\Box$ is informally explicated by telling that it can be used for symbolizing such expressions as ‘it is necessary that’, ‘it will always be the case that’, ‘agent α believes that’, ‘agent α knows that’, or ‘agent α perceives that’.[23] If one adopts a model-theoretic approach—instead of speaking of how to translate logical expressions into natural language—the conventional wisdom leads us to say that $\Box$ stands for a map from propositions to propositions (propositional modal logic) or, more generally, from propositional functions to propositional functions (FOML).[24] By a careless generalization, one may come to believe that it is somehow essential to the semantics of $\Box$ that it is a vehicle for logically expressing what in natural language is expressed using a (modally qualified) *that*-clause and that in the special case of epistemic logic its role is invariably to express a propositional attitude. By a further ill-judged generalization, one may come to think that other intentional states admitting an analysis in terms of modal logic—such as perceptual experience—must likewise be understood as propositional attitudes. Such generalizations are groundless: the semantic contribution of $\Box$ is restricted to its being a universal quantifier

[22] For definite noun phrases with a relative clause containing an intensional verb, such as *the book John needs to write*, cf. Sect. 6.7.

[23] Cf., e.g., Gamut [31, p. 16], Garson [32].

[24] If we wish to speak of propositional functions in connection with FOML, it cannot be required that all arguments of such a function belong to the domain of the same world. Instead, these arguments must be allowed to come from arbitrary domains.

over certain worlds. It is meaningful to apply $\Box$ syntactically to any expression to which a satisfaction condition can be assigned. In my framework, even each variable has a satisfaction condition. Since variables are singular terms—we make reference to world lines by means of variables—it follows that in the syntax, the complement of $\Box$ can be a singular term. This feature of my logical language allows it to represent a much larger variety of natural-language sentences than those representable in FOML. Formulas of the form $\Box_{\mathsf{a}} x$ are not naturally read according to the *that*-clause paradigm but on the model of intensional transitive verbs: a *thinks of* x, a *sees* x.

6.6 Robust Intensional Verbs

Priest [95, pp. 7–8] discusses such intensional verbs as *seek*, *worship*, *fear*, *believe*, and *dream*, stressing the syntactic distinction between intensional verbs with noun-phrase complements (i.e., intensional transitive verbs) and intensional verbs with sentential complements.[25] He calls verbs of the former type 'intentional predicates' and those of the latter type 'intentional operators'. He points out that some intensional verbs are syntactically ambiguous, in that they can be construed both as predicates and as operators. As an example, Priest mentions *fear*: 'Alice fears that Bob comes', 'Bob fears Alice'. Other examples would be *think*, *see*, and *know*: not only can we say 'Alice thinks that it is raining', 'Alice sees that the bus is coming', and 'Alice knows that Bob will be late' but equally well 'Alice thinks of Bob', 'Alice sees Bob', 'Cecile sees a lion', 'I know her', and 'I know someone who can help'.[26] There are also syntactically unambiguous intensional verbs: for example, *dream* and *hope* can only be read as operators, while *worship* and *seek* must be construed as predicates. Priest mentions in passing the question of whether, in connection with syntactically ambiguous intensional verbs V, there might exist a systematic connection between their two readings V_1 (used for forming intentional predicates) and V_2 (used for forming intentional operators) but declares that he fails to see any such connection. He has, however, a definite idea of how such a connection would look if there was one; see [ibid. p. 7, footnote 11]. His view is reductive: he takes it that a suitable connection would allow defining any verb phrase that employs V_1 in terms of a verb phrase that uses V_2 instead, so that without loss of expressive power, we could eliminate V_1 from our language and use exclusively V_2 (or vice versa).

Now, the possibility of contextually defining V_1 in terms of V_2 is not the only way in which the two readings of an intensional verb V can be semantically interconnected. They are interconnected also if verb phrases of the form 'a V_1s x' and those of the form 'a V_2s that ζ' can both be analyzed in terms of a modal operator $\Box$, the former having the logical form $\Box_{\mathsf{a}} x$ and the latter the logical form $\Box_{\mathsf{a}} \zeta$, where

[25] In Priest's vocabulary, these verbs are 'intentional' rather than 'intensional', but this terminological issue is of no consequence here.

[26] When construed as intensional transitive verbs, *see* and *know* take a direct object, whereas *think* takes a prepositional object with the preposition *about* or *of*.

ζ ranges over L-formulas of the unextended syntax defined in Sect. 3.4. I will call syntactically ambiguous intensional verbs satisfying this condition *robust intensional verbs*. Their semantics is robust, because both of their readings admit an analysis in terms of modal operators ranging over a set of worlds. The type of content ascribed to the agent's intentional state depends on the complement of the construction $\Box_{\mathsf{a}}$. Formulas $\Box_{\mathsf{a}}x$ ascribe no propositional content. If ϕ contains no free variables, the content ascribed by $\Box_{\mathsf{a}}\phi$ is exclusively propositional. Generally, the ascribed content has both a propositional and a non-propositional aspect.[27]

The class of all intensional verbs is large. I am not proposing a general logical analysis of all intensional verbs. For example, I do not claim to provide an analysis for verbs such as *need*, *seek*, *worship*, and *owe*. The verbs my analysis can cope with are precisely the robust intensional verbs. There are such verbs: cases in point are *see*, *think*, *imagine*, *perceive*, *perceptually experience*, and *remember*. However, not all syntactically ambiguous intensional verbs are robust. For instance, *know* is not. The sentence 'Alice knows Bob' means that Alice is acquainted with Bob in the sense of being familiar or friendly with him. This involves too radical a shift of meaning from *knowing that* to allow counting *know* as a credible instance of the phenomenon under discussion.[28] The negative fact that my semantic framework provides an analysis to some but not all intensional verbs may be perceived as disappointing by those who deem worthless anything short of full generality. However, this negative fact has a positive side: my approach reveals a hitherto unobserved semantic distinction among intensional verbs. In this book, I content myself with some remarks concerning robust intensional verbs, leaving for future research the question of how to deal with the remaining intensional verbs in my framework.

There is also another respect in which my analysis lacks straightforward applicability. Namely, I do not wish to claim that for arbitrary noun phrases 'n', all natural-language sentences whose grammatical form is 'a V_1s n' have the logical form $\Box_{\mathsf{a}}\xi$, where ξ ranges over formulas written in the syntax in which variables are admitted as formulas. In natural language, we have indeed binary intensional predicates, and they are represented by formulas of the form $\Box_{\mathsf{a}}x$, but grammatical form may not always reveal logical form. For example, since constant symbols do not stand for world lines in my modal language L, the logical form of 'Alice sees Bob' is not $\Box_{\mathsf{a}}\, b$; instead, its logical form is $\mathsf{E}_{\mathsf{a}}x\,\Box_{\mathsf{a}}\, x = b$, which can be thought of as having been obtained from $\Box_{\mathsf{a}}x \wedge \Box_{\mathsf{a}}\, x = b$ by existential closure.[29]

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[27] While I restrict attention to binary intentional predicates, $(n+1)$-ary intentional predicates could be represented by formulas $\Box_{\mathsf{a}}(x_1 \wedge \cdots \wedge x_n)$.

[28] Incidentally, Hintikka [46, pp. 50–2] in effect claims that *know* is a robust intensional verb.

[29] As explained in Sect. 3.4, I have wished to formulate my modal language so that interpretations of constant and predicate symbols are formulated in terms of entities of the same kind: relative to a given world, constant symbols stand for local objects and predicates stand for sets of tuples of local objects. One could, of course, extend the language L by allowing in addition to such *extensional* constant and predicate symbols even *intensional* constant and predicate symbols. These latter would, respectively, stand for world lines and sets of tuples of world lines. Using such an extended language, the logical form of 'Alice sees Bob' could be represented by the formula $\Box_{\mathsf{a}}\, \hat{b}$, with '$\hat{b}$' being an intensional constant symbol. Seen from this generalized perspective, sentences

Formulas $\Box_{\mathsf{a}} x$ can only be used for describing intentional relations that the agent bears to an intentional object conceived of as existing: if $\Box_{\mathsf{a}} x$ holds at w_0 when $x := \mathbf{I}$, then $\mathbf{I}$ is realized throughout the set $R_{\mathsf{a}}(w_0)$ of scenarios compatible with the agent's intentional state. Thus, taking verbs like *see* and *think* to be robust intensional verbs presupposes that when they are construed as intentional predicates, they express a specific sort of intentional relation between an agent α and an intentional object $\mathbf{I}$—namely, one that satisfies $R_{\mathsf{a}}(w_0) \subseteq marg(\mathbf{I})$. These intentional relations are well-behaved: in connection with them, there is no risk of confusion between the intrinsic and contextualized viewpoint on intentional objects (cf. Sect. 6.2). I assume, then, that the semantics of intentional predicates employs such well-behaved intentional relations, describable by formulas $\Box_{\mathsf{a}} x$. More complicated intentional relations can be represented by more complex formulas. In Sect. 4.7, we have seen that there are indeed intentional relations for which $R_{\mathsf{a}}(w_0) \not\subseteq marg(\mathbf{I})$. The formula $\Box_{\mathsf{a}}(x \rightarrow \phi(x))$ expresses that $\mathbf{I}$ is ϕ in *those* worlds $w \in R_{\mathsf{a}}(w_0)$ in which it exists.

I have put forward a positive understanding of the semantics of robust intensional verbs: they are verbs expressing a relation between an agent and an intentionally individuated world line—a relation that can be described by a formula of the form $\Box_{\mathsf{a}} x$. Let us compare the consequences of this understanding to ways in which intensional verbs are standardly characterized in the literature. Normally, failure of ontologically committing existential generalization (EG) and failure of substitutivity of co-referential expressions (SI) are both criteria that are considered sufficient for intensionality.[30] How do these negative features of intensional verbs manifest themselves in my setting? In order to answer this question, we must settle on suitable formulations of the principles SI and EG. Indeed, these principles admit different construals in my semantic framework; cf. Sect. 5.3. The co-referential expressions considered can be either constant symbols that denote one and the same local object in the context of evaluation w_0 or else variables whose current values are possibly distinct world lines that have one and the same local object as their common realization in w_0. Further, existential generalization can be effected with respect to constant symbols or variables. Finally, values of quantified variables can be either physically or intentionally individuated world lines. If the values are intentionally individuated, they can be required to be existent (realized) in w_0 or not.

In the following discussion, I will consider substitutivity of identicals with respect to constant symbols (SI-C) and weak existential generalization with respect to variables (EG-W-V):

EG-W-V $$\xi[\mu] \rightarrow \mathsf{E}\mu(\mu = \mu \wedge \xi[\mu]).$$

The instances of EG-W-V are the formulas $\theta \rightarrow \mathsf{E}x(x = x \wedge \theta)$, where x is a variable and θ is an L-formula containing at least one free occurrence of x. That is, weak

(Footnote 29 continued)
of the form 'a Vs b' have the form $R(a, b)$ if V is an extensional transitive verb, and their form is $\Box_{\mathsf{a}} \hat{b}$ if V is a robust intensional verb. In the former case, extensional predicates and extensional constant symbols are utilized, while in the latter case, we employ an agent-relative modal operator and an intensional constant symbol.

[30] See, e.g., [28].

existential generalization is applicable when we are warranted to infer from the fact that a certain value of x satisfies a formula $\theta(x)$ to the conclusion that this value is an *existent intentional object*—an intentionally individuated world line realized in the current circumstances. Recall that the formula $\mathsf{E}x(x = x \wedge \theta)$ is stronger than the formula $\mathsf{E}x\theta$. The latter formula merely states that there is an intentional object satisfying θ. Such an intentional object might be non-existent—it might be an intentionally individuated world line not realized in the current circumstances.[31]

If V is a robust intensional verb, let V_1 and V_2 be its two readings as specified above. (We recall that V_1 takes a non-propositional object, while V_2 combines with a *that*-clause.) I say that V is factive if 'a V_2s that p' entails 'p'; otherwise, V is said to be non-factive. Examples of non-factive robust intensional verbs are *think* and *perceptually experience*, whereas *see* and *perceive* are factive. If V is non-factive, according to my analysis, it is straightforward that V_1 does not support weak existential generalization: if x stands for an intentionally individuated world line, 'a V_1s x' does not entail 'x exists'. If V is factive, V_1 admits weak existential generalization, but it does not support substitutivity of identicals: the conjunction of 'a V_1s c' and 'c is identical to d' does not entail 'a V_1s d'. Let us proceed to check that robust intensional verbs, analyzed in terms of my semantic framework, indeed fail to obey the principles SI-C and EG-W-V—this being required by the standard understanding of the semantics of intensional verbs. As an illustration, consider (5) and (6):

5. Alice thinks of a pink elephant

6. Bob sees Marie's father.

From (5), we cannot infer that a pink elephant Alice is thinking of exists. That is, 'Alice thinks of x who is a pink elephant' does not entail 'x exists'. The value of x is an intentional object—an intentionally individuated world line—but not necessarily an *existent* intentional object: the world line need not be realized in the current circumstances. My logical analysis accounts for this semantic phenomenon. If $\Box_{\mathsf{a}}$ ranges over worlds compatible with what Alice believes, the formula

7. $[\Box_{\mathsf{a}} x \wedge \Box_{\mathsf{a}}(x \text{ is a pink elephant})] \rightarrow x = x$

is refutable. Namely, in order for an intentional object **I** to satisfy the antecedent in w_0, it must be realized in all worlds that are compatible with Alice's beliefs in w_0. Since believing is a non-factive state, w_0 need not be among these worlds, and therefore, **I** need not be realized in w_0. Consequently, **I** need not satisfy the consequent in w_0. If Alice thinks of a pink elephant, there is an intentional object that is characterized in Alice's thoughts as a pink elephant. This object is intentionally available to Alice as a possible value of an intentional quantifier, but this intentional object need not exist in the sense of being realized. The inference from $\mathsf{E}_{\mathsf{a}}x[\Box_{\mathsf{a}} x \wedge \Box_{\mathsf{a}}(x \text{ is a pink elephant})]$ to $\mathsf{E}_{\mathsf{a}}x[x = x \wedge \Box_{\mathsf{a}}(x \text{ is a pink elephant})]$ is blocked.[32] We may conclude that the

[31] For variants of the principle of existential generalization, cf. Sect. 5.3.

[32] From (5) we can, of course, infer that Alice thinks of an elephant and that she thinks of something pink, when these latter phrases are suitably construed. That is, we can infer $\mathsf{E}_{\mathsf{a}}x\, \Box_{\mathsf{a}}(x \text{ is an elephant})$,

principle EG-W-V is not generally applicable in connection with robust intensional verbs.

As for (6), suppose the expressions ‘Marie’s father’ and ‘the winner of the Nobel Prize for Literature in 2014’ are co-referential. There is a sense in which (6) does not entail that Bob sees the Nobel Prize laureate. It is this sense that explains why Bob might answer in the affirmative to the question ‘Have you run into Marie’s father recently?’ while firmly denying to have run into the 2014 Nobel Prize laureate in Literature, having perhaps even no idea of Marie’s father’s literary occupations. Let $\Box_{\mathsf{b}}$ range over scenarios compatible with Bob’s visual perception. Recall that in L, the identity symbol stands for extensional identity: relative to a world w, the symbol ‘$=$’ is interpreted as the identity relation on the set $dom(w)$ consisting of the local objects that belong to w. For the purposes of this example, let us agree to construe the noun phrases ‘Marie’s father’ and ‘the winner of the Nobel Prize for Literature in 2014’ logically as constant symbols. The formula

8. $$[\text{Marie's father} = \text{the winner of the Nobel Prize for Literature in 2014} \wedge \Box_{\mathsf{b}}x \wedge \Box_{\mathsf{b}}(x = \text{Marie's father})] \rightarrow \Box_{\mathsf{b}}(x = \text{the winner of the Nobel Prize for Literature in 2014})$$

is refutable. Namely, suppose the antecedent of (8) is satisfied in w_0 when the value of x is an intentional object $\mathbf{I}$. Recall that in L, the interpretation of a constant symbol in a world is a local object of that world—a constant symbol never denotes directly an entire world line. It should be observed, then, that the expressions ‘Marie’s father’ and ‘the winner of the Nobel Prize for Literature in 2014’ do not stand for $\mathbf{I}$. If they are related to $\mathbf{I}$ at all, they stand for realizations of $\mathbf{I}$ in specific worlds. Now, from the hypothesis that $\mathbf{I}$ satisfies the antecedent of (8), it follows, first, that in every $w \in R_{\mathsf{b}}(w_0)$, the world line $\mathbf{I}$ is realized and its realization $\mathbf{I}(w)$ in w equals the interpretation of the noun phrase ‘Marie’s father’ in w. Since *see* is a factive verb, we have $w_0 \in R_{\mathsf{b}}(w_0)$, whence the realization $\mathbf{I}(w_0)$ of $\mathbf{I}$ in w_0 equals the interpretation of the noun phrase ‘Marie’s father’ in w_0. Second, from our hypothesis, it also follows that there is a local object $b \in dom(w_0)$ such that the interpretation of the noun phrase ‘Marie’s father’ in w_0 equals b, which equals the interpretation of the noun phrase ‘the winner of the Nobel Prize for Literature in 2014’ in w_0. It ensues that $b = \mathbf{I}(w_0)$. We may conclude that both expressions ‘Marie’s father’ and ‘the winner of the Nobel Prize for Literature in 2014’ stand for the realization $\mathbf{I}(w_0)$ of the intentional object $\mathbf{I}$ in w_0. On the other hand, if $w \in R_{\mathsf{b}}(w_0)$ and w is distinct from w_0, we can infer that in w, the noun phrase ‘Marie’s father’ stands for $\mathbf{I}(w)$, but—because the co-referentiality of the two singular terms only means that their referents coincide in w_0—it remains perfectly possible that in w, the noun phrase ‘the winner of the Nobel Prize for Literature in 2014’ refers to an object other than $\mathbf{I}(w)$. That is, (8) is refutable, and the principle SI-C is not generally applicable in relation to robust intensional verbs. By contrast, because seeing is a factive state, it follows

(Footnote 32 continued)
as well as $\mathsf{E}_{\mathsf{a}}x\,\Box_{\mathsf{a}}(x \text{ is pink})$. But we certainly *cannot* infer $\mathsf{E}_{\mathsf{a}}x\,[x \text{ is an elephant} \wedge \Box_{\mathsf{a}}x]$, *nor* can we infer $\mathsf{E}_{\mathsf{a}}x\,[x \text{ is pink} \wedge \Box_{\mathsf{a}}x]$. This is because Alice’s object of thought need not exist (be actually realized).

that **I** satisfies in w_0 the formula ($x =$ Marie's father $\wedge \Box_{\mathsf{b}} x$). This fact allows us to infer—by the co-referentiality of the two singular terms—that the formula $\mathsf{E}_{\mathsf{b}} x(x =$ the Nobel prize laureate in Literature 2014 $\wedge \Box_{\mathsf{b}} x$) is true in w_0.

My analysis allows, then, representing certain intensional transitive verbs by formulas of the form $\Box_{\mathsf{a}} x$. Typically, an intentional object is introduced into the discourse by mentioning some representational properties. As noted in Sect. 6.5 in connection with formula (4), when such properties are expressed using existence-entailing predicates $\phi(x)$, it is superfluous to state $\Box_{\mathsf{a}} x \wedge \Box_{\mathsf{a}} \phi(x)$: in that case, $\Box_{\mathsf{a}} \phi(x)$ alone entails $\Box_{\mathsf{a}} x$. However, for the purpose of developing a systematic semantic analysis, it is clearly desirable to have a way of representing constructions 'a Vs x' as they stand, instead of having as a minimal representational unit a construction with a relative clause: 'a Vs x which is a ϕ'. Besides, not all predicates are existence-entailing (cf. Sect. 2.4). Further, we may wish to say things like 'Alice thinks of something' or 'Alice sees something'. If in these sentences 'something' is understood in a sufficiently limited sense, so that it is taken to range over world lines intentionally available to Alice (instead of ranging, say, indiscriminately over propositions, events, abstract objects like numbers, physical objects, and intentional objects available to one agent or another), then these sentences require in their logical representation the bare formula x. Their form is $\mathsf{E}_{\mathsf{a}} x \, \Box_{\mathsf{a}} x$.

Formulas $\Box_{\mathsf{a}} \phi(x)$ can be used for representing two types of natural-language constructions: attributions of a predicate to the object of a robust intensional verb (*a* V_1s x which/who is ϕ) and robust intensional verbs with a *that*-clause complement (*a* V_2s of x that x is ϕ). However, the latter option is available only in contexts in which it is specified that the value of x is an *existent* intentional object. Consider sentences (9) and (10):

9. Alice thinks of a spy.

10. There is someone of whom Alice believes that he or she is a spy.

Both sentences can be read as saying that Alice's intentional state has a material object: there is a real person who is a spy according to what Alice believes. Under this reading, (9) and (10) are logically equivalent. Their common form is given by the formula $\mathsf{E}_{\mathsf{a}} x \, \exists y [x = y \wedge \Box_{\mathsf{a}}(x \text{ is a spy})]$, which is equivalent to $\mathsf{E}_{\mathsf{a}} x [x = x \wedge \Box_{\mathsf{a}}(x \text{ is a spy})]$—given hypotheses H3 and H4 adopted in Sect. 3.4. Now, sentence (9) also has a reading of the form $\mathsf{E}_{\mathsf{a}} x \, \Box_{\mathsf{a}}(x \text{ is a spy})$: the sentence can be taken to report that Alice's intentional state has an intentional object represented as a spy.[33] By contrast, sentence (10) *cannot* be construed as having the form $\mathsf{E}_{\mathsf{a}} x \, \Box_{\mathsf{a}}(x \text{ is a spy})$. This is because according to (10), Alice's beliefs pertain to a certain physical individual, but the truth of the formula $\mathsf{E}_{\mathsf{a}} x \, \Box_{\mathsf{a}}(x \text{ is a spy})$ by no means requires that the intentional existential quantifier $\mathsf{E}_{\mathsf{a}} x$ be witnessed by an existing intentional object. A fortiori, the truth of this formula does not require that Alice's intentional state have a material

[33]There is even a third way of reading (9): its form can be taken to be $\mathsf{E}_{\mathsf{a}} x(x \text{ is a spy} \wedge \Box_{\mathsf{a}} x)$. According to this reading, Alice's thoughts pertain to someone who is actually a spy, but this reading gives no information about what Alice's thoughts concerning this object are. Sentence (10) does not admit such an interpretation.

object: the state need not pertain to any physical object. Regarding the formula $\mathsf{E}_{\mathsf{a}} x \,\Box_{\mathsf{a}}(x \text{ is a spy})$, it is important to note separately that it cannot be read as saying 'there is an intentional object of which Alice thinks that it is a spy'. Indeed, as I see it, it would be a reflection of a distorted way of viewing intentional objects to say that someone *thinks of an intentional object that* it is thus and so. The problem is not that one could not have non-existent objects thought. One can. The problem is that such a way of speaking suggests too strong a separation of intentional objects from the thoughts they serve to structure. It suggests viewing intentional objects as entities along the lines of 'object theories' of intentionality. In fact, objects of thought are objects of thought represented thus and so. Alice's intentional object being characterized as a spy does not mean that she *thinks that* an intentional object is a spy. It means that the content of her intentional state comprises a spy among its intentional objects—that one of her intentional objects is represented as a spy.

6.7 Intensional Verbs and World Line Semantics

Moltmann [88] argues that the linguistic analysis of certain natural-language constructions requires positing intentional objects and allowing quantification over them. She refers to those constructions as *intentional verbs* and claims that they must be sharply distinguished from intensional verbs. Moltmann gives *think about*, *refer to*, *describe*, and *imagine* as examples of intentional verbs, whereas *need*, *look for*, and *owe* are intensional verbs in her sense. Intentional verbs include verbs for linguistic intentional acts (*mention*, *refer to*, *describe*); my analysis is not meant to extend to such verbs. Moltmann likens certain predicates describing varieties of perceptual experience to intentional verbs; as an example, she gives *see* construed non-factively [ibid. p. 155]. Assuming the intentional theory of perceptual experience, further examples are the verbs *hallucinate* and *have an illusion of*.

Moltmann characterizes intentional verbs as transitive verbs describing a mental act or speech act directed toward something possibly non-existent. Both intentional and intensional verbs differ from extensional verbs; the distinction is based on the semantic behavior of their complements. Moltmann takes complements of *intentional* verbs to behave in the same way as ordinary referential or quantificational noun phrases, with the exception that their range includes objects that depend on the intentional act described by the verb. These are 'intentional objects' in Moltmann's non-standard sense—namely, *non-existent* objects of thought. By contrast, complements of *intensional* verbs are—depending on the adopted analysis—either properties or quantifiers, and therefore, their behavior is essentially different from that of complements of intentional verbs.[34] Moltmann [88, pp. 143–4] discerns three sorts of constructions whose semantic analysis cannot, according to her, do without quantification over intentional objects: *there*-constructions combined with occurrences

[34] As Moltmann mentions, Montague [91] and Moltmann herself [86] hold that the complement contributes a quantifier, while Zimmermann [128] maintains that it contributes a property.

of intentional verbs in a relative clause, quantificational noun phrases modified by relative clauses containing an intentional verb, and quantificational noun phrases functioning as complements of an intentional verb. Here is an example of each sort:

11. There is a woman John is thinking about who does not exist
12. Some women John mentioned do not exist
13. Mary made reference to a poet who does not exist.

As Moltmann notes [ibid. p. 144], these examples show that intentional objects can be introduced with the help of intentional verbs both when these occur as main verbs, as in (13), and when they occur in relative clauses, as in (11) and (12).

In Moltmann's analysis, an intentional verb describes an intentional act, allowing the object position of the verb to take as its semantic value an intentional object that depends on the described act. The presumed semantic fact that the complement of an intentional verb is evaluated relative to the event described by the verb is meant to reflect the conceptual fact that intentional objects are dependent on intentional acts.[35] This understanding of the semantics of complements of intentional verbs leads Moltmann to postulate a systematic polysemy among nouns and adjectives: in complements of intentional verbs, they take an event argument; elsewhere, they do not. In my analysis, logical representations of typical claims about intentional objects involve both intentional quantifiers and modal operators that serve to logically represent intentional states. Applied to natural language, this means that the availability of intentional objects requires the presence of intentional verbs or, at least, some constructions interpreted modally. Logic alone, of course, does not help us to predict in which sorts of syntactic settings intentional objects are introduced in natural-language sentences: there is, for example, no logical reason why a modal operator should be syntactically in the immediate neighborhood of an intentional quantifier. In my analysis, the need for distinguishing two modes of predication corresponds to Moltmann's postulation of two ways of construing nouns and adjectives, depending on whether or not they occur in complements of intentional verbs. On my account, certain natural-language predications carry a tacit modal operator while others do not. Intentionally ascribing $P(x)$ to $\mathbf{I}$ amounts to affirming that $\mathbf{I}$ satisfies the complex predicate $\Box[x \rightarrow P(x)]$.

Moltmann does *not* think of intentional objects in modal terms—in terms of a plurality of situations. According to her, intentional acts function as event arguments of intentional verbs, and in the presence of such event arguments, quantification over intentional objects also becomes possible [88, p. 149]. She understands the semantic structure of a sentence such as (14) to be (15):

[35] While Moltmann takes events introduced by the semantics of intentional verbs to be typical vehicles for triggering intentional objects, she points out that there are expressions whose semantic values are intentional objects but do not depend on events contributed by intentional verbs. As an example, she mentions *subject matter*, as in the sentence 'There is a subject matter we did not discuss—namely, the house John plans to build'. This leads her to suggest that what is special about intentional verbs is not so much the described event making available an intentional object but the ability of the complement to attribute a property constitutive of an intentional object.

14. John thinks of a woman

15. $\exists e \exists o(\text{thinks-of}(e, \text{John}, o) \wedge \text{woman}(o, e))$,

with ‘e’ ranging over events and ‘o’ ranging over intentional objects dependent on the event e. For my part, I take the structure of (14) to be (16):[36]

16. $\mathsf{E}_{\mathsf{John}} x \, \Box_{\mathsf{John}}(x \text{ is a woman})$.

In my modal analysis, it is essential for the value of x (an object of John’s thought construed as a world line) that it is realized over the modal margin contributed by the semantics of the operator $\Box_{\mathsf{John}}$. That is, also in my analysis, the intentional object is, in a relevant sense, dependent on John’s intentional state described by the verb *thinks*. Whereas my account provides an analysis of intentional objects as certain sorts of world lines, in Moltmann’s treatment, they are not analyzed in model-theoretic terms but merely postulated as constituting a novel domain of variables available in connection with the evaluation of certain linguistic expressions. She leaves their nature unanalyzed and their relation to actual objects unexplained. Indeed, Moltmann is committed to an ‘object theory’ of intentionality in the sense of Smith and McIntyre (cf. Sect. 4.7).

Moltmann’s avoidance of world lines in her semantic analysis of intentional verbs is a conscious decision: curiously enough, world lines are among the conceptualizations she uses in her semantic theorizing! She employs the notion of *variable object* to account for the semantics of ‘definite noun phrases with a relative clause containing an intensional verb’ (INPs), such as *the book John needs to write* or *the bike Lotta wanted for her fifth birthday*.[37] She intends her notion of variable object to be a generalization of Fine’s notion of variable embodiment (cf. Sect. 2.7.3): the function that is correlated with a variable object takes arbitrary situations or circumstances as its arguments (e.g., world–time pairs), not necessarily times considered as belonging to a fixed structured world. She stresses that a variable object may lack a manifestation in a given situation. Formally speaking, there is, then, no difference between variable objects and world lines. World lines of my account have a larger range of applicability than Moltmann’s variables objects, thanks to the two modes of individuation I distinguish. Further, adopting the transcendental interpretation of world lines

[36]For reasons explained at the end of Sect. 6.6, formula (16) cannot be alternatively read as saying ‘there is an intentional object that John thinks is a woman’—nor can it be taken to represent the form of the sentence ‘There is someone of whom John believes that he or she is a woman’. It may be noted that if the operator $\Box_{\mathsf{John}}$ stood for a *factive* modality (say, remembering), then a formula of the form $\mathsf{E}_{\mathsf{John}} x \, \Box_{\mathsf{John}} \, \phi(x)$ actually could be used to represent a sentence according to which John’s intentional state has a material object. The sentence ‘There is someone regarding whom John remembers that this person was a spy’ has the form $\mathsf{E}_{\mathsf{John}} x \, \Box_{\mathsf{John}}(x \text{ was a spy})$. Namely, given that $\Box_{\mathsf{John}}$ is used for representing the factive modality *remembering*, this formula is equivalent to $\mathsf{E}_{\mathsf{John}} x \, [x = x \wedge \Box_{\mathsf{John}}(x \text{ was a spy})]$, which, again, is equivalent to $\mathsf{E}_{\mathsf{John}} x \, \exists y [x = y \wedge \Box_{\mathsf{John}}(x \text{ was a spy})]$—by hypotheses H3 and H4 of Sect. 3.4.

[37]For a discussion of the latter phrase, see Jónsson’s ‘bike puzzle’ [57], cf. Jespersen [55]. In Moltmann’s classification, *want* is taken to be polysemous, allowing both an ‘intentional’ and an ‘intensional’ reading, see [88, pp. 153–4].

allows me to reduce the monstrous proliferation that variable objects and variable embodiments suffer from (cf. Sect. 2.7.3). For Moltmann's linguistic purposes, the convenience of variable objects comes from the fact that they can be categorized as individuals. It is commonplace to construe a phrase like *the president of the US* as an individual concept, but this would mean that its referent is an abstract object—a function from circumstances to individuals. By contrast, if we take functional noun phrases to stand for variable objects, their referents are not abstract entities.[38] Then again, variable objects may not be functions, but they correspond to partial functions from circumstances to manifestations; the real difference between the two referent candidates is not so much about avoiding functions as it is about re-interpreting the notion of individual.

Moltmann argues, then, that INPs should be taken to have variable objects as their semantic values. She points out that understanding the semantics of INPs in this way accounts for the *modal compatibility requirement* (MCR), according to which a modal is required in the main clause of a sentence with an INP.[39] In the sentence

17. The book John needs to write must have (or: may have) a greater impact than the book he has already written,

the presence of a modal in the main clause is obligatory (*must have*, *may have*). Replacing *must have* in (17) by *has* results in an ungrammatical sentence. Existence-entailing predicates, such as *having an impact*, can only be ascribed to a variable object in circumstances in which it is manifested. In the absence of a modal, the only semantically available situation is the circumstance of evaluation, in which the variable object is not manifested. A modal is needed to trigger a non-actual situation in which the variable object can get manifested and relative to which predicates can be ascribed to the variable object. Moltmann contrasts sentences like (17) with corresponding sentences containing an *intentional* verb:

18. The house John imagines is huge,

where the absence of a modal is obligatory: *is* cannot be replaced by *must be* or *would be*. Here, the semantic value of *the house John imagines* is an intentional object. By Moltmann's standards, intentional objects must be sharply distinguished from variable objects. According to her [87, p. 10], an intentional object is 'fully present in the world in which the act generating it occurs' and does not have manifestations in a number of worlds. (Cf. also [89, p. 21].) The grammatical and semantic differences between noun phrases with intensional verbs and those with intentional verbs may well be reflected in the ontological and/or psychological

[38] It is a separate question whether one really should wish to countenance variable objects as referents of such noun phrases as *the president of the US*: the idea that there is some one thing that was manifested as (a realization of) Bill Clinton and later on as (a realization of) George Bush Jr. does not seem to enjoy much ontological or psychological credibility. It is questionable to suppose that all or almost all functional noun phrases someone cares to formulate in our language stand for cross-world objects of some sort.

[39] For modal compatibility requirement, see Grosu and Krifka [36].

status of the corresponding semantic values, but the contrast between (17) and (18) does not by itself provide evidence against treating intentional objects as variable objects.[40] Namely, if the relevant intentional verbs themselves must be interpreted modally so that their evaluation introduces possibly non-actual situations—as I claim they must—it is only to be expected that the main clause in (18) contains no (further) modals. The predicate *is huge* is ascribed to the manifestations of the intentionally individuated world line in all situations belonging to the range of the 'modal' *imagines*.

Moltmann [88, pp. 150–1] observes that the semantic behavior of constructions with relative clauses containing an intentional verb is similar to that of relative clauses with past tense and with modals. All three types of constructions make available quantification over possibly non-existent objects: intentional, past, and possible objects, respectively. Consider the phrases

19. a woman John thought about
20. a building John could have built
21. a building built 700 years ago.

In each case, semantically, the head noun must take as one of its arguments the event described by the intentional verb in a lower position, inside the relative clause. This makes these constructions semantically peculiar. As Moltmann notes, the intensional modifiers like *John could have built* in (20) behave as modal operators that serve to shift the circumstance of evaluation to a non-actual situation. Similarly, temporal modifiers serve to introduce a non-present time. In my analysis, the parallel status of the three types of constructions is no coincidence: they *all* involve a modal operator, and the objects must in *all* cases be considered relative to non-actual circumstances. This applies in particular to (19), because *think* ranges over situations compatible with what John believes, and intentional objects are to be analyzed as world lines.

Incidentally, despite the parallels in the semantics of the three constructions, in each of the three cases separately, we can pose the question of what the specific nature of the objects is over which the construction ranges. Consider the sentences

22. There is a woman John thought about who does not exist
23. There is a building John could have built that does not exist
24. There is a building built 700 years ago that does not exist.

In all of sentences (22)–(24), the quantifier 'there is' is witnessed by a world line not realized in the context of evaluation. However, while (22) uses an intentional quantifier and has the form

[40]For my part, I take intentional objects to be a category we need in the analysis of constructions describing mental states. At the same time, I think that INPs like *the book John needs to write* and *the bike Lotta wanted for her fifth birthday* are artifacts of careless language-use, not worthy of being considered as standing for objects and eliminable in a deeper analysis in favor of descriptions of types of actions in which the agent is either actively or passively involved.

25. $\mathsf{E}_{\mathsf{John}}x(\neg x = x \ \wedge \ \Box_{\mathsf{John}}P(x))$,

it is not evident that we should take (23) and (24) to involve quantification over intentional objects. For example, the phrase 'there is a building' in (23) can be understood as ranging over physical objects available in a world distinct from w_0. That is, (23) can be construed as having the form (26):[41]

26. $\downarrow_{\mathsf{v}} \Diamond \exists x \left[R\big(\mathsf{John}, x\big) \ \wedge \ @_{\mathsf{v}}\neg x = x\right]$.

This way of analyzing (23) would mean that despite the fact that (22) and (23) are syntactically on a par, they differ semantically in an important way: in (22), we have an intentional quantifier ranging over intentional objects available in the actual world and in (23) a physical quantifier ranging over physical objects available in a non-actual world. Another option is to go for the idea that past and possible objects must indeed be construed as intentional objects. This would mean that (23) and (24) are tacitly relative to the beliefs of an agent α so that the objects quantified over are α's objects of thought. Understood in this way, (23) would have the form

27. $\mathsf{E}_{\mathsf{a}}x\big(\neg x = x \wedge \Diamond R\big(\mathsf{John}, x\big)\big)$.

At least in connection with past objects, the relevant intentional object must represent a (past) physical object in the sense of Sect. 4.8. Thus, (24) would have the form

28. $\downarrow_{\mathsf{v}} \mathsf{E}_{\mathsf{a}}x\big(\neg x = x \wedge \mathsf{Past} \downarrow_{\mathsf{w}} \exists y(x = y \ \wedge$
$@_{\mathsf{v}} \Box_{\mathsf{a}} @_{\mathsf{w}} [Q(x) \ \wedge \ y = y \wedge \blacklozenge x = y])\big)$,

which uses the resources of hybrid logic to keep track of two parameters: the time t of evaluation (value of the variable v) and the time s witnessing the operator Past (value of the variable w). Now, (28) says that α has, at the moment t of evaluation, an intentional object **J** such that at s, there exists a physical object **I** being represented by the intentional object **J** with respect to the beliefs that α has at t concerning s.

Viewing intentional objects as world lines explains nicely why we can meaningfully use *exists* both positively and negatively with an intensional verb:

29. There is a woman John is thinking about who exists (or: does not exist).

In Moltmann's analysis, 'intentional object' means 'non-existent object of thought' [88, p. 145], which is why she is led to artificial modifications of her theory in order to deal with such sentences as *John now lives in the house he had dreamt of*, according to which an object of thought exists. Her solution is to claim that in such cases, the relevant noun phrases take, after all, variable objects as their semantic values. My analysis is more robust. Moltmann appears to ignore that the whole point of the notion of an intentional object virtually vanishes unless intentional objects can, in suitable circumstances, in some sense 'be' physical objects. The philosophical motivation of

[41] I make use of expressive resources of hybrid logic to formulate the relevant condition. The semantic effect of the 'binder' $\downarrow_{\mathsf{v}}$ is to store the current world as the value of the variable v. The 'satisfaction operator' $@_{\mathsf{v}}$ can then be used in a syntactically subordinate position to change the world of evaluation: $@_{\mathsf{v}}\phi$ holds at w iff ϕ holds at the world assigned as the value of v.

intentional objects stems in the first instance from their use in a theory of perceptual experience, and if in that connection we stipulate that intentional objects can never ‘be’ actual objects, we have definitely and deliberately tied our hands so as to make it very difficult, if possible at all, to bridge the gap between objects of thought and the real objects out there. In my analysis, an intentionally individuated world line can ‘be’ a physically individuated world line in the sense that the two world lines have coincident realizations over a number of worlds.

Concluding Remarks

The starting point of this book was that the only unproblematic notion of identity applied to individuals is the notion of 'same individual' within one and the same world. I set as my task to develop a systematic answer to the question of what it means to speak of one and the same object in a variety of situations. I said in Chap. 1 that the *form* of my answer constitutes the common thread running through this book. I declared that in the framework I formulate, cross-world statements are seen as systematically involving two types of components: worlds and links between suitable world-bound objects. The form of my proposal is indeed that worlds and world lines are two complementary modal unities, both of which are operative whenever we think or talk about objects in modal settings.

In Chaps. 1 and 2, I pointed out that there are different ways of understanding world lines. Viewed epistemologically, they would be seen as codifying means of reidentification: methods of finding out how one and the same individual manifests itself in a variety of circumstances. Seen metaphysically, they would amount to variable embodiments in Fine's sense [25]. An anti-realist would maintain that we can only meaningfully speak of one and the same individual in several situations if we are in a position to recognize it in those situations: world lines would emerge as epistemically flavored applicability conditions of the notion of cross-world identity. My preferred understanding of world lines is, however, none of these three options: my proposal is that it is a transcendental precondition for our speaking and thinking about individuals in many-world settings that individuals are construed as world lines. This proposal admits two variants: it can be understood along the lines of transcendental idealism or, alternatively, in accordance with conceptualist realism in the sense of Cassam [13]. In the former case, the fact that objects are conceptualized as world lines is seen as originating in the constitution of our mind: according to this view, we can think and speak of objects as they appear to us only in terms of world lines. The conceptualist realist claims that if external objects indeed are temporally extended and have modal properties, then external objects must be thought of as world lines. The presumed truth of this conditional claim is seen as an explanation of our cognitive faculty to think of objects as world lines. It was not among the goals of this book to develop an argument that would allow us to decide between these two

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variants, though I do think that unless convincing overall evidence is presented for transcendental idealism, we had better adopt conceptualist realism. All that matters for this book, however, is the thesis that in cross-world settings, objects must be viewed as world lines. The book as a whole aims to defend this thesis—on the one hand by developing a specific semantic theory that incorporates the proposed understanding of worlds and world lines and on the other hand by relating this semantic theory to alternative semantic approaches and by spelling out its usefulness for discussing a variety of philosophically interesting phenomena.

In Chap. 2, I formulated *world line semantics*, according to which 'first-order quantifiers' range over a set of world lines dependent on the context of evaluation. Once the main ideas of world line semantics were laid down, it turned out that it provides us not only a way of analyzing what it means to speak of individuals (physical objects) existing in many scenarios but also an analysis of sentences ascribing object-directed intentional states to various agents (states with intentional objects). Indeed, I showed in Chaps. 3 and 4 that we can make a distinction between physically and intentionally individuated world lines and discuss in terms of this distinction interrelations between physical and intentional objects. I proposed to semantically model contents of intentional states as structures consisting of a world representation (a set of worlds) and a sequence of object representations (a sequence of intentionally individuated world lines). The multiplicity of worlds in a world representation—due to the agent's incapacity to make very fine distinctions among possible scenarios—finds its analog in the plurality of worlds over which an intentional object is realized. This latter plurality accounts for the possibility of indeterminate intentional objects.

In Chap. 5, I took up the study of the logical properties of the modal language L. Despite its higher-order flavor, this language was shown to be translatable into two-sorted first-order logic. Quantifiers of one sort range over first-order objects playing the role of worlds, while those of the other sort range over first-order objects that play the role of world lines. While values of these quantifiers are substantially speaking sets of local objects, we need not think of them as such for the purposes of the translation. Instead of considering local objects as elements of worlds and world lines, they can be recovered as pairs of worlds and world lines. Identity formulas of L (stating that two world lines share a realization) can be translated utilizing a special ternary predicate E applied to a world and a pair of world lines. I pointed out in Chap. 5 that L is anomalous in not admitting a well-behaved notion of logical form. This is due to the behavior of atomic predicates, which are existence-entailing, unlike arbitrary predicates. I argued on the one hand that L has a natural generalization S_L that is not anomalous in this way and on the other hand that the deviant behavior of L is a fact of life that must be accepted—not a reason to dismiss the study of a language reflecting conceptually interesting distinctions.

I discussed general theoretical consequences of world line semantics in Chap. 6. I stressed that any objects of thought (propositions and intentional objects alike) are naturally modeled in terms of situated contents—structures composed of a set of worlds and a number of intentionally individuated world lines, relativized to a fixed world. The notion of singular proposition finds its analog in my framework in the notion of *singular content*, which does not suffer from metaphysical problems of the sort associated with the idea of a structured proposition having physical objects as components. Singular contents are situated contents with a single world line component that counts as specific—i.e., indeterminate to a sufficiently small degree. Finally, I explored different ways of talking about objects of intentional states and indicated how my framework offers a novel perspective on the semantics of certain intensional transitive verbs: those I termed robust intensional verbs.

My analysis offers a surprising generalization of possible world semantics, by construing objects and worlds as things of the same general type (sets of local objects). This book demonstrates that this philosophically motivated semantic theory is fruitful in allowing us to semantically model language pertaining to intentional states and to clarify the similarities and dissimilarities between physical and intentional objects within the confines of a unified framework.

Appendix A
Proofs

A.1 Internally Indistinguishable Worlds and Physical Objects

Let us consider hypothesis H5 mentioned in Sect. 4.5:

H5. If there is a map $g : w \cong w'$, then $\mathcal{P}_w = \mathcal{P}_{w'}$, and there is in particular a map $f : w \cong w'$ such that $f(\mathbf{I}(w)) = \mathbf{I}(w')$ for all $\mathbf{I} \in \mathcal{P}_w$.

Assuming that no two physically individuated world lines overlap (H1) and that every local object is the realization of a physical object (H4), the function f mentioned in H5 is actually uniquely determined by the set $\mathcal{P}_w$. Let us see why. Suppose that w and w' are internally indistinguishable—i.e., there is at least one map g such that $g : w \cong w'$. For all $a \in dom(w)$, let $\mathbf{I}_a$ be the unique element of $\mathcal{P}_w$ mapping w to a. (By H4, there is at least one such world line and by H1, at most one.) By H5, we have $\mathcal{P}_w = \mathcal{P}_{w'}$, which entails that $\mathbf{I}_a$ is realized in w' as well: all elements of $\mathcal{P}_{w'}$ are realized in w'. Define a map $h : dom(w) \to dom(w')$ by setting $h(a) = \mathbf{I}_a(w')$. Let f be any map satisfying $f : w \cong w'$ and $f(\mathbf{I}(w)) = \mathbf{I}(w')$ for all $\mathbf{I} \in \mathcal{P}_w$. By H5, there is at least one such map. I claim that in fact $f = h$. Let $a \in dom(w)$ be arbitrary. It suffices to show that $f(a) = h(a)$. Because $\mathbf{I}_a(w) = a$ and $\mathbf{I}_a \in \mathcal{P}_w$, we have $f(a) = f(\mathbf{I}_a(w)) = \mathbf{I}_a(w')$. Since $\mathbf{I}_a(w') = h(a)$, it follows that $f(a) = h(a)$.

A.2 Bound Variables and Substitutivity of Identicals

Fact 5.1 *Let ϕ be an L-formula in which y is free for x. Let ϕ' be the result of replacing at least one free occurrence of x in ϕ by y. The following is a valid formula:*

$$\bigwedge_{\mathbf{i} \in P_\phi} \psi_{\mathbf{i}} \to \mathsf{A}x\mathsf{A}y([x = y \wedge \phi] \to \phi').$$

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Proof If $\chi \in L$, write $\Sigma(\chi, x/y)$ for the set of formulas obtained from χ by substituting y for at least one free occurrence of x in χ. (If χ contains no free occurrences of x, then $\Sigma(\chi, x/y) = \{\chi\}$.) Let ϕ be any L-formula in which y is free for x.

If ϕ is non-modal and therefore $P_\phi = \varnothing$, the formula $\mathsf{A}x\mathsf{A}y([x = y \wedge \phi] \rightarrow \phi')$ is valid for all $\phi' \in \Sigma(\phi, x/y)$. A fortiori, in this case all formulas $\bigwedge_{\mathbf{i}\in P_\phi} \psi_{\mathbf{i}} \rightarrow \mathsf{A}x\mathsf{A}y([x = y \wedge \phi] \rightarrow \phi')$ are valid. Suppose, then, that $md(\phi) \geq 1$ and $M, w, g \models x = y \wedge \bigwedge_{\mathbf{i}\in P_\phi} \psi_{\mathbf{i}}$, where M, w, and g are otherwise arbitrary except that $g(x) \in \mathfrak{I}^{\alpha_1}_w$ and $g(y) \in \mathfrak{I}^{\alpha_2}_w$ for some agents α_1 and α_2. This assumption entails that for all $\mathsf{i}_1 \ldots \mathsf{i}_m \in P_\phi$ and all u with $(R_{\mathsf{i}_1} \circ \cdots \circ R_{\mathsf{i}_m})(w, u)$, the following *Identity Fact* holds: if either $g(x)$ or $g(y)$ is realized in u, then they both are and $g(x)(u) = g(y)(u)$. Note that since y is free for x in ϕ, y is automatically free for x in all subformulas θ of ϕ. I prove by induction that the following claim $S(n)$ holds for all n with $0 \leq n \leq md(\phi)$:

> For all subformulas θ of ϕ with $md(\theta) = n$, formulas $\theta' \in \Sigma(\theta, x/y)$, and assignments h with $h(x) = g(x)$ and $h(y) = g(y)$, we have: $M, v, h \models \theta$ iff $M, v, h \models \theta'$, where v is any world such that $(R_{\mathsf{i}_1} \circ \cdots \circ R_{\mathsf{i}_m})(w, v)$ for some $\mathsf{i}_1 \ldots \mathsf{i}_m \in P_\phi$ with $m + n \leq md(\phi)$.

The special case in which $\theta = \phi$ and $n = md(\phi)$ and $h = g$ and $m = 0$ will entail, then, that all formulas $\bigwedge_{\mathbf{i}\in P_\phi} \psi_{\mathbf{i}} \rightarrow \mathsf{A}x\mathsf{A}y([x = y \wedge \phi] \rightarrow \phi')$ with $\phi' \in \Sigma(\phi, x/y)$ are valid, given that the empty composition amounts to identity: $(R_{\mathsf{i}_1} \circ \cdots \circ R_{\mathsf{i}_m}) = \{\langle w, w\rangle : w \in dom(M)\}$ when $m = 0$.[1]

For the base case $S(0)$, suppose first that θ is an *atomic* subformula of ϕ and $(R_{\mathsf{i}_1} \circ \cdots \circ R_{\mathsf{i}_m})(w, v)$. Let $\theta' \in \Sigma(\theta, x/y)$ be arbitrary and let h be any assignment such that $h(x) = g(x)$ and $h(y) = g(y)$. If $\theta := P(\mathsf{t}_1, \ldots, \mathsf{t}_k)$, then $\theta' := P(\mathsf{s}_1, \ldots, \mathsf{s}_k)$, where the term t_i can be distinct from the term s_i only if they both are variables and in particular $\mathsf{t}_i = x$ and $\mathsf{s}_i = y$. Now, $M, v, h \models \theta$ iff all values $\mathsf{t}_1^{M,v,h}, \ldots, \mathsf{t}_k^{M,v,h}$ are defined and $\langle \mathsf{t}_1^{M,v,h}, \ldots, \mathsf{t}_k^{M,v,h}\rangle \in Int(P, v)$ iff all values $\mathsf{s}_1^{M,v,h}, \ldots, \mathsf{s}_k^{M,v,h}$ are defined and $\langle \mathsf{s}_1^{M,v,h}, \ldots, \mathsf{s}_k^{M,v,h}\rangle \in Int(P, v)$ iff $M, v, h \models \theta'$. The reason why the second equivalence holds is as follows. If t_j is a constant symbol or a variable distinct from x, then $\mathsf{t}_j = \mathsf{s}_j$, and trivially either both values $\mathsf{t}_j^{M,v,h}$ and $\mathsf{s}_j^{M,v,h}$ are undefined or else $\mathsf{t}_j^{M,v,h} = \mathsf{s}_j^{M,v,h}$. By the Identity Fact we have, moreover, either $v \notin marg(h(x)) \cup marg(h(y))$ or else $v \in marg(h(x)) \cap marg(h(y))$ and $h(x)(v) = h(y)(v)$. Therefore, if $\mathsf{t}_j = x$ and $\mathsf{s}_j \in \{x, y\}$, then either both values $\mathsf{t}_j^{M,v,h}$ and $\mathsf{s}_j^{M,v,h}$ are undefined or else $\mathsf{t}_j^{M,v,h} = \mathsf{s}_j^{M,v,h}$. By similar reasoning, it follows that $M, v, h \models \theta$ iff $M, v, h \models \theta'$ when θ is an identity formula. The remaining subformulas of ϕ having modal depth 0 are obtained from atomic formulas by applications of the operators $\wedge, \neg, \exists$, and E_{a}. By the compositionality of L (see Fact 5.9), the satisfaction of an L-formula of modal depth 0 at a world v depends only on the satisfaction of its atomic subformulas at v. Now, there is a one-one correspondence between atomic subformulas $\chi_1, \ldots, \chi_k$ of θ and atomic subformulas $\chi_1', \ldots, \chi_k'$ of θ' such that θ'

[1] In fact, the proof establishes the following stronger claim: whenever $\phi' \in \Sigma(\phi, x/y)$, the formula $\bigwedge_{\mathbf{i}\in P_\phi} \psi_{\mathbf{i}} \rightarrow \mathsf{A}x\mathsf{A}y[x = y \rightarrow (\phi \leftrightarrow \phi')]$ is valid.

is built from the formulas χ_i' in exactly the same way as θ is built from the χ_i. Since $M, v, h \models \chi_i$ iff $M, v, h \models \chi_i'$, it follows that $M, v, h \models \theta$ iff $M, v, h \models \theta'$.

Assume, then, inductively that the claim $S(n)$ holds for $n < md(\phi)$. (Recall that we are working under the hypothesis that $md(\phi) \geq 1$.) I move on to prove the claim $S(n+1)$. I begin by considering the case $\theta := \Box_{\mathsf{i}}\chi$ with $md(\chi) = n$. Let $\theta' \in \Sigma(\theta, x/y)$ and let h be any assignment such that $h(x) = g(x)$ and $h(y) = g(y)$. Consequently, there is $\chi' \in \Sigma(\chi, x/y)$ such that $\theta' = \Box_{\mathsf{i}}\chi'$. Suppose $M, v, h \models \Box_{\mathsf{i}}\chi$, where $(R_{\mathsf{i}_1} \circ \cdots \circ R_{\mathsf{i}_m})(w, v)$ with $m + md(\theta) = m + md(\chi) + 1 \leq md(\phi)$. Let u with $R_{\mathsf{i}}(v, u)$ be arbitrary. It follows that $M, u, h \models \chi$, where $(R_{\mathsf{i}_1} \circ \cdots \circ R_{\mathsf{i}_m} \circ R_{\mathsf{i}})(w, u)$. Because $(m+1) + md(\chi) \leq md(\phi)$, the inductive hypothesis yields $M, u, h \models \chi'$ and we may conclude that $M, v, h \models \Box\chi'$. Similarly, the inductive hypothesis guarantees that if $M, v, h \models \Box\chi'$, then $M, v, h \models \Box\chi$. Now, write Ξ for the class of all subformulas of ϕ that are either of modal depth at most n or else of the syntactic form $\Box_{\mathsf{i}}\chi$ with $md(\chi) = n$ for some index i. By the above reasoning, we know that the claim $S(n+1)$ holds if attention is restricted to formulas $\theta \in \Xi$. Write Ξ^* for the class of all subformulas of ϕ whose modal depth is at most $n+1$. It remains to argue why the claim $S(n+1)$ holds for an *arbitrary* formula $\theta \in \Xi^*$.

Any formula of Ξ^* is obtained from formulas of Ξ by some finite number of applications of the operators $\neg, \wedge, \exists$, or $\mathsf{E_a}$. We have already shown that $S(n+1)$ holds for all formulas in Ξ. Let us assume inductively that ξ and ζ are formulas in Ξ^* satisfying $S(n+1)$. We must show that if $\beta \in \mathit{Var}$ and $\theta \in \{(\xi \wedge \zeta), \neg\xi, \exists\beta\xi, \mathsf{E_a}\beta\xi\}$, we have: $M, v, h \models \theta$ iff $M, v, h \models \theta'$, where h is any assignment such that $h(x) = g(x)$ and $h(y) = g(y)$, v is any world such that $(R_{\mathsf{i}_1} \circ \cdots \circ R_{\mathsf{i}_m})(w, v)$ with $m + md(\theta) \leq md(\phi)$, and $\theta' \in \Sigma(\theta, x/y)$ is arbitrary. By the inductive hypothesis, it is immediate that the claim holds for $(\xi \wedge \zeta)$ and $\neg\xi$. Let us move on to consider the formula $\exists\beta\xi$. If x does not occur free in $\exists\beta\xi$, there is nothing to prove, so suppose it does. Now, because x occurs free in $\exists\beta\xi$, we have $\beta \neq x$. Further, since y is free for x in ϕ and therefore in its subformula $\exists\beta\xi$, we have $\beta \neq y$. Suppose, then, that $M, v, h \models \exists\beta\xi$ and let $\xi' \in \Sigma(\xi, x/y)$ be arbitrary. By assumption there is $\mathbf{I} \in \mathcal{P}_v$ such that $M, v, h[\beta := \mathbf{I}] \models \xi$. Because $x \neq \beta \neq y$, we may infer that $h[\beta := \mathbf{I}](x) = h(x) = g(x)$ and $h[\beta := \mathbf{I}](y) = h(y) = g(y)$, whence the inductive hypothesis applies and yields $M, v, h[\beta := \mathbf{I}] \models \xi'$. Thus, $M, v, h \models \exists\beta\xi'$. Conversely, the inductive hypothesis guarantees that if $M, v, h \models \exists\beta\xi'$, then $M, v, h \models \exists\beta\xi$. The case of $\mathsf{E_a}\beta\theta$ can be proven similarly. □

A.3 Constant Symbols and Substitutivity of Identicals

Fact 5.3 *Let ϕ be an L-formula. Let ϕ' be the result of replacing at least one occurrence of c in ϕ by d. The following is a valid formula:*

$$\bigwedge_{\mathbf{i} \in P_\phi} (\theta_{\mathbf{i}}^c \wedge \theta_{\mathbf{i}}^d) \rightarrow ((c = d \wedge \phi) \rightarrow \phi').$$

Proof If $\chi \in L$, write $\Sigma(\chi, c/d)$ for the set of formulas obtained from χ by substituting d for at least one occurrence of c in χ. (If χ contains no occurrences of c, then $\Sigma(\chi, c/d) = \{\chi\}$.) If $md(\phi) = 0$, the formula $((c = d \wedge \phi) \rightarrow \phi')$ is valid for all $\phi' \in \Sigma(\phi, c/d)$. A fortiori, therefore, all formulas $\bigwedge_{\mathbf{i}\in P_\phi}(\theta^c_{\mathbf{i}} \wedge \theta^d_{\mathbf{i}}) \rightarrow ((c = d \wedge \phi) \rightarrow \phi')$ are valid. Suppose, then, that $md(\phi) \geq 1$.

Since physically individuated world lines do not overlap, if e is any constant symbol, the formula $\bigwedge_{\mathbf{i}\in P_\phi} \theta^e_{\mathbf{i}} = \bigwedge_{\mathsf{i}_1\ldots\mathsf{i}_m\in P_\phi} \exists x(x = e \wedge \Box_{\mathsf{i}_1} \cdots \Box_{\mathsf{i}_m} x = e)$ is equivalent to $\exists x \bigwedge_{\mathsf{i}_1\ldots\mathsf{i}_m\in P_\phi}[x = e \wedge \Box_{\mathsf{i}_1} \cdots \Box_{\mathsf{i}_m} x = e]$. Namely, if the former formula is satisfied at w, and $\mathbf{J}_1$ and $\mathbf{J}_2$ are witnesses of $\exists x$ in distinct conjuncts $\mathbf{i_1}$ and $\mathbf{i_2}$, then $\mathbf{J}_1(w) = Int(e, w) = \mathbf{J}_2(w)$. By hypothesis H1 of Sect. 3.4, this entails that $\mathbf{J}_1 = \mathbf{J}_2$. We may conclude that the formula $\bigwedge_{\mathbf{i}\in P_\phi}(\theta^c_{\mathbf{i}} \wedge \theta^d_{\mathbf{i}})$ is equivalent to the formula

$$\exists x \exists y[x = c \wedge y = d \wedge \bigwedge_{\mathsf{i}_1\ldots\mathsf{i}_m\in P_\phi} \Box_{\mathsf{i}_1} \cdots \Box_{\mathsf{i}_m} (x = c \wedge y = d)].$$

Now, let M be an arbitrary model, let $w \in dom(M)$ be any world, and let g be any assignment in M. Suppose that $M, w, g \models \bigwedge_{\mathbf{i}\in P_\phi}(\theta^c_{\mathbf{i}} \wedge \theta^d_{\mathbf{i}})$. Thus, there are physically individuated world lines $\mathbf{I}, \mathbf{J} \in \mathcal{P}_w$ such that

$(*)$ $M, w, x := \mathbf{I}, y := \mathbf{J} \models x = c \wedge y = d \wedge \bigwedge_{\mathsf{i}_1\ldots\mathsf{i}_m\in P_\phi} \Box_{\mathsf{i}_1} \cdots \Box_{\mathsf{i}_m}(x = c \wedge y = d)$.

Assume, then, that $M, w, g \models c = d \wedge \phi$. Together with $(*)$ this assumption yields $\mathbf{I}(w) = Int(c, w) = Int(d, w) = \mathbf{J}(w)$, which again entails by hypothesis H1 of Sect. 3.4 that $\mathbf{I}$ and $\mathbf{J}$ are one and the same world line. Therefore $(*)$ allows us to infer that $M, w, x := \mathbf{I} \models \bigwedge_{\mathsf{i}_1\ldots\mathsf{i}_m\in P_\phi} \Box_{\mathsf{i}_1} \cdots \Box_{\mathsf{i}_m}(x = c \wedge x = d)$ and consequently

$(\dagger)$ $M, w \models c = d \wedge \bigwedge_{\mathsf{i}_1\ldots\mathsf{i}_m\in P_\phi} \Box_{i_1} \cdots \Box_{i_m} c = d$.

It remains to show that $M, w, g \models \phi'$ for all $\phi' \in \Sigma(\phi, c/d)$. Observe, first, that if θ is *atomic* and $\theta' \in \Sigma(\phi, c/d)$, then by $(\dagger)$ we have: $M, v, g \models \theta$ iff $M, v, g \models \theta'$ whenever $v = w$ or $(R_{\mathsf{i}_1} \circ \cdots \circ R_{\mathsf{i}_m})(w, v)$ for $\mathsf{i}_1 \ldots \mathsf{i}_m \in P_\phi$. It is, then, straightforward to prove by induction on the number n the following claim: for any numbers n and m such that $n + m \leq md(\phi)$, string $\mathsf{i}_1 \ldots \mathsf{i}_m \in P_\phi$, world v with $v = w$ or $(R_{\mathsf{i}_1} \circ \cdots \circ R_{\mathsf{i}_m})(w, v)$, formula θ with $md(\theta) = n$, and formula $\theta' \in \Sigma(\theta, c/d)$, we have: $M, v, g \models \theta$ iff $M, v, g \models \theta'$. In the special case of $\theta = \phi$ and $n = md(\phi)$ and $m = 0$ and $v = w$, it follows that $M, w, g \models \phi$ iff $M, w, g \models \phi'$ for all $\phi' \in \Sigma(\phi, c/d)$. Since by the hypothesis, we have $M, w, g \models \phi$, may conclude that $M, w, g \models \phi'$. $\square$

A.4 Variants of the Barcan Formula and Its Converse

A.4.1 Relation to Versions of Monotonicity and Anti-Monotonicity

Fact 5.5 *Let F be any frame.*

(a) $F \models$ BF-P $\Leftrightarrow$ *F is physically anti-monotonic.*
(b) $F \models$ CBF-P $\not\Rightarrow \Leftarrow$ *F is physically monotonic.*
(c) $F \models$ BF-I $\not\Rightarrow \Leftarrow$ *F is intentionally anti-monotonic relative to agent α.*
(d) $F \models$ CBF-I $\not\Rightarrow \Leftarrow$ *F is intentionally monotonic relative to agent α.*

Proof The right–left entailments of all four claims are obvious; let us consider the left–right entailments. As for claim (a), suppose for contradiction that $F = \langle W, \{R\}, \mathcal{P}, \mathcal{I}\rangle$ is a frame satisfying $F \models$ BF-P, though F is not physically anti-monotonic. Because the physical anti-monotonicity fails, there are worlds w and w' with $R(w, w')$ such that for some $\mathbf{I} \in \mathcal{P}_{w'}$, we have $\mathbf{I} \notin \mathcal{P}_w$. Since $\mathbf{I}$ is physically individuated and $\mathbf{I} \in \mathcal{P}_{w'}$, there is a local object $b \in dom(w')$ such that $b = \mathbf{I}(w')$. Define an interpretation function *Int* by setting $Int(Q, w') = \{b\}$ and $Int(Q, w'') = \varnothing$ for all $w'' \in W \backslash \{w'\}$. Letting $M = \langle F, Int\rangle$, it follows that $M, w \models \Diamond\exists x Q(x)$. Since $F \models$ BF-P, it ensues that $M, w \models \exists x \Diamond Q(x)$. That is, there is $\mathbf{J} \in \mathcal{P}_w$ such that for some w'' with $R(w, w'')$, we have $\mathbf{J}(w'') \in Int(Q, w'')$. Given the definition of *Int*, we may conclude that $w'' = w'$ and $\mathbf{J}(w'') = b$. As no two physically individuated world lines overlap, we have $\mathbf{J} = \mathbf{I}$ and $\mathbf{I} \in \mathcal{P}_w$. This is a contradiction. Regarding claim (b), we recall that CBF-P is valid. Therefore it is not valid *only* in physically monotonic frames.

For claim (c), in order to show that the validity of BF-I in a frame fails to entail its intentional anti-monotonicity, consider a frame $F_0 = \langle W, \{R\}, \mathcal{P}, \mathcal{I}\rangle$ with an arbitrarily chosen $\mathcal{P}$ such that $W = \{w_1, w_2\}$ with $w_1 \neq w_2$, $R = \{\langle w_1, w_2\rangle\}$, and $\mathcal{I} = \{\mathcal{I}^\alpha_{w_1}, \mathcal{I}^\alpha_{w_2}\}$—given that $\mathcal{I}^\alpha_{w_1} = \{\mathbf{I}\}$ and $\mathcal{I}^\alpha_{w_2} = \{\mathbf{J}\}$, where $marg(\mathbf{I}) = \{w_1, w_2\}$ and $marg(\mathbf{J}) = \{w_2\}$ and $\mathbf{I}(w_2) = \mathbf{J}(w_2)$. Now, F_0 is not intentionally anti-monotonic relative to agent α: we have $R(w_1, w_2)$ but $\{\mathbf{I}\} = \mathcal{I}^\alpha_{w_1} \not\supseteq \mathcal{I}^\alpha_{w_2} = \{\mathbf{J}\}$, because $\mathbf{I} \neq \mathbf{J}$. I proceed to show that BF-I is, however, valid in F_0. Namely, let $M = \langle F_0, Int\rangle$ be any model based on F_0, let $w \in \{w_1, w_2\}$ be arbitrary, and suppose $M, w \models \Diamond\mathsf{E}_{\mathsf{a}} x Q(x)$. In fact, w must equal w_1, as w_2 has no R-successor: the formula $\Diamond\mathsf{E}_{\mathsf{a}} x Q(x)$ could not be true at w_2. Because w_2 is the only world R-accessible from w_1, it follows from the truth of $\Diamond\mathsf{E}_{\mathsf{a}} x Q(x)$ at w_1 that the unique world line $\mathbf{J}$ intentionally available for α in w_2 satisfies $\mathbf{J}(w_2) \in Int(Q, w_2)$. Because $\mathbf{I}(w_2) = \mathbf{J}(w_2)$, we have $M, w_1, x := \mathbf{I} \models \Diamond Q(x)$. Since $\mathbf{I}$ is available for α in w_1, we may conclude that $M, w_1 \models \mathsf{E}_{\mathsf{a}} x \Diamond Q(x)$. It ensues that $F_0 \models$ BF-I. Concerning claim (d), we may note that the frame F_0 is not intentionally monotonic: we have $R(w_1, w_2)$ but $\{\mathbf{I}\} = \mathcal{I}^\alpha_{w_1} \not\subseteq \mathcal{I}^\alpha_{w_2} = \{\mathbf{J}\}$, because $\mathbf{I} \neq \mathbf{J}$. Yet, it can be checked that CBF-I is valid in F_0. $\square$

A.4.2 Noncharacterizability of Intentional Monotonicity and Intentional Anti-Monotonicity

Theorem 5.1 *(a) There is no L-formula that is valid in a frame iff the frame is intentionally monotonic. (b) Neither is there an L-formula that is valid in a frame iff the frame is intentionally anti-monotonic.*

Proof Let us begin with statement (a). I construct two frames of which one is intentionally monotonic but the other is not, and show that any L-formula valid in the former is also valid in the latter. Let w_1, w_2, and w_3 be three worlds. For each $1 \leq i \leq 3$, let $\mathcal{P}_{w_i}$ be a singleton $\{\langle w_i, a_i \rangle\}$, where $a_i \in dom(w_i)$. Let $\mathcal{I}^{\alpha}_{w_1} = \mathcal{I}^{\alpha}_{w_2} = \mathcal{I}^{\alpha}_{w_3} = \{\mathbf{I}_0, \mathbf{J}_0\}$, where $marg(\mathbf{I}_0) = marg(\mathbf{J}_0) = \{w_1\}$ and $\mathbf{I}_0(w_1) \neq \mathbf{J}_0(w_1)$. Define a frame $F = \langle W, \{R\}, \mathcal{P}, \mathcal{I} \rangle$ by setting $W = \{w_1, w_2, w_3\}$, $R = \{\langle w_1, w_2 \rangle, \langle w_2, w_3 \rangle\}$, $\mathcal{P} = \{\mathcal{P}_{w_1}, \mathcal{P}_{w_2}, \mathcal{P}_{w_3}\}$, and $\mathcal{I} = \{\mathcal{I}^{\alpha}_{w_1}, \mathcal{I}^{\alpha}_{w_2}, \mathcal{I}^{\alpha}_{w_3}\}$. It is trivial that the frame F is intentionally monotonic. Next, define another frame $F' = \langle W', \{R'\}, \mathcal{P}', \mathcal{I}' \rangle$ by setting $W' = W$ and $R' = R$ and $\mathcal{P}' = \mathcal{P}$ and $\mathcal{I}' = \{\mathcal{I}'^{\alpha}_{w_1}, \mathcal{I}'^{\alpha}_{w_2}, \mathcal{I}'^{\alpha}_{w_3}\}$—given that $\mathcal{I}'^{\alpha}_{w_1} = \{\mathbf{I}_0, \mathbf{J}_0\}$ and $\mathcal{I}'^{\alpha}_{w_2} = \{\mathbf{I}_0\}$ and $\mathcal{I}'^{\alpha}_{w_3} = \{\mathbf{I}_0, \mathbf{J}_0\}$, where $\mathbf{I}_0$ and $\mathbf{J}_0$ are the same world lines as in F. The frame F' is not intentionally monotonic since $R(w_1, w_2)$ but $\mathcal{I}'^{\alpha}_{w_1} \not\subseteq \mathcal{I}'^{\alpha}_{w_2}$. The only difference between F and F' is that unlike in F, in F' the world line $\mathbf{J}_0$ is not intentionally available in w_2. Let Int be any interpretation over W. Let $M = \langle F, Int \rangle$ and $M' = \langle F', Int' \rangle$, where $Int' = Int$. Let us prove the following auxiliary claim:

Claim 1 For all $\phi \in L$, all $g : Free(\phi) \to \{\mathbf{I}_0, \mathbf{J}_0\}$, and all $w \in W$, we have:

$$M, w, g \models \phi \quad \text{iff} \quad M', w, g \models \phi.$$

The claim can be proven by induction on the structure of the L-formula ϕ. The base case concerns atomic predications $Q(\mathsf{t}_1, \ldots, \mathsf{t}_n)$ and identities $\mathsf{t}_1 = \mathsf{t}_2$. Let $g : Free(Q(\mathsf{t}_1, \ldots, \mathsf{t}_n)) \to \{\mathbf{I}_0, \mathbf{J}_0\}$ be an arbitrary assignment. We have $M, w, g \models Q(\mathsf{t}_1, \ldots, \mathsf{t}_n)$ iff all values $\mathsf{t}_1^{M,w,g}, \ldots, \mathsf{t}_n^{M,w,g}$ are defined and $\langle \mathsf{t}_1^{M,w,g}, \ldots, \mathsf{t}_n^{M,w,g} \rangle \in Int(Q, w) = Int'(Q, w)$ iff $M', w, g \models Q(\mathsf{t}_1, \ldots, \mathsf{t}_n)$. Further, for any assignment $g : Free(\mathsf{t}_1 = \mathsf{t}_n) \to \{\mathbf{I}_0, \mathbf{J}_0\}$, we have $M, w, g \models \mathsf{t}_1 = \mathsf{t}_2$ iff both values $\mathsf{t}_1^{M,w,g}$ and $\mathsf{t}_2^{M,w,g}$ are defined and $\mathsf{t}_1^{M,w,g}$ equals $\mathsf{t}_2^{M,w,g}$ iff $M', w, g \models \mathsf{t}_1 = \mathsf{t}_2$.

Suppose, then, inductively that for fixed formulas ψ and θ, we have: if $\chi \in \{\psi, \theta\}$, then $M, w, g \models \chi$ iff $M', w, g \models \chi$ for all $g : Free(\chi) \to \{\mathbf{I}_0, \mathbf{J}_0\}$ and all $w \in W$. Evidently, the formula $\neg\psi$ satisfies the claim. For the remaining inductive cases, let w be any world in W. Concerning $(\psi \wedge \theta)$, let $g : Free(\psi) \cup Free(\theta) \to \{\mathbf{I}_0, \mathbf{J}_0\}$ be arbitrary. Now, $M, w, g \models (\psi \wedge \theta)$ iff $M, w, g \models \psi$ and $M, w, g \models \theta$ iff $M, w, g\restriction_{Free(\psi)} \models \psi$ and $M, w, g\restriction_{Free(\theta)} \models \theta$ iff (ind. hyp.) $M', w, g\restriction_{Free(\psi)} \models \psi$ and $M', w, g\restriction_{Free(\theta)} \models \theta$ iff $M', w, g \models \psi$ and $M', w, g \models \theta$ iff $M', w, g \models (\psi \wedge \theta)$.

The case of $\Box\psi$: Let $g : Free(\psi) \to \{\mathbf{I}_0, \mathbf{J}_0\}$ be arbitrary. Now, $M, w, g \models \Box\psi$ iff for all w' with $R(w, w')$, we have $M, w', g \models \psi$ iff (ind. hyp.) for all w' with $R(w, w')$, we have $M', w', g \models \psi$ iff $M', w, g \models \Box\psi$. Observe that actually, the inductive hypothesis is not needed in the special case that $w = w_3$. This is because w_3 has no R-successors—it satisfies vacuously all formulas of the form $\Box\zeta$.

The case of $\exists x\psi$: Because there is a world line $\mathbf{I}$ with $marg(\mathbf{I}) = \{w\}$ such that $\mathcal{P}_w = \{\mathbf{I}\}$, we have $M, w, g \models \exists x\psi$ iff $M, w, g[x := \mathbf{I}] \models \psi$ iff (ind. hyp.) $M', w, g[x := \mathbf{I}] \models \psi$ iff $M', w, g \models \exists x\psi$ for any suitable assignment g.

The case of $\mathsf{E}_{\mathsf{a}}x\psi$: Let g be any map $Free(\psi)\setminus\{x\} \to \{\mathbf{I}_0, \mathbf{J}_0\}$. Suppose $M, w, g \models \mathsf{E}_{\mathsf{a}}x\psi$. Thus, there is $\mathbf{I} \in \mathcal{I}^{\alpha}_{w}$ such that $M, w, g[x := \mathbf{I}] \models \psi$. Therefore, by the inductive hypothesis, $M', w, g[x := \mathbf{I}] \models \psi$. Unless $w = w_2$ and $\mathbf{I} = \mathbf{J}_0$, we may conclude—given how the frame F' is defined—that $\mathbf{I} \in \mathcal{I}'^{\alpha}_{w}$, whence $M', w, g \models \mathsf{E}_{\mathsf{a}}x\psi$. Let us consider separately the special case that $M, w_2, g[x := \mathbf{J}_0] \models \psi$. Because the sets $\mathcal{I}^{\alpha}_{w_j}, \mathcal{I}'^{\alpha}_{w_j}$ with $j := 2, 3$ are non-empty and none of their elements is realized in w_j, the following general fact can be easily shown by induction: for all $n \geq 0$, formulas $\zeta(x_1, \ldots, x_n)$ of n free variables, assignments $f, f' : \{x_1, \ldots, x_n\} \to \{\mathbf{I}_0, \mathbf{J}_0\}$, and indices $j \in \{2, 3\}$, we have $M, w_j, f \models \zeta$ iff $M', w_j, f' \models \zeta$. Therefore, we can infer from $M, w_2, g[x := \mathbf{J}_0] \models \psi$ that $M', w_2, g[x := \mathbf{I}_0] \models \psi$, which entails $M', w_2, g \models \mathsf{E}_{\mathsf{a}}x\psi$, because $\mathbf{I}_0 \in \mathcal{I}'^{\alpha}_{w_2}$. The direction from $M', w, g \models \mathsf{E}_{\mathsf{a}}x\psi$ to $M, w, g \models \mathsf{E}_{\mathsf{a}}x\psi$ can be proven similarly, without resorting to the last-mentioned additional fact. ⧄

Having now proven Claim 1, it can be used to establish the following:

Claim 2 Any formula valid in F is valid in F', as well.

Suppose $\phi \in L$ and $F \models \phi$. Let $M' = \langle F', Int' \rangle$ be any model based on F'. Let $w \in dom(M')$ and $g : Free(\phi) \to \{\mathbf{I}_0, \mathbf{J}_0\}$ be arbitrary. Consider the model $M = \langle F, Int \rangle$ with $Int = Int'$. Since ϕ is valid in F, we have $M, w, g \models \phi$. By Claim 1, it follows $M', w, g \models \phi$. We may conclude that ϕ is valid in F'. ⧅

In order to prove statement (a), suppose for contradiction that there is an L-formula ϕ_0 such that for all frames F_0, the formula ϕ_0 is valid in F_0 iff F_0 is intentionally monotonic. Since F is intentionally monotonic, ϕ_0 is valid in F. By Claim 2, ϕ_0 is valid in F', as well. However, F' is not intentionally monotonic. This contradicts the assumption that ϕ_0 is valid only in intentionally monotonic frames.

Concerning statement (b), observe that F is not only intentionally monotonic, but also intentionally anti-monotonic. Further, not only does F' fail to be intentionally monotonic, but it also fails to be intentionally anti-monotonic: we have $R(w_2, w_3)$ but $\mathcal{I}'^{\alpha}_{w_2} \not\supseteq \mathcal{I}'^{\alpha}_{w_3}$. If there was an L-formula characterizing anti-monotonicity, it would be valid in F and—by Claim 2—in F'. So it would, after all, not be valid only in anti-monotonic frames. □

A.5 Existence-Entailment Problem Is Undecidable

Let FOL* be the fragment of FOL without identity symbol or constant symbols. Its only logical symbols are $\neg$, $\wedge$, $\exists$, and the variables in the set *Var*. It is well known that the validity problem of FOL* is undecidable; see [59]. Note that syntactically, FOL* is a fragment of L. I prove that there is no algorithm for solving the *existence-entailment problem* of L—the problem of determining whether an L-formula is existence-entailing. I do this by showing that if this problem were decidable, so would be the validity problem of FOL*.

An FOL*-formula ϕ is *FOL*-valid* if $\mathcal{M}, \Gamma \models_{\mathrm{FOL}} \phi$ for all first-order models $\mathcal{M}$ and assignments Γ. The same formula, viewed as an L-formula, is valid if $M, w, g \models \phi$ for all models M, worlds $w \in dom(M)$, and assignments g in M. If g is an assignment and $X \subseteq dom(g) \subseteq Var$, then g is said to be *w-realized for X* provided that for all variables $x \in X$, the world line $g(x)$ is realized in w. The assignment g is *w-realized* if it is w-realized for $dom(g)$. The following lemma is useful.

Lemma *Let $\phi(x_1, \ldots, x_n) \in FOL^*$ be arbitrary. The formula $\phi(x_1, \ldots, x_n)$ is FOL*-valid iff the L-formula $\left(\bigwedge_{1 \leq i \leq n} x_i = x_i\right) \to \phi(x_1, \ldots, x_n)$ is valid.*

Proof Let $\phi(x_1, \ldots, x_n) \in$ FOL* and assume, first, that ϕ is FOL*-valid. Suppose for contradiction that there is a model M, a world w_0, and an assignment g_0 such that nevertheless $M, w_0, g_0 \not\models \left(\bigwedge_{1 \leq i \leq n} x_i = x_i\right) \to \phi(x_1, \ldots, x_n)$. This means that the assignment g_0 is w_0-realized for $Free(\phi)$ and $M, w_0, g_0 \not\models \phi(x_1, \ldots, x_n)$. If Int is the interpretation function of M, define an interpretation function $\mathbf{Int}$ by setting $\mathbf{Int}(Q) := Int(Q, w_0)$ for all predicate symbols Q occurring in ϕ. Let $\mathcal{M} := \langle dom(w_0), \mathbf{Int} \rangle$. If g is any w_0-*realized* assignment in M, let Γ_g be the function defined on the set $dom(g)$ as follows: $\Gamma_g(y) := g(y)(w_0)$ for all $y \in dom(g)$. We can prove by induction on the complexity of an FOL*-formula θ the following claim: for all w_0-realized assignments g in M with $dom(g) = Free(\theta)$, we have $M, w_0, g \models \theta$ iff $\mathcal{M}, \Gamma_g \models \theta$. The base case of atomic formulas $Q(y_1, \ldots, y_m)$ holds, because $M, w_0, g \models Q(y_1, \ldots, y_m)$ iff $\langle g(y_1)(w_0), \ldots, g(y_m)(w_0) \rangle \in Int(Q, w_0)$ iff $\langle \Gamma_g(y_1), \ldots, \Gamma_g(y_m) \rangle \in \mathbf{Int}(Q)$ iff $\mathcal{M}, \Gamma_g \models Q(y_1, \ldots, y_m)$. The inductive cases for $\neg$ and $\wedge$ are trivial. For the remaining case of physical quantifiers, we need hypothesis H4, according to which every local object is the realization of some physical object. If $\mathcal{M}, \Gamma_g \models \exists x \psi$, there is $a \in dom(w_0)$ such that $\mathcal{M}, \Gamma_g[x := a] \models \psi$. By H4, there is $\mathbf{I}_a$ in M such that $\mathbf{I}_a(w_0) = a$ and $\Gamma_g[x := a] = \Gamma_{g[x := \mathbf{I}_a]}$, where $g[x := \mathbf{I}_a]$ is w_0-realized. By the inductive hypothesis, we have $M, w_0, g[x := \mathbf{I}_a] \models \psi$, whence it follows that $M, w_0, g \models \exists x \psi$. Further, if $M, w_0, g \models \exists x \psi$, there is $\mathbf{J}$ realized in w_0 such that $M, w_0, g[x := \mathbf{J}] \models \psi$, where $g[x := \mathbf{J}]$ is w_0-realized. The inductive hypothesis entails $\mathcal{M}, \Gamma_{g[x := \mathbf{J}]} \models \psi$, where $\Gamma_{g[x := \mathbf{J}]} = \Gamma_g[x := \mathbf{J}(w_0)]$. It follows that $\mathcal{M}, \Gamma_g \models \exists x \psi$. Since g_0 is w_0-realized for $Free(\phi)$, we may conclude that $M, w_0, g_0 \models \phi$ iff $\mathcal{M}, \Gamma_{g_0} \models \phi$. As ϕ is FOL*-valid, we have $\mathcal{M}, \Gamma_{g_0} \models \phi$ and therefore $M, w_0, g_0 \models \phi$, which is a contradiction.

Conversely, assume that the L-formula $\left(\bigwedge_{1 \leq i \leq n} x_i = x_i\right) \to \phi(x_1, \ldots, x_n)$ is valid and suppose for contradiction that $\mathcal{M}, \Gamma_0 \not\models \phi$ for some $\mathcal{M}$ and Γ_0. If $\mathcal{M} = \langle D, \mathbf{Int} \rangle$, let w_0 be a world such that $dom(w_0) = D$. Let $\mathcal{P}_{w_0} := \{\mathbf{I}_a : a \in D\}$, where each $\mathbf{I}_a = \{\langle w_0, a \rangle\}$. Note that each world line $\mathbf{I}_a$ is realized in w_0. Observe also that $\mathbf{I}_a \neq \mathbf{I}_{a'}$ whenever $a \neq a'$. Define an interpretation Int by letting $Int(Q, w_0) := \mathbf{Int}(Q)$. Finally, let $M := \langle W, \mathcal{R}, \mathcal{P}, \mathcal{I} \rangle$, where $W = \{w_0\}$ and $\mathcal{R} = \varnothing = \mathcal{I}$ and $\mathcal{P} = \{\mathcal{P}_{w_0}\}$. If Γ is any assignment with values in D, let g_Γ be the function defined on the set $dom(\Gamma)$ as follows: $g_\Gamma(y) := \mathbf{I}_{\Gamma(y)}$ for all $y \in dom(\Gamma)$. Thus, g_Γ is w_0-realized for $dom(\Gamma)$ and satisfies $g_\Gamma(y)(w_0) = \Gamma(y)$. We may, then, prove by induction on the complexity of an FOL*-formula θ that for all assignments $\Gamma : Free(\theta) \to D$, we have $\mathcal{M}, \Gamma \models \theta$ iff $M, w_0, g_\Gamma \models \theta$. The base case of atomic formulas $Q(y_1, \ldots, y_m)$ is in force, since $\mathcal{M}, \Gamma \models Q(y_1, \ldots, y_m)$ iff $\langle \Gamma(y_1), \ldots, \Gamma(y_m) \rangle \in \mathbf{Int}(Q)$ iff $\langle g_\Gamma(y_1)(w_0), \ldots, g_\Gamma(y_m)(w_0) \rangle \in Int(Q, w)$ iff $M, w_0, g_\Gamma \models Q(y_1, \ldots, y_m)$. The inductive cases for $\neg$, $\wedge$, and physical quantifiers readily follow. We may conclude that $\mathcal{M}, \Gamma_0 \models \phi$ iff $M, w_0, g_{\Gamma_0} \models \phi$. Since $\left(\bigwedge_{1 \le i \le n} x_i = x_i \right) \to \phi(x_1, \ldots, x_n)$ is valid, we have $M, w_0, g_{\Gamma_0} \models \left(\bigwedge_{1 \le i \le n} x_i = x_i \right) \to \phi(x_1, \ldots, x_n)$. Now, the assignment g_{Γ_0} is w_0-realized for $Free(\phi)$, whence $M, w_0, g_{\Gamma_0} \models \bigwedge_{1 \le i \le n} x_i = x_i$ and therefore $M, w_0, g_{\Gamma_0} \models \phi(x_1, \ldots, x_n)$. Consequently, $\mathcal{M}, \Gamma_0 \models \phi$. This is a contradiction. □

We can now prove the undecidability of the existence-entailment problem of L.

Theorem 5.2 *The problem of determining whether an L-formula is existence-entailing (safe for substitution) is undecidable.*

Proof We observe, first, that if ϕ is any L-formula such that $Free(\phi) \neq \varnothing$, we have:

$$\neg\Box_{\mathrm{j}}\phi \text{ is existence-entailing iff } \neg\Box_{\mathrm{j}}\phi \text{ is contradictory (not satisfiable).}$$

To begin with, if $\neg\Box_{\mathrm{j}}\phi$ is satisfiable, then so is $\neg\phi$: there is a model $M = \langle W, \mathcal{R}, \mathcal{P}, \mathcal{I} \rangle$, a world $w \in W$, and an assignment g in M such that $M, w, g \models \neg\phi$. Construct a model $M' = \langle W', \mathcal{R}', \mathcal{P}', \mathcal{I}' \rangle$ from M as follows. Let $\mathcal{P}' := \mathcal{P}$, $\mathcal{I}' := \mathcal{I}$, and $W' := W \cup \{w^*\}$, where $w^* \notin W$. Further, let $\mathcal{R}'$ contain exactly the same relations as $\mathcal{R}$, except that its relation R'_{j} corresponding to the index j equals $R_{\mathrm{j}} \cup \{\langle w^*, w \rangle\}$. Consequently, w^* cannot be accessed from w along (results of composing) relations belonging to $\mathcal{R}$, though w can be accessed from w^* along R'_{j}. Evidently $M', w, g \models \neg\phi$ and therefore $M', w^*, g \models \Diamond_{\mathrm{j}}\neg\phi$. Note that there is no variable $x \in Free(\phi)$ such that $g(x)$ is defined on w^*. (The world w^* was brought in from outside M.) It follows that the formula $\Diamond_{\mathrm{j}}\neg\phi$ and therefore its equivalent $\neg\Box_{\mathrm{j}}\phi$ are not existence-entailing: these formulas contain at least one free variable x and they are satisfied in M' at w^* under g, but $w^* \notin marg(g(x))$. We have shown (by contraposition) that if $\neg\Box_{\mathrm{j}}\phi$ is existence-entailing, it is contradictory. Conversely, if $Free(\phi) = \{x_1, \ldots, x_n\}$ and $\neg\Box_{\mathrm{j}}\phi$ is contradictory, it is trivially existence-entailing—it satisfies trivially the condition that in *all* structures in which it is satisfied (namely, in none), also the formula $\bigwedge_{1 \le i \le n} x_i = x_i$ is satisfied.

Let us proceed to observe that for every $\phi \in L$ with at least one free variable,

$$\neg\Box_{\mathrm{j}}\phi \text{ is contradictory iff } \phi \text{ is valid.}$$

First, if ϕ is not valid, then $\neg\phi$ and therefore $\Diamond_{\mathrm{j}}\neg\phi$ are satisfiable, whence $\neg\Box_{\mathrm{j}}\phi$ is not contradictory. Second, if $\neg\Box_{\mathrm{j}}\phi$ is not contradictory, then $\Diamond_{\mathrm{j}}\neg\phi$ and therefore $\neg\phi$ are satisfiable, whence ϕ is not valid.

Suppose, then, for contradiction that the existence-entailment problem of L is decidable. Let $\theta(x_1, \ldots, x_n)$ be an arbitrary FOL*-formula containing at least one free variable (i.e., $n \geq 1$). Now, $\big(\bigwedge_{1\leq i\leq n} x_i = x_i\big) \rightarrow \theta$ is an L-formula with at least one free variable, so it follows in particular that

$$\neg\Box_{\mathrm{j}}[\big(\bigwedge\nolimits_{1\leq i\leq n} x_i = x_i\big) \rightarrow \theta] \text{ is existence-entailing iff}$$
$$\big(\bigwedge\nolimits_{1\leq i\leq n} x_i = x_i\big) \rightarrow \theta \text{ is valid.}$$

By the Lemma above, we may infer that

$$\neg\Box_{\mathrm{j}}[\big(\bigwedge\nolimits_{1\leq i\leq n} x_i = x_i\big) \rightarrow \theta] \text{ is existence-entailing iff } \theta \text{ is FOL}^*\text{-valid.}$$

Since by assumption there is a decision algorithm for the existence-entailment problem of L, we can apply it to the formula $\neg\Box_{\mathrm{j}}[\big(\bigwedge_{1\leq i\leq n} x_i = x_i\big) \rightarrow \theta]$ and thereby determine whether θ is FOL*-valid. That is, the mentioned decision algorithm induces a decision method for the validity problem of FOL*, which is impossible. □

A.6 Translation of *L* into First-Order Logic

Theorem 5.3 (First-order translation of L) *For all $\phi(x_1, \ldots, x_n) \in L$, models M, worlds $w \in dom(M)$, and assignments $g : \{x_1, \ldots, x_n\} \rightarrow \mathrm{WL}(M)$, we have:*

$$M, w, g \models \phi \textit{ iff } \mathcal{M}, \Gamma_{t,w,g} \models T_t[\phi].$$

Proof Let M be an arbitrary model. I prove by induction on the complexity of ϕ the following slightly more general claim: for all suitable worlds w, assignments g, and variables s of sort 2, we have $M, w, g \models \phi(x_1, \ldots, x_n)$ iff $\mathcal{M}, \Gamma_{s,w,g} \models T_s[\phi]$. The base cases of induction concern atomic predications and identities.

Atomic formulas. Let us consider predications. Suppose $M, w, g \models Q(x_1, \ldots, x_n)$ with $g(x_j) := \mathbf{I}_j$. Thus, $\langle \mathbf{I}_1(w), \ldots, \mathbf{I}_n(w)\rangle \in Int(Q, w)$. Let $m := dom(w)$ and $i_j := Im(\mathbf{I}_j)$. Because $\mathbf{I}_j(w) \in m \cap i_j$ for all $1 \leq j \leq n$, we have $\langle m, i_1, \ldots, i_n\rangle \in \mathbf{Int}(\mathsf{Q})$. Since $\Gamma_{s,w,g}(s) = m$ and $\Gamma_{s,w,g}(x_j) = i_j$, we may infer that $\mathcal{M}, \Gamma_{s,w,g} \models \mathsf{Q}(s, x_1, \ldots, x_n)$. Conversely, suppose $\mathcal{M}, \Gamma_{s,w,g} \models T_s[Q(x_1, \ldots, x_n)]$. Letting $m := dom(w)$ and $i_j := Im(g(x_j))$, there are elements $b_1, \ldots, b_n \in m$ such that $b_j \in i_j$ and $\langle b_1, \ldots, b_n\rangle \in Int(Q, w)$. Since each $b_j = g(x_j)(w)$, it follows that $M, w, g \models Q(x_1, \ldots, x_n)$. We must still consider identities. Suppose $M, w, g \models x_1 = x_2$, where $g(x_j) := \mathbf{I}_j$. So, there is $b \in dom(w)$ such that $\mathbf{I}_1(w) = b = \mathbf{I}_2(w)$. Thus, the set $dom(w) \cap Im(\mathbf{I}_1) \cap Im(\mathbf{I}_2)$ is non-empty, whence $\mathcal{M}, \Gamma_{s,w,g} \models \mathsf{E}(s, x_1, x_2)$. Conversely, if $\mathcal{M}, \Gamma_{s,w,g} \models T_s[x_1 = x_2]$, there is b such that $\Gamma_{s,w,g}(s) \cap \Gamma_{s,w,g}(x_1) \cap \Gamma_{s,w,g}(x_2) = \{b\}$. This means that $g(x_1)(w) = b = g(x_2)(w)$ and $b \in dom(w)$, and therefore $M, w, g \models x_1 = x_2$.

Having now established the base cases, let us assume inductively that formulas $\psi(y_1, \ldots, y_m)$ and $\chi(z_1, \ldots, z_k)$ satisfy the claim. The inductive case of negation holds trivially. I proceed to consider the remaining cases.

Conjunction. Write $g_1 := g\lceil_{\{y_1,\ldots,y_m\}}$ and $g_2 := g\lceil_{\{z_1,\ldots,z_k\}}$. Now, $M, w, g \models \psi \wedge \chi$ iff: $M, w, g_1 \models \psi$ and $M, w, g_2 \models \psi$ iff: (ind. hyp.) $\mathcal{M}, \Gamma_{s,w,g_1} \models T_s[\psi]$ and $\mathcal{M}, \Gamma_{s,w,g_2} \models T_s[\chi]$ iff: $\mathcal{M}, \Gamma_{s,w,g_1} \cup \Gamma_{s,w,g_2} \models T_s[\psi \wedge \chi]$. Here, the union $\Gamma_{s,w,g_1} \cup \Gamma_{s,w,g_2}$ is a well-defined assignment: if there are variables $u \in \{y_1, \ldots, y_m\} \cap \{z_1, \ldots, z_k\}$, we have $\Gamma_{s,w,g_1}(u) = g_1(u) = g(u) = g_2(u) = \Gamma_{s,w,g_2}(u)$. Indeed, $\Gamma_{s,w,g_1} \cup \Gamma_{s,w,g_2} = \Gamma_{s,w,g}$.

Quantifiers. I prove the claim about physical quantifiers; the case of intentional quantifiers can be dealt with similarly. Suppose $M, w, g \models \exists x\psi$. There is, then, $\mathbf{I} \in \mathcal{P}_w$ such that $M, w, g[x := \mathbf{I}] \models \psi$. By the inductive hypothesis, $\mathcal{M}, \Gamma_{s,w,g[x:=\mathbf{I}]} \models T_s[\psi]$. Write $m := dom(w)$ and $i := Im(\mathbf{I})$. Since $\mathbf{I} \in \mathcal{P}_w$, we have $\langle m, i\rangle \in \mathcal{P}^*$, wherefore $\mathcal{M}, \Gamma_{s,w,g}[x := i] \models \mathsf{P}(s, x)$. Here, $\Gamma_{s,w,g}[x := i] = \Gamma_{s,w,g[x:=\mathbf{I}]}$ and it follows that $\mathcal{M}, \Gamma_{s,w,g} \models T_s[\exists x\psi]$. For the converse direction, suppose $\mathcal{M}, \Gamma_{s,w,g} \models \exists x(\mathsf{P}(s, x) \wedge T_s[\psi])$. Thus, there is $i \in U_2$ such that $\mathcal{M}, \Gamma_{s,w,g}[x := i] \models \mathsf{P}(s, x) \wedge T_s[\psi]$. Given how the predicate P is interpreted, there is $\mathbf{I} \in \mathcal{P}_w$ such that $i = Im(\mathbf{I})$. Note that $\Gamma_{s,w,g}[x := i] = \Gamma_{s,w,g[x:=\mathbf{I}]}$. By the inductive hypothesis, we may conclude that $M, w, g[x := \mathbf{I}] \models \psi$, which entails that $M, w, g \models \exists x\psi$.

Boxes. Suppose $M, w, g \models \Box_{\mathsf{i}}\psi$. I wish to show that $\mathcal{M}, \Gamma_{s,w,g} \models T_s[\Box_{\mathsf{i}}\psi]$. To this end, let m' be any element of U_1 such that $\mathcal{M}, \Gamma_{s,w,g}[u := m'] \models \mathsf{R}_{\mathsf{i}}(s, u)$. I must show that $\mathcal{M}, \Gamma_{s,w,g}[u := m'] \models T_u[\psi]$. Write $m := \Gamma_{s,w,g}(s)$. Because $\mathcal{M}, \Gamma_{s,w,g}[u := m'] \models \mathsf{R}_{\mathsf{i}}(s, u)$, we have $\langle m, m'\rangle = \langle dom(w), m'\rangle \in R_{\mathsf{i}}^*$, whence $m' = dom(w')$ for some w' with $R_{\mathsf{i}}(w, w')$. Since $M, w, g \models \Box_{\mathsf{i}}\psi$, we have $M, w', g \models \psi$, which yields $\mathcal{M}, \Gamma_{u,w',g} \models T_u[\psi]$ by the inductive hypothesis. Now, $\Gamma_{u,w',g}[s := m] = \Gamma_{s,w,g}[u := m']$ and s does not occur free in $T_u[\psi]$, so we may conclude that $\mathcal{M}, \Gamma_{s,w,g}[u := m'] \models T_u[\psi]$. Conversely, suppose $\mathcal{M}, \Gamma_{s,w,g} \models T_s[\Box_{\mathsf{i}}\psi]$. Let w' with $R_{\mathsf{i}}(w, w')$ be arbitrary. I wish to show that $M, w', g \models \psi$. Write $m := dom(w)$ and $m' := dom(w')$. We have, then, $\mathcal{M}, \Gamma_{s,w,g}[u := m'] \models \mathsf{R}_{\mathsf{i}}(s, u)$. Because $\mathcal{M}, \Gamma_{s,w,g} \models T_s[\Box_{\mathsf{i}}\psi]$, we have $\mathcal{M}, \Gamma_{s,w,g}[u := m'] \models T_u[\psi]$. Since $\Gamma_{s,w,g}[u := m'] = \Gamma_{u,w',g}[s := m]$ and s does not occur free in $T_u[\psi]$, it follows that $\mathcal{M}, \Gamma_{u,w',g} \models T_u[\psi]$. By the inductive hypothesis, it ensues that $M, w', g \models \psi$. $\square$

Appendix B
Overview of Definitions

B.1 FOL and FOML

The reader is expected to be familiar with first-order logic and with first-order modal logic interpreted according to the standard Kripke semantics. However, for clarity of exposition I recall certain aspects of these languages here.

Formulas of 'first-order logic' of a relational vocabulary τ, or FOL[τ], are those formulas of $L_0[\tau]$ that do not employ the modal operator $\Box$. Its semantics uses *models* $\langle D, \mathbf{Int}\rangle$, where D is a non-empty set and **Int** is a function such that $\mathbf{Int}(Q) \subseteq D^n$ for every positive n and n-ary predicate symbol Q in τ. If clarity so demands, these models may be referred to as 'first-order models'. An *assignment* is any function of type $Var \to D$. The standard semantics of FOL[τ] specifies recursively what it means for a formula ϕ to be *satisfied* in a model $\mathcal{M}$ under an assignment Γ, denoted $\mathcal{M}, \Gamma \models_{\mathrm{FOL}} \phi$. An occurrence of x is *free* in ϕ if in ϕ it does not lie in the scope of the quantifier $\exists x$, the notion of scope being defined in the usual way. In fact, the satisfaction of ϕ under Γ depends only on the values of Γ on the free variables of ϕ: a formula ϕ can be evaluated relative to a partial assignment defined only on the free variables of ϕ. The syntax of FOL could be generalized by including constant symbols in the vocabulary and its semantics could be formulated so as to allow non-referring constant symbols. Such generalizations are not needed in this book.

The syntax of 'first-order modal logic' of a relational vocabulary τ, or FOML[τ], is that of $L_0[\tau]$. Here the notion of free variable can be defined as in FOL. The semantics of FOML[τ] can be explained as follows. If W is a non-empty set, R is a binary relation on W, and $w \mapsto D_w$ is a map assigning to each $w \in W$ a non-empty set D_w, the structure $\langle W, R, (D_w)_{w\in W}\rangle$ is a *Kripke frame*. If F is a Kripke frame, a *Kripke model* based on F is a pair $\langle F, \mathbf{Int}\rangle$, where **Int** is a function such that for every positive n and n-ary predicate symbol $Q \in \tau$, we have $\mathbf{Int}(Q, w) \subseteq (\bigcup_{w\in W} D_w)^n$. Interpretations of predicate symbols are *not* subject to the domain constraint: if $\langle a_1, \ldots, a_n\rangle \in \mathbf{Int}(Q, w)$, it is *not* required that the a_i belong to D_w. It is only required that for every a_i there be $w_i \in W$ such that $a_i \in D_{w_i}$. By contrast, the ranges of quantifiers in a world w are restricted to D_w. If M is a Kripke model, an *assignment*

T. Tulenheimo, *Objects and Modalities*, Logic, Epistemology, and the Unity of Science 41, DOI 10.1007/978-3-319-53119-9

is any function of type $Var \to \bigcup_{w \in W} D_w$. The semantics of FOML[$\tau$] is defined by recursively specifying what it means for a formula ϕ to be *satisfied* in a Kripke model M at a world under an assignment g, denoted $M, w, g \models_K \phi$:

- $M, w, g \models_K Q(x_1, \ldots, x_n)$ iff $\langle g(x_1), \ldots, g(x_n) \rangle \in \mathbf{Int}(Q, w)$.
- $M, w, g \models_K x = y$ iff $g(x) = g(y)$.
- $M, w, g \models_K \neg\phi$ iff $M, w, g \not\models_K \phi$.
- $M, w, g \models_K (\phi \wedge \psi)$ iff $M, w, g \models_K \phi$ and $M, w, g \models_K \psi$.
- $M, w, g \models_K \Box\phi$ iff for all w' with $R(w, w')$ we have: $M, w', g \models_K \phi$.
- $M, w, g \models_K \exists x\phi$ iff there is $a \in D_w$ such that $M, w, g[x := a] \models_K \phi$.

The satisfaction of a formula ϕ in a world w under an assignment g depends only on the values of g on the free variables of ϕ.

The syntax of FOML can be generalized by allowing the use of multiple modal operators. Such a generalization is not needed for the purposes of this book. Further, the syntax can be generalized by allowing constant symbols in the same way as in the syntax of L. If c is a constant symbol, it can be stipulated that $\mathbf{Int}(c, w) \in \{*\} \cup \bigcup_{w \in W} D_w$, where $* \notin \bigcup_{w \in W} D_w$. Intuitively, $\mathbf{Int}(c, w) = *$ means that in w, c lacks a referent. A constant symbol c is a *rigid designator* if it refers to the same object in every world in which that object exists (cf. Kripke [71, pp. 48–9]). We may take this to mean that the following three conditions are met for all worlds w and v:

(i) If c refers at all in w, it refers to something that exists in w: $\mathbf{Int}(c, w) \in D_w \cup \{*\}$.
(ii) If $\mathbf{Int}(c, w) \neq * \neq \mathbf{Int}(c, v)$, then $\mathbf{Int}(c, w) = \mathbf{Int}(c, v)$.
(iii) If $\mathbf{Int}(c, w) \in D_v$, then $\mathbf{Int}(c, v) \neq *$.

By clause (ii), if c has a referent in two worlds, it refers to the same object in both worlds. By clause (i), this means that the domains of the two worlds have a non-empty intersection. By clause (iii), again, if the object to which c refers in w is present in v, then c has a referent in v, as well. By (ii), this referent $\mathbf{Int}(c, v)$ actually equals $\mathbf{Int}(c, w)$.

B.2 Modal Languages L_0 and L

The quantified modal language L_0 is introduced in Sect. 2.3, and its extension L is introduced in Sect. 3.4.

B.2.1 Syntax of L_0

Let *Var* be a set of variables and τ a set of predicate symbols, each with an associated positive arity. The language $L_0[\tau]$ of vocabulary τ is built according to the following syntax:

$$\phi ::= Q(x_1, \ldots, x_n) \mid x_1 = x_2 \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box\phi \mid \exists x\phi,$$

where n is a positive integer, the symbols $x, x_1, x_2, \ldots, x_n$ all belong to *Var*, and Q is an n-ary predicate belonging to τ.

B.2.2 Syntax of L

For all $n \geq 0$, let τ_n be a set of n-ary predicate symbols. Constant symbols are elements of τ_0. The set $\tau := \bigcup_{i \in \mathbb{N}} \tau_i$ is a *vocabulary*. Variables and constant symbols are collectively referred to as *terms*. I write *Term* for the set $Var \cup \tau_0$. Further, let $\mathbb{A}$ and $\mathbb{I}$ be finite non-empty sets of *agent markers* and *indices*, respectively. The extended quantified modal language L is recursively defined like L_0, except that it is closed under applications of modal operators $\Box_{\mathsf{i}}$ with $\mathsf{i} \in \mathbb{I}$ and applications of quantifiers E_{a} with $\mathsf{a} \in \mathbb{A}$. Moreover, atomic formulas can employ arbitrary terms. That is, the language $L[\tau]$ of vocabulary τ is built according to the following syntax:

$$\phi ::= Q(\mathsf{t}_1, \ldots, \mathsf{t}_n) \mid \mathsf{t}_1 = \mathsf{t}_2 \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_{\mathsf{i}}\phi \mid \exists x\phi \mid \mathsf{E}_{\mathsf{a}}x\phi,$$

where $n \geq 1$ and $Q \in \tau_n$ and $\mathsf{t}_1, \mathsf{t}_2, \ldots, \mathsf{t}_n \in Term$ and $x \in Var$ and $\mathsf{i} \in \mathbb{I}$ and $\mathsf{a} \in \mathbb{A}$. I refer to $\exists$ as a *physical quantifier* and to the E_{a} as *intentional quantifiers*.

B.2.3 Semantics of L_0

Models of vocabulary τ are structures $M = \langle W, R, \mathfrak{I}, Int\rangle$. Here, W is a non-empty set. Every member w of W has a specified non-empty domain $dom(w)$ whose elements are referred to as *local objects*. Further, R is a binary relation on W, and Int is a function assigning to every n-ary predicate Q of τ and element w of W a subset $Int(Q, w)$ of $dom(w)^n$. Finally, $\mathfrak{I}$ is a collection of sets $\mathfrak{I}_w$ with $w \in W$. Each element of $\mathfrak{I}_w$ is a non-empty partial function on W, assigning an element of $dom(w')$ to every w' on which this partial function is defined.

The elements of the sets $\mathfrak{I}_w$ are referred to as *world lines over* W, although strictly speaking these partial functions are not world lines in the sense discussed in Sect. 1.5. (Yet, there is a one-to-one correspondence between world lines and such partial functions.) A world line $\mathbf{I}$ is *available* in w iff $\mathbf{I} \in \mathfrak{I}_w$. And it is *realized* in w iff there is a local object $b \in dom(w)$ such that $\mathbf{I}(w) = b$. These features are, generally, independent. A world line may be available in w without being realized therein, and realized in w without being available in w. Quantifiers evaluated relative to w range over world lines available in w. Atomic formulas $Q(x_1, \ldots, x_n)$ are evaluated with reference to realizations of those world lines that have been assigned as values of the variables $x_1, \ldots, x_n$.

I refer to the domain of a world line $\mathbf{I}$ as its *modal margin*, denoted $marg(\mathbf{I})$. The set $\mathrm{WL}(M)$ is defined as the union $\bigcup_{w \in W} \mathfrak{I}_w$. An *assignment* in M is a function of type $Var \to \mathrm{WL}(M)$. If g is an assignment defined on x, then $g(x)$ is a world line. If this world line is realized in world w, the result $g(x)(w)$ of applying the function $g(x)$ to the world w is a local object that belongs to the domain of w. If g is an assignment and $\mathbf{I}$ is a world line, $g[x := \mathbf{I}]$ stands for the assignment that differs from g at most in that it assigns $\mathbf{I}$ to x. The satisfaction relation $M, w, g \models \phi$ is defined recursively for suitable models M, worlds w, assignments g, and L_0-formulas ϕ as follows:

- $M, w, g \models Q(x_1, \ldots, x_n)$ iff for all $1 \leq i \leq n$, the world line $g(x_i)$ is realized in the world w, and the tuple $\langle g(x_1)(w), \ldots, g(x_n)(w)\rangle$ belongs to $Int(Q, w)$.
- $M, w, g \models x_1 = x_2$ iff the world lines $g(x_1)$ and $g(x_2)$ are both realized in the world w, and the local object $g(x_1)(w)$ is the same as the local object $g(x_2)(w)$.
- $M, w, g \models \neg\phi$ iff $M, w, g \not\models \phi$.
- $M, w, g \models (\phi \wedge \psi)$ iff $M, w, g \models \phi$ and $M, w, g \models \psi$.
- $M, w, g \models \Box\phi$ iff for all worlds w' with $R(w, w')$, we have: $M, w', g \models \phi$.
- $M, w, g \models \exists x\phi$ iff there is $\mathbf{I} \in \mathcal{I}_w$ such that $M, w, g[x := \mathbf{I}] \models \phi$.

B.2.4 Semantics of L

If $\mathsf{a} \in \mathbb{A}$, then α is the agent denoted by 'a' and A is the set of agents denoted by markers in the set $\mathbb{A}$. *Frames* are structures $\langle W, \mathcal{R}, \mathcal{P}, \mathcal{I}\rangle$, where $\mathcal{R} = \{R_\mathsf{i} : \mathsf{i} \in \mathbb{I}\}$ is a family of accessibility relations, and $\mathcal{P} = \{\mathcal{P}_w : w \in W\}$ and $\mathcal{I} = \{\mathcal{I}_w^\alpha : w \in W, \alpha \in A\}$ are families of world lines over W. *Models* of vocabulary τ are structures $M = \langle F, Int\rangle$, where F is a frame and Int is an interpretation function defined otherwise as in connection with L_0, except that it associates every constant symbol c and world w with an element of the set $dom(w) \cup \{*\}$, where $* \notin \bigcup_{v \in W} dom(v)$. We define $\mathrm{WL_P}(M) := \bigcup_{w \in W} \mathcal{P}_w$ and $\mathrm{WL_I}(M) := \bigcup_{w \in W, \alpha \in A} \mathcal{I}_w^\alpha$. Further, $\mathrm{WL}(M) := \mathrm{WL_P}(M) \cup \mathrm{WL_I}(M)$. I refer to the elements of $\mathrm{WL_P}(M)$ as *physically individuated world lines* and to those of $\mathrm{WL_I}(M)$ as *intentionally individuated world lines*. The *value* $\mathsf{t}^{M,w,g}$ of term t in model M at world w under assignment $g : Var \to \mathrm{WL}(M)$ is defined as follows depending on whether t is a constant symbol or a variable:

$$\mathsf{t}^{M,w,g} = \begin{cases} Int(\mathsf{t}, w) & \text{if } \mathsf{t} \in \tau_0 \text{ and } Int(\mathsf{t}, w) \neq * \\ g(\mathsf{t})(w) & \text{if } \mathsf{t} \in Var \text{ and } g(\mathsf{t}) \text{ is realized in } w. \end{cases}$$

It is assumed that an element of the set $\mathrm{WL_P}(M)$ is available in a world iff it is realized therein, whereas for elements of $\mathrm{WL_I}(M)$, availability and realization are mutually independent properties. The following further hypotheses H1 through H4 are made concerning the sets $\mathrm{WL_P}(M)$ and $\mathrm{WL_I}(M)$:

H1. *No two physically individuated world lines overlap:* If $\mathbf{I}, \mathbf{J} \in \mathrm{WL_P}(M)$ and there is w such that $w \in marg(\mathbf{I}) \cap marg(\mathbf{J})$ and $\mathbf{I}(w) = \mathbf{J}(w)$, then for all $v \in W$ we have: [either $v \notin marg(\mathbf{I}) \cup marg(\mathbf{J})$, or $v \in marg(\mathbf{I}) \cap marg(\mathbf{J})$ and $\mathbf{I}(v) = \mathbf{J}(v)$].

H2. *Realizations of physically individuated world lines are local objects:* If $w \in W$ and $\mathbf{I} \in \mathcal{P}_w$, then $\mathbf{I}(w) \in dom(w)$.
H3. *Realizations of intentionally individuated world lines are local objects:* If $\alpha \in A$ and $w, v \in W$ and $\mathbf{I} \in \mathcal{I}_w^\alpha$ and $\mathbf{I}$ is realized in v, then $\mathbf{I}(v) \in dom(v)$.
H4. *Every local object is the realization of some physical object:* If $w \in W$ and $b \in dom(w)$, then there is $\mathbf{I} \in \mathcal{P}_w$ such that $b = \mathbf{I}(w)$.

The clauses used for defining the semantics of L are otherwise as in L_0, except that in a world w, the physical quantifier $\exists$ ranges over the set $\mathcal{P}_w$ and the intentional quantifier $\mathsf{E_a}$ over the set $\mathcal{I}_w^\alpha$. Further, the accessibility relation associated with the modal operator $\Box_\mathsf{i}$ depends on the index i. Finally, the clauses for atomic formulas must be modified, since atomic formulas may contain constant symbols. Here are the clauses that need modifications:

- $M, w, g \models Q(\mathsf{t}_1, \ldots, \mathsf{t}_n)$ iff for all $1 \leq i \leq n$, the value $\mathsf{t}_i^{M,w,g}$ of the term t_i in M at w under g is defined, and the tuple $\langle \mathsf{t}_1^{M,w,g}, \ldots, \mathsf{t}_n^{M,w,g} \rangle$ belongs to $Int(Q, w)$.
- $M, w, g \models \mathsf{t}_1 = \mathsf{t}_2$ iff for all $i \in \{1, 2\}$, the value $\mathsf{t}_i^{M,w,g}$ of the term t_i in M at w under g is defined, and $\mathsf{t}_1^{M,w,g}$ equals $\mathsf{t}_2^{M,w,g}$.
- $M, w, g \models \exists x \phi$ iff there is $\mathbf{I} \in \mathcal{P}_w$ such that $M, w, g[x := \mathbf{I}] \models \phi$.
- $M, w, g \models \mathsf{E_a} x \phi$ iff there is $\mathbf{I} \in \mathcal{I}_w^\alpha$ such that $M, w, g[x := \mathbf{I}] \models \phi$.
- $M, w, g \models \Box_\mathsf{i} \phi$ iff for all worlds w' with $R_\mathsf{i}(w, w')$ we have: $M, w', g \models \phi$.

B.3 Semantic Values and Features of Intensional Predicates

Formulas of the quantified modal languages L_0 and L give rise to intensional predicates. Any such formula $\phi(x_1, \ldots, x_n)$ with n free variables can be considered as an intensional n-ary predicate that applies in a model M at a world w to those n-tuples of world lines that satisfy it in M at w. In Sect. 2.4, this observation leads to the following definition of the semantic value of a formula.

Definition 2.1 (*Semantic value*) Let M be a model, and let $\phi(x_1, \ldots, x_n)$ be a formula of the language L_0. The *semantic value* $|\phi(x_1, \ldots, x_n)|^M$ of ϕ in M is the set of all $(n+1)$-tuples $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in dom(M) \times \mathrm{WL}(M)^n$ such that

$$M, w, x_1 := \mathbf{I}_1, \ldots, x_n := \mathbf{I}_n \models \phi(x_1, \ldots, x_n).$$

If ϕ is a sentence, then $|\phi|^M$ is a (possibly empty) subset of $dom(M)$—namely, the set of worlds w at which ϕ is true in M. □

We say that y is *free for* x in ϕ iff x does not occur free in the scope of the quantifier $\exists y$ in ϕ. If $\phi(x_1, \ldots, x_n)$ is an L_0-formula and $y_1, \ldots, y_n$ are variables, all of which are free for every variable $x_1, \ldots, x_n$ in ϕ, then $\phi[x_1 /\!/ y_1, \ldots, x_n /\!/ y_n]$

stands for the result of uniformly replacing all free occurrences of the variable x_i in ϕ by the variable y_i for all $1 \leq i \leq n$ (see Sect. 2.4). A unary intensional predicate is *existence-entailing* if it can be satisfied in a world w only by a world line that is realized in w. It is *pro mundo* if its satisfaction in w by a world line depends only on the realization (if any) of the world line in w. More generally, these features of intensional predicates are defined as follows.

Definition 2.2 Let $\phi(x_1, \ldots, x_n)$ be a predicate in L_0. It is *existence-entailing* if the formula $\phi(x_1, \ldots, x_n) \rightarrow \bigwedge_{1 \leq i \leq n} x_i = x_i$ is valid. It is *pro mundo* if the formula $\big(\phi(x_1, \ldots, x_n) \wedge \bigwedge_{1 \leq i \leq n} x_i = y_i\big) \rightarrow \phi[x_1 /\!\!/ y_1, \ldots, x_n /\!\!/ y_n]$ is valid, given that each variable y_i is free for every variable x_j. It is *quasi-extensional* if it is both existence-entailing and *pro mundo*. □

A quasi-extensional predicate can be satisfied in a world w only by a tuple of world lines all of which are realized in w, and the satisfaction of such a predicate in w depends only on the realizations of those world lines in w. Semantic values of quasi-extensional predicates can be encoded by interpretations of extensional predicates—i.e., predicates satisfied by tuples of local objects.

B.4 Modes of Predication

Two *modes of predication* are distinguished in Sect. 4.2: the physical and the intentional. Let M be a model with the interpretation function *Int*. Suppose w_0 is a world, $\mathbf{I}_1, \ldots, \mathbf{I}_n$ are (physically or intentionally individuated) world lines, and $\phi(x_1, \ldots, x_n)$ is an intensional predicate. Further, let $R_{\mathsf{i}}(w_0)$ be the set of worlds compatible with the intentional state i at w_0.

- *Physical predication:* Ascribing $\phi(x_1, \ldots, x_n)$ to the tuple of world lines $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ in w_0 under the physical mode is to affirm that $\langle w_0, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in |\phi|^M$.
- *Intentional predication:* Ascribing $\phi(x_1, \ldots, x_n)$ to the tuple of world lines $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ in w_0 under the intentional mode relative to state i is to affirm that $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in |\phi|^M$ for all $w \in R_{\mathsf{i}}(w_0) \cap \bigcap_{1 \leq j \leq n} marg(\mathbf{I}_j)$.

B.5 Contents

In terms of the following general concept of *content*, the further notions of *situated content* and *internal modal margin* of a world line are defined (Sect. 2.5). Contents may but need not be propositional.

Definition 2.3 (*Content, situated content, internal modal margin*) Let M be a model. Let $V \subseteq dom(M)$ and $\mathbf{I}_1, \ldots, \mathbf{I}_n \in \mathrm{WL}(M)$. The structure $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ is an *n-ary content* over M. The set V is its *propositional component*, and the $\mathbf{I}_j$ are its *world* *line components*. A content is *propositional* if $n = 0$, otherwise it is said to have a propositional and a non-propositional aspect. If R is a binary relation on $dom(M)$, $w^* \in dom(M)$, and $V = R(w^*)$, the structure $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n, w^* \rangle$ is an *R-situated n-ary content*. The set $V \cap marg(\mathbf{I}_j)$ is the *internal modal margin* of $\mathbf{I}_j$. □

Let $Cont_n[M]$ be the set of all n-ary contents over M. Relative to any given model, any formula generates a set of contents.

Definition 2.4 (*Contents generated by a formula*) Let M be a model. The set $Cont(\phi, M)$ of *contents generated by $\phi(x_1, \ldots, x_n)$ in M* is the smallest subset of $Cont_n[M]$ satisfying the following condition: if V is non-empty and $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in |\phi|^M$ for all $w \in V$, then $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in Cont(\phi, M)$. □

We discern two ways in which a content may be related to a formula: by *locally* or *uniformly supporting* the formula.

Definition 2.5 (*Formulas supported by a content*) Let $C = \langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n, w^* \rangle$ be a situated n-ary content over M, and let $\phi(x_1, \ldots, x_n)$ be an L_0-formula. *C locally supports ϕ* (in symbols $C \Vdash_{loc} \phi$) if $\langle w^*, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in |\phi|^M$. It *uniformly supports ϕ* (in symbols $C \Vdash_{uni} \phi$) if $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in Cont(\phi, M)$. □

Sometimes it is more convenient to speak of the converses of these relations. In Sect. 6.5 the following terminology is adopted: we say that ϕ is *true of C* iff C locally supports ϕ, and that ϕ *describes C* iff C uniformly supports ϕ.

B.6 Syntactic Properties of Modal Formulas

As defined in Sect. 5.2, the *modal depth* of an L-formula ϕ, denoted $md(\phi)$, is the maximum number of nested modal operators occurring in ϕ. The *degree* of ϕ is the number indices of modal operators occurring in ϕ. The following definition introduces the syntactic notions of modal character and modal profile.

Definition 5.1 (*Modal character, modal profile*) If $\phi \in L$, let $\mathsf{i}_1, \ldots, \mathsf{i}_k$ be the indices of modal operators occurring in ϕ. If $m \in \mathbb{N}$, let $\langle \mathsf{j}_1, \ldots, \mathsf{j}_m \rangle$ be a tuple whose members are among the elements of the set $\{\mathsf{i}_1, \ldots, \mathsf{i}_k\}$. (The tuple may contain several occurrences of one and the same index: if $k \geq 1$, we may have $m > k$.) The tuple $\langle \mathsf{j}_1, \ldots, \mathsf{j}_m \rangle$ is a *modal character in ϕ* if $m \geq 1$ and it satisfies the following: there are in ϕ modal operator tokens $\bigcirc^1, \ldots, \bigcirc^m$ with respective indices $\mathsf{j}_1, \ldots, \mathsf{j}_m$ such that for all $1 \leq r < m$, $\bigcirc^{r+1}$ is the immediate successor of $\bigcirc^r$ along the relation of syntactic subordination among modal operator tokens in ϕ, and $\bigcirc^1$ is not subordinate to any modal operator in ϕ. The *modal profile* of ϕ (denoted P_ϕ) is the set of all modal characters in ϕ. □

B.7 Relative Rigid Designators

According to the semantics of L, a constant symbol does not stand for a name of a physically individuated world line; it stands for a *realization* of such a world line. If 'rigid designators' are constant symbols that stand for the same object in all worlds in which that object exists, then by the semantics of L, there are no non-trivial cases of rigid designators: a constant symbol cannot refer to the same object in two worlds. Namely, in each world w, the interpretation of a constant symbol is a local object belonging to $dom(w)$, and no local object appears in the domains of several worlds. The notion of rigid designator can, however, be simulated as follows (Sect. 5.3).

Definition 5.2 (*Relative rigid designator*) If $\mathsf{i}_1 \dots \mathsf{i}_m$ is a finite string of indices and c is a constant symbol, let us write $\theta^c_{\mathsf{i}_1\dots\mathsf{i}_m} := \exists x(x = c \wedge \Box_{\mathsf{i}_1} \cdots \Box_{\mathsf{i}_m} x = c)$. We say that c is a *relative rigid designator of type* $\mathsf{i}_1 \dots \mathsf{i}_m$ in M at w iff $M, w \models \theta^c_{\mathsf{i}_1\dots\mathsf{i}_m}$. □

That is, c is a rigid designator relative to the string $\mathsf{i}_1 \dots \mathsf{i}_m$ of types of modalities and relative to the world w if there is a physically individuated world line $\mathbf{I} \in \mathcal{P}_w$ whose realization is named by 'c' in w and in all worlds w' such that $(R_{\mathsf{i}_1} \circ \cdots \circ R_{\mathsf{i}_m})(w, w')$ but possibly not in any further world.

B.8 Substitutions and Validity

The following definition given in Sect. 5.5 provides a notion of substitution that consists of replacing atomic formulas by specified arbitrary formulas within an L-formula. (Recall the notation $\phi[x_1 /\!/ y_1, \dots, x_n /\!/ y_n]$ for uniform substitution of y_i for x_i; see Sect. B.3 of this appendix.)

Definition 5.3 (*Substitution, base of substitution*) Let τ_1 and τ_2 be disjoint vocabularies. Let V_1 and V_2 be disjoint subsets of *Var*, with $V_2 = \{v_i : i \geq 1\}$. A *base of substitution* is a map $\upsilon : \tau_1 \to L[\tau_1 \cup \tau_2, V_2]$ that assigns to every n-ary predicate P of τ_1 an $L[\{P\} \cup \tau_2, V_2]$-formula $\upsilon(P)$ whose free variables are $v_1, \dots, v_n$. A map $\sigma : L[\tau_1, V_1] \to L[\tau_1 \cup \tau_2, V_1 \cup V_2]$ is an υ*-substitution* (or *substitution based on* υ) if it satisfies the following:

- $\sigma[P(x_1, \dots, x_n)] := \upsilon(P)[v_1 /\!/ x_1, \dots, v_n /\!/ x_n]$
- $\sigma[x = y] := x = y$
- $\sigma[\neg\psi] := \neg\sigma[\psi]$
- $\sigma[(\psi_1 \wedge \psi_2)] := (\sigma[\psi_1] \wedge \sigma[\psi_2])$
- $\sigma[\Box_{\mathsf{i}}\psi] := \Box_{\mathsf{i}}\sigma[\psi]$
- $\sigma[\mathsf{Q}x\psi] := \mathsf{Q}x\sigma[\psi]$ for $\mathsf{Q} \in \{\exists\} \cup \{\mathsf{E}_{\mathsf{a}} : \mathsf{a} \in \mathbb{A}\}$. □

The following variant of the above notion of substitution is considered, as well.

Definition 5.4 (*Strong substitution, strong base of substitution*) Let the sets τ_1, τ_2, V_1, and V_2 be as in Definition 5.3. A *strong base of substitution* is a base of substitution $\rho : \tau_1 \to L[\tau_1 \cup \tau_2, V_2]$ such that for all $P \in \tau_1$, there is $\chi_P \in L[\{P\} \cup \tau_2, V_2]$ satisfying $\rho(P) = \chi_P(v_1, \ldots, v_n) \wedge \bigwedge_{1 \leq i \leq n} v_i = v_i$. A *strong ρ-substitution* is a ρ-substitution, where ρ is a strong base of substitution. □

Two notions of validity are distinguished in Sect. 5.5. An L-formula ϕ is *model-theoretically valid* if it is satisfied in all models M in all worlds $w \in dom(M)$ under all assignments in M. The formula ϕ is *schematically valid* if for all substitutions σ, the formula $\sigma[\phi]$ is model-theoretically valid. It is shown that in L, model-theoretic validity is not preserved under uniform substitution, though it is preserved under strong uniform substitution.

References

1. Anscombe, Elizabeth (1965). 'The Intentionality of Sensation: A Grammatical Feature,' in R. J. Butler (ed.), *Analytical Philosophy: Second Series* (Oxford: Blackwell), pp. 158–80.
2. Anscombe, Elizabeth (1979). 'Under a Description,' *Noûs*, 13, pp. 219–33.
3. Areces, Carlos, and ten Cate, Balder (2007). 'Hybrid Logics,' in P. Blackburn *et al.* (eds.), *Handbook of Modal Logic* (Amsterdam: Elsevier), pp. 821–68.
4. Belnap, Nuel, and Müller, Thomas (2014). 'CIFOL: Case-Intensional First Order Logic,' *Journal of Philosophical Logic*, 43, pp. 393–437.
5. Braüner, Torben, and Ghilardi, Silvio (2007). 'First-Order Modal Logic,' in P. Blackburn *et al.* (eds.), *Handbook of Modal Logic* (Amsterdam: Elsevier), pp. 549–620.
6. Bressan, Aldo (1972). *A General Interpreted Modal Calculus* (New Haven, CT: Yale University Press).
7. Bricker, Phillip (1993). 'The Fabric of Space: Intrinsic vs. Extrinsic Distance Relations,' *Midwest Studies in Philosophy*, 18, pp. 271–94.
8. Burgess, John (1999). 'Which Modal Logic Is the Right One?,' *Notre Dame Journal of Formal Logic*, 40, pp. 81–93.
9. Burgess, John (2003). 'Which Modal Logics Are the Right Ones (for Logical Necessity)?,' *Theoria* (San Sebastián), 18, pp. 145–58.
10. Byrne, Alex (2001). 'Intentionalism Defended,' *Philosophical Review*, 110, pp. 199–240.
11. Carnap, Rudolf (1928). *Der logische Aufbau der Welt* (Berlin: Weltkreis). Translated by Rolf A. George as *The Logical Structure of the World. Pseudoproblems in Philosophy* (Berkeley: University of California Press, 1967).
12. Carnap, Rudolf (1947). *Meaning and Necessity: A Study in Semantics and Modal Logic* (Chicago: Chicago University Press).
13. Cassam, Quassim (1999). 'Self-Directed Transcendental Arguments,' in R. Stern (ed.), *Transcendental Arguments: Problems and Prospects* (Oxford: Oxford University Press), pp. 83–110.
14. Chisholm, Roderick (1967). 'Identity through Possible Worlds: Some Questions,' *Noûs*, 1, pp. 1–8.
15. Church, Alonzo (1956). *Introduction to Mathematical Logic* (Princeton, NJ: Princeton University Press).
16. Clark, Austen (1996). 'Three Varieties of Visual Field,' *Philosophical Psychology*, 9, pp. 477–95.
17. Crane, Tim (1992). 'The Nonconceptual Content of Experience,' in T. Crane (ed.), *The Contents of Experience: Essays on Perception* (Cambridge: Cambridge University Press), pp. 136–57.

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18. Crane, Tim (2009). ‘Is Perception a Propositional Attitude?,’ *The Philosophical Quarterly*, 59, pp. 452–69.
19. Crane, Tim (2009). ‘Intentionalism,’ in A. Beckermann and B. McLaughlin (eds.), *Oxford Handbook to the Philosophy of Mind* (Oxford: Oxford University Press), pp. 474–93.
20. Crane, Tim (2011). ‘The Problem of Perception,’ in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*: Spring 2011 edition.
21. Crane, Tim (2013). *The Objects of Thought* (Oxford: Oxford University Press).
22. Crane, Tim (2013). ‘The Given,’ in J. Schear (ed.) *Mind, Reason and Being-in-the-World: The McDowell-Dreyfus Debate* (London: Routledge), pp. 229–49.
23. Dummett, Michael (1978). *Truth and Other Enigmas* (London: Duckworth).
24. Evans, Gareth (1982). *The Varieties of Reference* (Oxford: Oxford University Press).
25. Fine, Kit (1999). ‘Things and Their Parts,’ *Midwest Studies in Philosophy*, 23, pp. 61–74.
26. Fitch, Greg, and Nelson, Michael (2013). ‘Singular Propositions,’ in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*: Winter 2013 edition.
27. Fitting, Melvin, and Mendelsohn, Richard (1998). *First-Order Modal Logic* (Dordrecht: Kluwer).
28. Forbes, Graeme (2013). ‘Intensional Transitive Verbs,’ in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*: Fall 2013 edition.
29. Frege, Gottlob (1974) [1884]. *The Foundations of Arithmetic: A Logico-Mathematical Enquiry into the Concept of Number*, translation by J. L. Austin (Oxford: Blackwell, second revised edition).
30. Frege, Gottlob (1980). *Philosophical and Mathematical Correspondence* (Oxford: Blackwell).
31. Gamut, L. T. F. (1991). *Logic, Language, and Meaning, vol. 2: Intensional Logic and Logical Grammar* (Chicago: The University of Chicago Press).
32. Gardner, Sebastian (1999). *Kant and the Critique of Pure Reason* (London: Routledge).
33. Garson, James (2014). ‘Modal Logic,’ in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*: Summer 2014 edition.
34. Geach, Peter (1967). ‘Intentional Identity,’ *The Journal of Philosophy*, 64, pp. 627–32.
35. Grandy, Richard (2007). ‘Sortals,’ in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*: Summer 2007 edition.
36. Grosu, Alexander, and Krifka, Manfred (2007). ‘The Gifted Mathematician that You Claim to Be: Equational Intensional “Reconstruction” Relatives,’ *Linguistics and Philosophy*, 30, pp. 445–85.
37. Hall, Ned (2010). ‘David Lewis’s Metaphysics,’ in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*: Fall 2010 edition.
38. Hawley, Katherine (2001). *How Things Persist* (Oxford: Clarendon Press).
39. Hawley, Katherine (2015). ‘David Lewis on Persistence,’ in B. Loewer, and J. Schaffer (eds.), *A Companion to David Lewis* (Oxford: Wiley-Blackwell), pp. 237–49.
40. Hintikka, Jaakko (1956). ‘Identity, Variables, and Impredicative Definitions,’ *Journal of Symbolic Logic*, 21, pp. 225–45.
41. Hintikka, Jaakko (1962). *Knowledge and Belief* (Ithaca, NY: Cornell University Press).
42. Hintikka, Jaakko (1969). ‘On the Logic of Perception,’ in N. Care, and R. Grimm (eds.), *Perception and Personal Identity* (Cleveland, OH: Case Western Reserve University Press), pp. 140–73. References are to the reprint in [44, pp. 151–83]
43. Hintikka, Jaakko (1969). ‘Semantics for Propositional Attitudes,’ in W. Davis *et al.* (eds.), *Philosophical Logic* (Dordrecht: Reidel), pp. 21–45. References are to the reprint in [44, pp. 87–111]
44. Hintikka, Jaakko (1969). *Models for Modalities* (Dordrecht: Reidel).
45. Hintikka, Jaakko (1973). ‘Carnap’s Semantics in Retrospect,’ *Synthese*, 25, pp. 372–97.
46. Hintikka, Jaakko (1975). *The Intentions of Intentionality and Other New Models for Modalities* (Dordrecht: Reidel).
47. Hintikka, Jaakko (1998). ‘Perspectival Identification, Demonstratives and “Small Worlds”,’ *Synthese*, 114, pp. 203–32.

48. Hintikka, Jaakko (1998). *Paradigms for Language Theory and Other Essays*, Selected Papers, vol. 4 (Dordrecht: Kluwer).
49. Hintikka, Jaakko (2006). 'Intellectual Autobiography' and Hintikka's replies to the contributors, in R. E. Auxier, and L. E. Hahn (eds.), *The Philosophy of Jaakko Hintikka, The Library of Living Philosophers*, vol. 30 (Chicago: Open Court).
50. Hintikka, Jaakko, and Hintikka, Merrill B. (1982). 'Towards a General Theory of Individuation and Identification,' in W. Leinfellner *et al.* (eds.), *Language and Ontology: Proceedings of the Sixth International Wittgenstein Symposium* (Vienna: Hölder-Pichler-Tempsky), pp. 137–50.
51. Hintikka, Jaakko, and Sandu, Gabriel (1995). 'The Fallacies of the New Theory of Reference,' *Synthese*, 104, pp. 245–83.
52. Hintikka, Jaakko, and Symons, John (2003). 'Systems of Visual Identification in Neuroscience: Lessons from Epistemic Logic,' *Philosophy of Science*, 70, pp. 89–104.
53. Hornstein, Norbert (1990). *As Time Goes By: Tense and Universal Grammar* (Cambridge, MA: MIT Press).
54. Humberstone, Lloyd (1992). 'Direction of Fit,' *Mind*, 101, pp. 59–83.
55. Jespersen, Bjørn (2011). 'An Intensional Solution to the Bike Puzzle of Intentional Identity,' *Philosophia*, 39, pp. 297–307.
56. Jespersen, Bjørn (2015). 'Qualifying Quantifying-in,' in A. Torza (ed.), *Quantifiers, Quantifiers, and Quantifiers: Themes in Logic, Metaphysics, and Language* (Heidelberg: Springer), pp. 241–69.
57. Jónsson, Ólafur Páll (2005). 'The Bike Puzzle,' *Mind*, 114, pp. 929–32.
58. Jubien, Michael (1993). *Ontology, Modality, and the Fallacy of Reference* (Cambridge: Cambridge University Press).
59. Kalmár, László, and Surányi, János (1950). 'On the Reduction of the Decision Problem, Third Paper: Pepis Prefix, a Single Binary Predicate,' *Journal of Symbolic Logic*, 15, pp. 161–73.
60. Kant, Immanuel (1933) [first edition 1781 (A), second edition 1787 (B)]. *Critique of Pure Reason*, translation by N. Kemp Smith (London: Macmillan).
61. Kaplan, David (1989). 'Demonstratives,' in J. Almog, H. Wettstein, and J. Perry (eds.), *Themes From Kaplan* (Oxford: Oxford University Press), pp. 481–563.
62. Kirjavainen, Heikki (2008). 'How is God-Talk Logically Possible? A Sketch for an Argument on the Logic of "God",' *International Journal for Philosophy of Religion*, 64, pp. 75–88.
63. Koslicki, Kathrin (2008). *The Structure of Objects* (Oxford: Oxford University Press).
64. Kraut, Robert (1979). 'Attitudes and Their Objects,' *Journal of Philosophical Logic*, 8, pp. 197–217.
65. Kraut, Robert (1983). 'There Are No *De Dicto* Attitudes,' *Synthese*, 54, pp. 275–94.
66. Kracht, Marcus, and Kutz, Oliver (2007). 'Logically Possible Worlds and Counterpart Semantics for Modal Logic,' in D. Jacquette (ed.), *Philosophy of Logic* (Amsterdam: Elsevier), pp. 943–95.
67. Kripke, Saul (1963). 'Semantical Considerations in Modal Logic,' *Acta Philosophica Fennica*, 16, pp. 83–94.
68. Kripke, Saul (1971). 'Identity and Necessity,' in M. Munitz (ed.), *Identity and Individuation* (New York: New York University Press), pp. 135–64.
69. Kripke, Saul (1972). 'Naming and Necessity,' in D. Davidson, and G. Harman (eds.), *Semantics of Natural Language* (Dordrecht: Reidel), pp. 253–355.
70. Kripke, Saul (1976). 'Is There a Problem About Substitutional Quantification?,' in G. Evans, and J. McDowell (eds.), *Truth and Meaning* (Oxford: Oxford University Press), pp. 324–419.
71. Kripke, Saul (1980). *Naming and Necessity*. Oxford: Blackwell.
72. Kripke, Saul (2013). *Reference and Existence: The John Locke Lectures* (Oxford: Oxford University Press).
73. Lewin, Kurt (1922). *Der Begriff der Genese in Physik, Biologie und Entwicklungsgeschichte* (Berlin: Springer).
74. Lewin, Kurt (1923). 'Die zeitliche Geneseordnung,' *Zeitschrift für Physik*, 13, pp. 62–81.
75. Lewis, David (1968). 'Counterpart Theory and Quantified Modal Logic,' *The Journal of Philosophy*, 65, pp. 113–26.

76. Lewis, David (1986). *On the Plurality of Worlds* (Oxford: Blackwell).
77. Lewis, David (1986). *Philosophical Papers*, vol. 2 (Oxford: Oxford University Press).
78. Lewis, David (1994). 'Humean Supervenience Debugged,' *Mind*, 103, pp. 473–90.
79. Lewis, David (1999). *Papers in Metaphysics and Epistemology* (Cambridge: Cambridge University Press).
80. Lowe, E. Jonathan (1989). 'What Is a Criterion of Identity?,' *The Philosophical Quarterly*, 39, pp. 1–21.
81. Lowe, E. Jonathan (1989). *Kinds of Being: A Study of Individuation, Identity, and the Logic of Sortal Terms* (Oxford: Basil Blackwell).
82. Marcus, Ruth Barcan (1961). 'Modalities and Intensional Languages,' *Synthese*, 13, pp. 303–22.
83. Markosian, Ned (2000). 'What are Physical Objects?,' *Philosophy and Phenomenological Research*, 61, pp. 375–95.
84. McDowell, John (1977). '*De Re* Senses,' *The Philosophical Quarterly*, 34, pp. 283–94.
85. McDowell, John (1998). *Meaning, Knowledge and Reality* (Cambridge, MA: Harvard University Press).
86. Moltmann, Friederike (1997). 'Intensional Verbs and Quantifiers,' *Natural Language Semantics*, 5, pp. 1–52.
87. Moltmann, Friederike (2011). 'Tropes, Intensional Relative Clauses, and the Notion of a Variable Object'. Draft retrieved February 11, 2015, from http://semanticsarchive.net/Archive/zg0ZmYyN. Appeared with modifications in M. Aloni *et al.* (eds.), *Logic, Language and Meaning. Proceedings of the 18th Amsterdam Colloquium (2011)*, (Berlin: Springer, 2012), pp. 431–40.
88. Moltmann, Friederike (2015). 'Quantification with Intentional and with Intensional Verbs,' in A. Torza (ed.), *Quantifiers, Quantifiers, and Quantifiers: Themes in Logic, Metaphysics, and Language* (Heidelberg: Springer), pp. 141–68.
89. Moltmann, Friederike 3(2015). 'Variable Objects and Truth-Making'. To appear in M. Dumitru (ed.), *Metaphysics, Meaning, and Modality: Themes from Kit Fine* (Oxford: Oxford University Press). References are to the preprint retrieved February 11, 2015, from http://semanticsarchive.net/Archive/zQ2YjQzM.
90. Montague, Michelle (2007). 'Against Propositionalism,' *Noûs*, 41, pp. 503–18.
91. Montague, Richard (1973). 'The Proper Treatment of Quantification in Ordinary English,' in J. Hintikka *et al.* (eds.), *Approaches to Natural Language* (Reidel: Dordrecht), pp. 221–42.
92. Niiniluoto, Ilkka (1982). 'Remarks on the Logic of Perception,' *Acta Philosophica Fennica*, 35, pp. 116–29.
93. Pereboom, Derek (2013). 'Kant's Transcendental Arguments,' in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*: Winter 2013 edition.
94. Plantinga, Alvin (1974). *The Nature of Necessity* (Oxford: Oxford University Press).
95. Priest, Graham (2005). *Towards Non-Being: The Logic and Metaphysics of Intentionality* (Oxford: Clarendon Press).
96. Prior, Arthur (1957). *Time and Modality* (Oxford: Clarendon Press).
97. Prior, Arthur (1967). *Past, Present and Future* (Oxford: Clarendon Press).
98. Prior, Arthur (1968). *Papers on Time and Tense* (Oxford: Clarendon Press).
99. Prior, Arthur (1971). *Objects of Thought*, edited by P. Geach, and A. Kenny (Oxford: Clarendon Press).
100. Prior, Arthur (1976). *Papers in Logic and Ethics*, edited by P. Geach, and A. Kenny (London: Duckworth).
101. Quine, Willard Van Orman (1970). *Philosophy of Logic* (Englewood Cliffs, NJ: Prentice-Hall).
102. Rebuschi, Manuel, and Tulenheimo, Tero (2011). 'Between *De Dicto* and *De Re*: *De Objecto* Attitudes,' *The Philosophical Quarterly*, 61, pp. 828–38.
103. Recanati, François (2012). *Mental Files* (Oxford: Oxford University Press).
104. Russell, Bertrand (1910). 'Knowledge by Acquaintance and Knowledge by Description,' *Proceedings of the Aristotelian Society*, 11, pp. 108–28.
105. Russell, Bertrand (1912). *The Problems of Philosophy* (Oxford: Oxford University Press).

106. Russell, Bertrand (1914). *Our Knowledge of the External World* (London: George Allen & Unwin).
107. Saarinen, Esa (1982). 'Propositional Attitudes Are Not Attitudes Towards Propositions,' *Acta Philosophica Fennica*, 35, pp. 130–62.
108. Salmon, Nathan (1986). *Frege's Puzzle* (Cambridge, MA: MIT Press).
109. Schurz, Gerhard (1997). *The Is-Ought Problem* (Dordrecht: Kluwer).
110. Searle, John (1983). *Intentionality* (Cambridge: Cambridge University Press).
111. Sider, Theodore (1996). 'All the World's a Stage,' *Australasian Journal of Philosophy*, 74, pp. 433–53.
112. Sider, Theodore (2001). *Four-Dimensionalism: An Ontology of Persistence and Time* (Oxford: Oxford University Press).
113. Smith, David, and McIntyre, Ronald (1982). *Husserl and Intentionality. A Study of Mind, Meaning, and Language* (Dordrecht: Reidel).
114. Soames, Scott (1985). 'Lost Innocence,' *Linguistics and Philosophy*, 8, pp. 59–71.
115. Soames, Scott (1987). 'Direct Reference, Propositional Attitudes, and Semantic Content,' *Philosophical Topics*, 15, pp. 47–87.
116. Stroud, Barry (1968). 'Transcendental Arguments,' *Journal of Philosophy*, 65, pp. 241–56.
117. Stroud, Barry (1999). 'The Goal of Transcendental Arguments,' in R. Stern (ed.), *Transcendental Arguments: Problems and Prospects* (Oxford: Oxford University Press), pp. 155–72.
118. Tulenheimo, Tero (2009). 'Remarks on Individuals in Modal Contexts,' *Revue Internationale de Philosophie*, 63, pp. 383–94.
119. Tulenheimo, Tero (2015). 'Cross-World Identity, Temporal Quantifiers and the Question of Tensed Contents,' in A. Torza (ed.), *Quantifiers, Quantifiers, and Quantifiers: Themes in Logic, Metaphysics, and Language* (Heidelberg: Springer), pp. 409–61.
120. Tulenheimo, Tero (2016). 'Worlds, Times and Selves Revisited,' *Synthese*, 193, pp. 3713–25.
121. Tye, Michael (2000). *Consciousness, Color, and Content* (Cambridge, MA: MIT Press).
122. van Inwagen, Peter (1990). *Material Beings* (Ithaca: Cornell University Press).
123. Wasserman, Ryan (2013). 'Material Constitution,' in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*: Summer 2013 edition.
124. Weatherson, Brian (2015). 'Humean Supervenience,' in B. Loewer, and J. Schaffer (eds.), *A Companion to David Lewis* (Oxford: Wiley-Blackwell), pp. 101–15.
125. Wehmeier, Kai (2012). 'How to Live Without Identity—And Why,' *Australasian Journal of Philosophy*, 90, pp. 761–77.
126. Wiggins, David (2001). *Sameness and Substance Renewed* (Cambridge: Cambridge University Press).
127. Williamson, Timothy (2013). *Modal Logic as Metaphysics* (Oxford: Oxford University Press).
128. Zimmerman, Thomas Ede (1993). 'On the Proper Treatment of Opacity in Certain Verbs,' *Natural Language Semantics*, 1, pp. 149–79.

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T. Tulenheimo, *Objects and Modalities*, Logic, Epistemology, and the Unity of Science 41, DOI 10.1007/978-3-319-53119-9