☐ *What is Mathematical Argument?*

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There are several possible perspectives towards this question:

☐ *In the experimental approach, we consider the actual proofs suggested by mathematicians for mathematical theorems and try to compare them and categorize them and then philosophically describe them by understanding their assumptions and their level of validity and access to the truth and the level of faith they could provide us and trying to see how rigorous they actually are.*

☐ *In the philosophical approach, we try to philosophically predict and characterize the logical capacity of mathematical arguments and their basic assumptions and the level of certainty they can provide considering the philosophical understanding we already have from human languages and the level of truth they can carry.*

☐ *In the intuitionist approach, we try to utilize our intuition in order to evaluate the true capacity of human logical mind and its basic assumptions and its perspectives towards the truth, without limiting our minds to the philosophical language and standards of thought. Intuition has a more direct access to the truth, since philosophical approach only understands by making models of truth in our mind.*

☐ *These are the well known perspectives towards understanding the role of arguments in human cognition. To us, there are not three but seven layers of cognition for humans consisting of: formal language, mind and thought, inspiration, revelation, wisdom, enlightenments and finally directly tasting the truth. The layer of wisdom coincides with the philosophical approach above and the layer of enlightenment coincide with the intuitionist approach above and the layer of mind and thought coincides with the experimental approach above. The level of formal language is the understanding computers can have of what an argument is. Revelation is a spiritual way of understanding and inspiration is another metaphysical type of cognition related to the realm of heart. In each of these layers of cognition we have a different understanding of what a mathematical argument is.*

☐ *Humanistic experimental approach is introduced by Aristotle. Hierarchies of cognition are introduced by Plato. His hierarchy of consist of layers of wisdom, soul, and body. Hierarchy of several layers of darkness and light was introduced by Ibn-e-Arabi. Hierarchy of several layers of light was introduced by Sohrevardi. These are purely intuitionist cognition structures. Hierarchy of several layers of existence was introduced by Ibn-e-Sina. This is a purely philosophical cognition structure with several layers of cognition. In our approach, the hierarchy of cognition consists of personality, light, wisdom, spirit, heart, soul and body.*

Possible approaches to communication with audience

☐ *The most important thing about an argument is that it should be communicated, and that puts all intuitionists' approaches on the side. Because, communication of intuition means forcing one's vision to other people's cognition which is a rare skill. The other approach is top-down incarnation, from theology to mathematics and then to physics, which was the approach of Einstein; and then the down-top elevation approach, from physics to mathematics and then to theology, which was the approach of Newton. Top-down approach means that, one communicates in the realm of theology, and then this communication incarnates in the realm of mathematics and then in the realm of physics. Down-top approach means that, one communicates in the realm of physics and then it is elevated to the realm of mathematics and then to the realm of theology. Nowadays, people limit the realm of communication to inside the world of mathematics. This means that one should make a hierarchy of cognition inside the world of mathematics.*

What is the nature of argument?

In the Aristotelian point of view, which is verbal and analyzer, arguments are humanistic and imperfect. But in the Platonic point of view, which is pictorial and wholistic, arguments are perfect and of a supernatural source and are independent of humans. In the Leibnizian point of view, arguments could be written in formal language and in the Kantian point of view, arguments are of a predetermined nature implanted in our brain. In the lingual point of view, arguments could only be verbal and there is no pictorial version of an argument. In the point of view of social sciences, validity of an argument is a social phenomenon, and in the point of view of information age, the flow of data determines if an argument is valid.

The role of arguments in problem solving

☐ *Arguments play an important role for problem solvers. Arguments clarify the logical structure of the problem. For example, one can subdivide the solution to small pieces, by notifying important steps in form of lemmas simplifying the argument. The logical structure of arguments, also organize the process of coming to a solution. For example, arguments promote divergent thinking or criticize conjectures. Looking for equivalent formulations help coming up with a new argument and therefore a new solution to the given problem. Also looking for simpler models simplifies arguments. Decomposition and reduction to simpler problems is also performed by arguments. Arguments help us estimate how much progress has been made towards coming to a final solution of the problem. It also helps formulating good conjectures using the perspective given by arguments.*

The role of arguments in making assumptions and development of theories

Not only arguments are important for problem solver, but also they play a crucial role for theorizers. Arguments test assumptions suggested by theorizers, and generalize assumptions to wider scopes. There are natural barriers to generalization of assumptions revealed by arguments. Sometimes one can't unite two given theories. This could be because of not being formulated in compatible formulations or because of the paradigms not being connected.

☐ *Arguments help reformulation of assumptions. For example, arguments do surgery on assumptions in order to repair implications. Surgery and repair performed by arguments could lead to unification of assumptions. Arguments could also help assessment of assumptions. For example, strength and weakness of assumptions are assessed by fluency and naturality of implications.*

☐ *Arguments test theories and generalize theories to wider scopes. By generalization one can unite the realms of two theories. Recognition of relations between assumptions via arguments usually leads to unification of theories. Recognition of relations between theories via arguments forms a paradigm. One is interested to find relations between two theories for the further development of mathematics.*

Search for the truth

☐ *Truth in mathematics is understood by analogies. Analogies, on the other hand, are revealed by arguments. In mathematics, one compares two or three theories and find dictionaries between them in order to look for background truth. In mathematics, concepts are just a model of the truth. So arguments discuss relations between the models of truth.*

Psychology of problem solving against psychology of theorization

☐ *Problem solvers race against time. But theorizers race against truth. Problem solvers take advantage of a clean mind. Theorizers take advantage of a clear wisdom. Problem solvers manage the process of thought. Theorizers manage the realm of wisdom. Problem solvers manage the language. Theorizers manage the process of formulation. Problem solvers manage the feelings and mathematical behavior. Theorizers manage the personality and mathematical qualities. Problem solvers develop intuition and inspiration. Theorizers develop vision and enlightenment. All these aspects show up naturally in the language of arguments.*

Dual pairs of arguments

☐ *There are geometric versus algebraic arguments. Geometric arguments are pictorial and algebraic arguments are verbal and symbolic. There are continuous versus discrete arguments. Continuous arguments use topological structures, where discrete arguments are combinatorial. There are local versus global arguments. Local arguments discuss a neighborhood in some topology, where global arguments deal with global objects and global concepts. There are categorical versus interstructural arguments. Categorical arguments use morphisms between objects to understand them, but interstructural arguments discuss the internal structure of objects under consideration. There are rigid versus deforming arguments. Rigid arguments deal with a rigid space or structure, where deforming arguments try to deform objects or spaces in order to understand them.*

Pairs of formulations

☐ *Arguments can usually be formulated in several languages. When this happens, there chance to have a deeper understanding of the truth behind the argument, by searching for a*

language beyond the two formulations. There are many examples of pairs of formulations, where arguments could be formulated in both languages. For example, Lagrangian versus Hamiltonian formulations of mechanics, differential forms versus vector formulation of electromagnetism, differentials versus finite differences, infinitesimals versus limits, Hopf-algebra versus geometric formulation of affine algebraic groups and geometric versus arithmetic formulation of algebraic curves.

Paradigms back up arguments

☐ *Sometimes particular arguments in particular intuitive situations could be formally extended to a much wider scope, where the intuition continues to hold. This is what we call a paradigm. Examples of paradigms where a particular argument can be extended to a wider scope is listed below: Paradigm of wave equation; Paradigm of energy; Paradigm of mechanics; Paradigm of center of gravity; Paradigm of intersection theory; Paradigm of algebraic variety; Paradigm of derivative; Paradigm of integral and area.*

Language is the ultimate tool for forming and communicating paradigms.

☐ *Do you believe that one could conceptually relate two theories, without joining their lingual formulations? No one believes in communication outside the realm of language. This means that people trust language but not intuition. Because, they can speak but they can't even pretend to communicate mental images directly. Therefore, it comes very naturally that arguments are governed by language.*

Marriage of arguments

☐ *There are many instances where two paradigms of arguments join and let's say marry to form a new wider realm of argument. For example, relativity-quantum marriage, give rise to field theory. Real-imaginary marriage, give rise to complex numbers. Geometric versus arithmetic thinking, give rise to algebraic thinking. Integral versus summation, give rise to fundamental theorem of calculus. Boundary versus differential, give rise to Poincare duality. Fruit of these marriages is a stronger, more general, deeper paradigm of argument. There are in fact dual roles played by the father and mother argument.*

☐ *The role of father argument is that it provides ideas and intuitions, manages relations with other arguments, provides the global structure, and determines how to generalize. Father argument furnishes the soul.*

☐ *The role of mother argument is that it provides appropriate formulation and language manages internal relations between sub-arguments, provides the local structures, and determines how to solve problems. Mother argument furnishes the body.*