What is the Meaning of Deep for a Mathematician?

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A mathematician could be thought of, both as a problem solver and also as a theoretician; as a scientist and also as an artist; as an arguer and also as a conjecturer; as a coach and also as a teacher; As an educator and also as an evaluator; As a philosopher and also as a physician. So, there are several roles a mathematician is playing in the mathematical society and each of these roles has its own definition and relevant skills and each has a different perspective towards what is called a deep for a mathematician.

Mathematician as a problem solver

Mathematician as a problem solver, lists strategies to attack the problem and chooses the most appropriate one; reduces the problem to simpler components and tries to solve each of them separately; estimates how much progress has been made in the process of coming to a solution; criticizes conjectures and conjectural approaches to solve the problem; tries to recognize easy from difficult and simple from complicated in the process of coming to a solution.

What is the meaning of deep for a problem solver? Powerful strategies for solving problems are deep. Important lemmas which widely appear in the structure of arguments are deep. Those steps in the argument which make much progress are deep. Conjectures could be deep the same way that arguments could be deep. Deep is simple not complicated; because it can combine with many ideas. Deep is not always easy. Deep could be difficult.

Mathematician as a theoretician

Mathematician as a theoretician makes assumptions and does repair and surgery of assumptions and theories based on experience; tests assumptions and theories; generalizes of approved assumptions and theories to wider scopes; recognizes the relation between two assumptions or two theories; compares the

strength and weakness of different assumptions and theories; and is searching for the truth.

What is the meaning of deep for a theoretician? Deep are assumptions and phenomena that widely appear. Deep are a series of related concepts, assumptions or phenomena. Deep assumptions and theories are appropriate for generalizations. Deep assumptions and theories are strong assumptions and theories. They have many implications. Deep is close to the truth.

Mathematician as a scientist

Mathematician as a scientist communicates and consults with other scientists; criticizes scientific assumptions and theories and tries to think divergent; relates the previous knowledge to the new problem considering; tries to control the nature; tries to explain and describe the nature; tries to solve everyday problems.

What is the meaning of deep for a scientist? It is important for the deep to be easily and fluently communicatable. Deep must be easily describable. It is important for the deep to be criticizable. Deep must be divergent. Deep is related to several assumptions, theories and phenomena. Deep is helpful to control the nature. Deep is related to nature. Deep is helpful to solve everyday problems.

Mathematician as an artist

Mathematician as an artist looks for beauty; looks for simplicity; looks for coherence; borrows many ideas from nature and also from metaphysics; has a hidden message and meaning in mind; is the creator of the art work; uses what is available to create what is new; creates new needs and therefore new problems to solve.

What is the meaning of deep for an artist? Deep is beautiful. Deep is simple. Deep is coherently present everywhere. Deep could be natural or metaphysical. Deep has a hidden message and meaning. Deep is created by the mathematician. Deep is made of what is available and consists of what is new. Deep creates new needs and therefore new problems to solve.

Mathematician as an arguer

Mathematician as an arguer evaluates the given reasoning; tries to correlate given information; controls and evaluates the process of discovery from outside the process of solution; tries to make conclusions presentable; correlates local to global, or parts to the whole; argues by means of postulates and their natural implications.

What is the meaning of deep for an arguer? Deep is highly evaluated among other reasonings. Deep correlates given information. Deep is recognizable from outside the process of solution. It is important for deep to be presentable. In recognition of deep one correlates local to global, or parts to the whole. Postulates and their natural implications reveal depth of arguments.

Mathematician as a conjecturer

Mathematician as a conjecturer approve assumptions and theories. As a conjecturer generalize assumptions and theories to wider scopes. By generalization one can unite the realms of two theories. Recognition of relations between assumptions via nice conjectures usually leads to unification of theories. Recognition of relations between theories via conjectures forms a paradigm. Mathematician as a conjecturer does surgery on assumptions in order to repair implications; could unify assumptions and theories by surgery and repair performed by conjectures; assesses strength and weakness of assumptions by fluency and naturality of conjectures.

What is the meaning of deep for a conjecturer? Deep approves assumptions and theories. Deep generalizes assumptions and theories to wider scopes. It is important to do surgery on deep in order to repair implications. Deep is a kind of a surgery and repair which could unify assumptions and theories. Deep assesses strength and weakness of assumptions by fluency and naturality.

Mathematician as a coach

Mathematician as a coach tries to teach doing mathematics by imitation; tries to correct the students according to their behavior; teaches skills and considers pre-skills to teach a given skill; designs practical exercises to prepare students to be taught particular pre-skills; tries to communicate with students in a practical level.

What is the meaning of deep for a coach? Deep could be taught by imitation. Deep is described according to behaviors. Teaching deep skills needs considering pre-skills. One should design practical excercizes to prepare for teaching deep. Deep could be communicated with students in a practical level. Deep could be a skill.

Mathematician as a teacher

Mathematician as a teacher tries to teach doing mathematics by conceptualization; tries to correct the students according to their thought and perspectives; teaches concepts and considers pre-concepts to teach a given concept; designs mental exercises to prepare students to be taught particular pre-concept; tries to communicate with students in a conceptual level.

What is the meaning of deep for a teacher? Deep could be taught by conceptualization. Deep is described according to thoughts and perspectives. Teaching deep concepts needs considering pre-concepts. One should design mental excercizes to prepare for teaching deep. Deep could be communicated with students in a conceptual level. Deep could be a concept.

Mathematician as an educator

Mathematician as an educator educates the patterns and structures of cognition; manages the relations between different abstract layers of cognition; studies the scientific personality of the student; correlates the knowledge of the student in different disciplines; correlates personality of the student with his knowledge.

What is the meaning of deep for an educator? Deep is defined in terms of the patterns and structures of cognition. Deep appears in and between different abstract layers of cognition. Deep is independent of the scientific personality of the student. Deep correlates the knowledge of the student in different disciplines.

Mathematician as an evaluator

As an evaluator mathematician evaluates the patterns and structures of cognition; evaluates the relations between different abstract layers of cognition; evaluates the correlation of the knowledge of the student in different disciplines;

evaluates the correlation of the knowledge of the student with his personality; evaluates the capacity of student in attending group work.

What is the meaning of deep for an evaluator? Depth can be evaluated by the patterns and structures of cognition. Fluency of appearance of depth in different abstract layers of cognition can be evaluated. Independent of depth from the scientific personality of the student can be evaluated. Depth has the capacity to engage students in group work.

Mathematician as a philosopher

Mathematician as a philosopher looks for hierarchies; tries to create hierarchies; tries to create a new language to study a package of concepts; tries to create new concepts in order to correlate unrelated concepts; tries to understand theories in light of new postulates or new assumptions; tries to discover the language of truth.

What is the meaning of deep for a philosopher? Deep stands in a hierarchy. Creating depth needs creation of hierarchies. Deep is creating a new language to study a package of concepts. Deep is creating new concepts in order to correlate unrelated concepts. Deep is to understand theories in light of new postulates or new assumptions. Deep is the language of truth.

Mathematician as a physician

Mathematician as a physician or as a pathologist recognizes diseases in different scales, such as organic, molecular, etc.; provides a language to explain the disease which is fluent for its analysis; studies the causal relations between different diseases; controls the side effects of diseases and drugs used for curing them; gives time to the body to be cured naturally.

What is the meaning of deep for a physician. Smaller the scale of pathology, deeper the disease. A language which explains the disease more fluently, is deeper. The deeper the disease is, causal affects are more serious. Deep disease has more side effects. Deep disease takes more time to be cured naturally.

Depth is the main concern of a mathematician

A mathematician's concern is not only understanding depth, but also evaluating depth, discovering depth, communicating depth, teaching depth, creating depth, designing depth are all what a mathematicians makes his or her own concern. We shall study each of these concerns separately to see what a mathematician has to do with the concept of depth.

How do mathematicians try to understand depth?

Sometimes a mathematical object is deep. In other words, depth shows itself in form of an object. Sometimes a theorem is deep. Other times a proof is deep. Deep could be a strategy; a concept; a language; a theory; or a phenomenon. In fact, deep is truth, not a model of truth. It is truth that shows up in the form of an object; a theorem; a strategy; a concept; a language; a theory; or a phenomenon. Obviously this is a very Platonic point of view towards mathematics. But, there is no way to run away. The concept of deep is Platonic in its nature. It is Platonic anthropology which lets us assume several layers of abstractness for cognition, which is a prerequisite for the concept of depth. Platonic anthropology is pictorial and wholistic. Therefore, verbals and analyzers are not supposed to be as good as the pictorials and wholists in understanding depth. Pictorials and wholists try to define depth by recognition of several layers of abstractness.

is deep, if it has several layers of abstractness. These layers of abstractness could have different formulations; be in different generalities; have different levels of computational sophistication and so on.

How do mathematicians try to evaluate depth?

Sometimes, one evaluates depth by considering a hierarchy. On can make a hierarchy of several different layers of abstractness and compare the depth of two objects, theorems, a strategies, concepts, languages, theories or a phenomena. For example, one can evaluate by comparison of concept relations. One can evaluate by comparison of influence between two objects, theorems, a strategies, concepts, languages, theories or a phenomena. Another example is evaluating depth by considering generalizability.

Sometimes, a standard of comparison of depths works for only one of objects, theorems, strategies, concepts, languages, theories or a phenomena. For

example,The more relations with neighboring concepts, the deeper that concept is. One can evaluate depth by considering simplicity, or beauty of the language which a piece of mathematics is formulated within it.

How do mathematicians try to discover depth?

For discovering depth in theorization, mathematicians look for central facts in a theory, and also look for dictionaries between theories, in order to deduce depth of a theory from depth of another. For discovering depth in problem solving, mathematicians look for powerful strategies for solving problems. For discovering depth in the language a piece of mathematics is formulated in, a mathematician looks for facts interpreted in several languages. For discovering depth in concepts, a mathematician looks for concepts related with several concepts. For discovering depth in theorems, a mathematician looks for generalizable theorems. Looking for hierarchies, is the general strategy for discovering depth.

How do mathematicians try to communicate depth with their colleagues?

Not every piece of research work tries to communicate depth. But, communicating hierarchies is the strongest method for communicating depth. Hierarchy objects, theorems, strategies, concepts, languages, theories or phenomena are the best means to communicate why a piece of mathematics is deep. For example, one can communicate generalizations of theories or communicate a package of correlated concepts. Communicating analogous theories and dictionaries between theories is another way to imply depth of a theory assuming depth of an analogous theory.

How do mathematicians try to teach depth?

Mathematicians could teach depth by teaching a piece of mathematics in several languages and formulations, or by teaching several generalizations. Mathematics teacher emphasize on correlated concepts. They teach hierarchies and incarnation of truth in several layers of abstractness. They emphasize on analogous theories and dictionaries between several theories. Mathematicians also teach phenomena and their several incarnations in order to communicate depth with their students.

How do mathematicians try to create depth?

A mathematician could work hard and try to create hierarchies of concepts, theorems or theories. One could translate a and reformulate a piece of mathematics to several different languages, or try to correlate a concept to several different concepts, or try to create several theories analogous a given theory, or try to prove several generalizations for a given theorem. All these are ways to create depth. But the main point in creating depth is incarnating truth in several layers of abstractness. Therefore one is bound to make a hierarchy.

Platonic to creating depth

Platonists believe that depth is not created. Deep mathematics is predetermined. Deep means being close to the truth and truth is created by God, not by human. Although, truth could incarnate in manmade creatures, it can be found in humanities, nature and metaphysics. Since the concept of depth is strongly tied with the concept of truth, knowledge in the light of depth is knowledge in the light of truth. In fact, knowledge is a model for the truth. Scientific languages are created to communicate models of truth. Mathematics is also a model for the truth. Only part of the truth could incarnate in mathematics. In brief, evaluating depth of knowledge is evaluation of closeness to the truth.