

What Kind of Mathematics is Valuable?

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Abstract: One could do mathematics for different purposes and this affects the concept of value for any mathematician. Also there are different styles of doing mathematics like: problem solving, theorizing, cognitive mathematics, or truth finding. The concept of value also depends on the cognitive style of mathematician doing mathematics. Although the possible advantages of cognitive abilities are predetermined, possible manipulation of the truth are not. We explore the limits of these manipulations.

Introduction

Mathematicians make different decisions when it comes to choosing the kind of mathematics they want to do. This is based on different conceptions of value. These are fundamental questions: what is the value of mathematics? What is the value of research in mathematics? What is the value of doing mathematics? These questions have different answers in the eyes of problem solvers, theoreticians, cognitive mathematicians, or truth finders. This is because they have different purpose in doing mathematics. The concept of purpose affects the concept of value. At the end of the paper, we explore the value of discovering the truth and interaction with it and changing the truth. This will force us to consider the question of essence and free-will and limits of human ability in changing the direction of human mind and human knowledge.

On the concept of value of mathematics

Some mathematicians think of doing mathematics only as a job. But majority of mathematicians think of mathematics as a way of life. Still there are differences in people's perspectives and their contribution of mathematics to mathematician's

lifestyle. Some people like doing mathematics because it is a challenge, and some do it gain respect and some for becoming wiser and more knowledgeable. The value of mathematics is different for all these different people. As a result the valuable mathematics has different characteristics for them. Also there are different styles of doing mathematics. Valuable mathematics is defined differently for a problem solver, a theoretician, a cognitive mathematician or truth finder. People have different purposes and why they do mathematics and the style they do mathematics also affects their values. Because different styles developed different skills and people value mathematics according to their own skills. Of course, all this changes in time. The purpose of a students in school doing mathematics is different from the purpose of a mature mathematician and this difference happens by going through continuous change. This change implies that the concept of valuable mathematics is a changing concept for every mathematician. Still these different concepts of value of mathematics could be characterized and categorized and studied. Let us see what different purposes of doing mathematics could be distinguished.

On the concept of purpose of doing mathematics

Mathematicians have different purposes of doing mathematics. Among them I can name a few: mathematics for the sake of application, and some do it for the sake of curiosity. Some do mathematics for education of others and some for developing their own cognitive structure, and some try to discover truth. Why is that people have different purposes in doing mathematics? Partly this difference comes from general difference of purpose of learning and knowledge. Some try to address their or others basic needs and some try to develop their minds or other people's minds. Some search for higher truth in learning and among them some find it more accessible through mathematics. Others have different perspective to mathematics than to general learning and obtaining knowledge. Because mathematics is just a cognitive tool for them. Not that it's really matters in its own right. People with different maturities in mathematics pick different purposes for their act of doing mathematics. In our point of view, thinking of mathematics as a tool for cognition is the most basic purpose and searching for the hidden truth in mathematics is the most advanced purpose of doing mathematics. This makes us

start our study from applications and move step by step through different levels of depth and study their implications on what kind of mathematics is valuable.

Mathematics for the sake of application

All scientists and Engineers dealing with applications of mathematics to their personal fields of study or among people who do mathematics for the sake of application. But there are professional mathematicians who shared the same purpose with them. Usually these mathematicians are called applied mathematicians, but it is a delicate question what applied mathematics really is? This concept of application could be directed to everyday life, or directed to engineering or to fundamental sciences like physics. Trying to apply mathematics to mathematics which has a very similar personality to pure mathematicians. This concept applied mathematician is understood against a concept of pure mathematician which has also engaged through the history. For example, the kind of applied mathematician who applies mathematics in mathematics is understood against a modern concepts of pure mathematician. The former tries to see the implications of existing methods and theorems in mathematics. The latter tries to originate new methods which could contribute to the way people do mathematics. These new methods could be proposed to solve problem or address theorizing needs or other styles of doing mathematics. This is the subject of another paper of ours with title "On the concept of pure and applied mathematics". Application is the most elementary purpose of doing mathematics.

Mathematics for the sake of curiosity

Many mathematicians do mathematics for the sake of curiosity and the circle of these mathematicians is wider than pure mathematicians. For example, applied mathematicians who seeks applications of mathematics to itself may also do mathematics for the purpose of curiosity. Mathematics which is produced by this purpose is more abstract than the mathematics produced by the purpose of application. It is evident what kind of mathematics is valuable for applied mathematicians. But mathematicians doing mathematics for curiosity are curious in different things. Some mathematicians are satisfied by knowing true theorems

and some by knowing visionary conjectures, some by illuminating truth and some look for hidden truth. This is why pure mathematicians doing mathematics with different styles value different kinds of mathematics although all of them do mathematics out of curiosity. We will list these different styles and will study the concept of valuable mathematics for each style. These styles include problem solvers, theoreticians, cognitive mathematicians and truth finder. Others do mathematics for the sake of education. They are mathematics teachers or PhD advisors whose research is circling around educating people and preparing them for research. Next we shall see what is the concept of valuable mathematics in the eyes of educators.

Mathematics for the sake of education

Some teachers try to teach concepts and some try to teach skills and some try to develop cognitive abilities. In this section, we study the first two and leave the third one for the next section. The mathematics which has more conceptual relations among its concepts in mind is more valuable for educators who are trying to teach concepts. Mathematics which involve more mathematical skills is more valuable for educators who are trying to teach mathematical skills. An example of an appropriate field to teach for the first group is calculus and an example for the second group is Euclidean geometry. Many believe that the educational rules of these two topics are irreplaceable. Algebra and trigonometry are also studied under the umbrella of these two topics. There have been attempts to teach other advanced mathematics to high school students, but none survived over the years. Mathematics taught at undergraduate level has a similar role and purpose. Although, geometry, analysis, number theory and combinatorics are seen as the main topics of problem solving in high school and university. There's not much taste of theorizing at the level of undergraduate mathematics. Mathematics which is conceptually central and contains important skills of doing mathematics is paid more attention in education of undergraduate students. Education of mathematics in graduate level is more specialized to particular concepts and particular mathematical skills.

Mathematics for the sake of developing cognitive structure

Developing cognitive structure could be directed to self or directed to others. What is the appropriate mathematics to be done for the sake of developing cognitive structure? The fact is that different cognitive styles should do different kinds of mathematics. Algebra and analysis better fit for verbals and geometry and combinatorics better fit for pictorials. But the problem is that there are several classification of cognitive styles which should lead to several borders between different kinds of mathematics. For example, one of the other most dominant cognitive styles is being analytic or holistic. "Holistic" algebra is category theory and "analytic" geometry is differential geometry. "Holistic" analysis is fundamental analysis and "analytic" combinatorics is counting. What is the concept of valuable mathematics for each cognitive style of thinking and learning? The answer is different for different cognitive styles. But they all search for truth. There is a truth behind the face of mathematics which is Independent of the cognitive style of the mathematician. And the mathematician eventually tries to define the value of the mathematics he/she does in terms of this truth which is universal and not in terms of cognitive style which is subjective. Existence of such a universal realm for truth is not trivial and even assumption of its existence has many philosophical prerequisites.

Mathematics for the sake of discovering the truth

Belief in ultimate truth which is very abstract and incarnates in lower abstraction levels layer-by-layer is developed after many years of development from its Platonic origin. I became familiar with these developments through Islamic philosophy. Of course Platonic ideas took a different path in the west which we are not interested here. In the eastern philosophy, truth has many layers of abstractions and there is manifestation and elevation relations between these layers. Every truth seeker understands only certain layers of truth and the number of these layers is infinite. But the concept of ultimate truth still exists as a limit and all these layers of abstraction are compatible and correlated. In such a system, the truth has movements and is purified under a process of development which is

the same as its elevation. The fact that, there is an identity to mathematics which is independent of cognitive style in itself can be considered as an elevation. Sometimes the mathematical style of doing mathematics is determined by the purpose of doing math. Sometimes it is not the case for example, problem solvers and theoreticians may have different purposes in doing math, but cognitive mathematicians or truth finder mathematician all have the same purpose and therefore one should be able to talk about their values in a unified language. Even problem solvers and theoreticians because of the similarities in the style of doing mathematics share similar values.

On the concept of value for a problem solver

What is a valuable mathematics for a problem solver? The answer should be in the language of valuable problems and valuable solutions. For example, a valuable problem could be a problem which answering to it, gives us a better and deeper picture of objects, relations between objects, or theories or mathematical phenomena. Of course the solution to a problem could also be illuminating or not. In fact, there are many kinds of problems in mathematics. A problem could be a conjecture giving a clear vision of what could be true and what are hidden phenomena in the field of mathematics. It can even be a big web of related conjectures like "Langland program" for example. Proposing such problems is the job of theoreticians which we will discuss next. For problem solver, solving a valuable problem even with an invaluable solution is very important. They care about truth of a theorem more than understanding why a statement is true. They don't look for a deeper understanding necessarily but for an accurate proof. Of course a valuable proof of an invaluable problem is also important for them. This could be because a valuable solution could be illuminative or could contain techniques which are useful for solving other valuable problems. Problem solvers even treat theories as problems and evaluate theories the same way as they evaluate problems. But theoreticians have different standards while evaluating theories and also theoretical conjectures. Being a theoreticians affects the personality of mathematicians in a way that it affects their values.

The concept of value for a theoretician

For a theoretician the more conceptual relations in a theory are entangled, the more valuable the theories are. And not all branches of mathematics are very appropriate for such a theoretical treatment. In some fields of mathematics there is more potential for a theoretical treatment than others. Number theory and algebraic geometry are the heaven for theoreticians. A web of entangled concepts and a web of entangled skills are of different values for a theoretician. A web of entangled skills is more valuable from the problem solving perspective toward mathematics. Solutions to problems which could help many other problems to be solved are such an example. For theoreticians analogies between theories are of similar value. A web of analogous theories which make relations between different fields of mathematics is of out most value in the eyes of theoreticians. Theories could be reformulated and the new formulations are of different values with respect to the old ones. Some new formulations are better fit with analogies and some are better fit with problem solving and computational skills. So some formulations of the same theories are of more theoretical value and some of are more value in the eyes of problem solvers. A web of entangled cognitive relations is valuable in the eyes of the cognitive mathematicians. Both theoretical value and problem-solving values could be understood in the language of cognitive relations. But cognitive relations are not limited to conceptual relations and computational relations.

On the concept of value for a cognitive mathematician

For cognitive mathematician the value of mathematics is because of its effects on development of cognitive structure of the mathematician who learns or does a kind of math. A web of entangled cognitive relations is what makes mathematics valuable in the eyes of the cognitive mathematician. The same way that concept have precepts and skills have preskills, cognitive steps have pre-steps. So it is dependent on a mathematician's history what cognitive steps are available to him/her or what the cognitive value of learning or doing a piece of mathematics could be. Even the cognitive value of a piece of mathematics depends on the

history of development of cognitive structure of a mathematician. This means that the same piece of mathematics could be of different values to different mathematicians. Therefore in learning mathematics a lot of cognitive decisions should be made. For example, learning analysis before algebra or after has consequences in the cognitive structure of the mathematician following these steps. For example, the concept of proof for an analyst is different from the concept of proof for an algebraist. Although they are both verbal concepts. Therefore, this concept of value is very case dependent and trying to get something not subjective out of these values leads to the concept of truth finder mathematician. The value of the mathematics for a truth finder mathematician is independent of his/her cognitive structure and cognitive history.

On the concept of value for truth finder mathematician

The very assumption that there exists a value associated to a piece of mathematics which is Independent of the cognitive structure of the mathematician is under dispute. It would be a highly Platonic picture of mathematics if we assume such a value exists. An implication of this assumption would be existence of the concept of truth, independent of its being discovered. This in itself is a highly Platonic idea, since it assumes existence of mathematics independent of human mind. We assume existence of the collective realm of consciousness in which mathematical ideas could live independent of the creator when at the same time they can be created by human minds. This would be a more accessible structure to existence of truth which is not Platonic, since it is creation of human mind. The contents of this collective consciousness are called "truth" in our terminology. This defines the concept of truth finding mathematician and introduces us to the concept of value for such a mathematician. Although a platonic mathematician could say that there exists ultimate truth which could be mathematical or non-mathematical which incarnates in our mathematical mind and thoughts. We do not assume existence of such higher truth in this paper. Although we could assume that this is a level of abstraction valuable in itself and it may be accessible to human cognition and even subject to change by human interfere. When we end up in a theory more or less similar to "truth" in collective consciousness.

The possible advances of cognitive abilities are predetermined

Although people are different in cognitive structures, the possible advances in the cognitive abilities is predetermined in their DNA. It is not the case that one is free to deform one's cognitive structure as he/she wills. Even the will of the mathematician trying to educate him/herself is subject to rules predetermined in his/her DNA. The question that is it possible to make changes in our own DNA in a way that it affects are cognitive structure, is answered negatively for now. Therefore it is not possible to be creatively affect the change of the cognitive structure. It will never be the case that human race has cognitive abilities that could not be imagined beforehand. One must correct the meaning of this sentence in one's mind by comparing human today with prehistoric human's abilities of cognition. The cognitive structure is essentially the same but more complex. The complexity is not due to structure, but due to the content of cognition. Of course, the cognitive structure of the mathematician is not fully available to him. The cognitive structure could be cleaned to correct cognitive beliefs and false understandings. In fact stopping to do mathematics for a short period is one of the possible ways to clean the cognitive structure of the mathematician. This means that everyone has hidden cognitive abilities which must be discovered and used. This makes mathematics valuable as cleaning tool which helps discovery of hidden cognitive abilities more effective than other practices of learning.

The value of discovery of "Truth"

The value of discovery of "truth" for a truth finder mathematician is not limited to cleaning the mathematician's cognitive structure and developing it and making him or her wiser. The mathematical "truth" is valuable in itself. This value in the essence of mathematics, is not just because of applications and implications. But because of the perspective it gives us on the nature of "truth" as nature of

anything which is subject to human understanding. This can be a reason to justify the value of knowledge as a whole. The question is, why shall we insist on the value of mathematical truth? One should have insisted on the value of truth for all scientists and all types of “knowledge”. But the fact is, by experience it has been observed that mathematics is more effective to understand the nature of truth against other subjects of cognition because of being more abstract. You can think of mathematics as the science of recognizing structures, and construction of structures. This is a reason why we can talk about a truth finder mathematician and not a truth finder physicist. In fact, the truth finder physicist is a truth finder mathematical physicist. Above all these considerations the value of the subject of cognition is not only because of it being known. One can interact with the truth and in fact one is able to change the truth as a result of this interaction. We are interested in the value of interaction with truth and the value of changing the truth.

The value of interaction with truth

Experience of the nature of truth tells us that the truth is live. You can in fact interact with the truth and change the truth. But what is the value of interaction with the truth or in other words having a dialogue with the truth? First of all, interaction with the truth is a cognitive ability which is not satisfied in any other means. So, this is a level of cognitive achievements. For the cognitive mathematician this is a value in itself. But for a truth finder mathematician this is level of knowing the truth. This means that truth is unpredictable in certain sense and has free-will in certain sense. This means that there is concept of hidden truth which could be revealed if you could interact with. Or maybe there is a concept of essence of truth which could be revealed to us by the will of truth. And then comes the question if the truth can recognize things and structures. This is a prerequisite of having free-will. On another level even animals and insects are unpredictable and therefore you can assume a level of free-will which implies a level of cognition. But why it is valuable to interact with truth and not with animals and insects? The reason is you can change the truth by interaction and this way you can influence the mind of people and even direction of humanity.

This is not the case for animals and insects since they don't have a collective consciousness.

The value of changing the truth

You can train animals and you can train humans and you can change the truth. But truth is more universal. This is why changing the truth is more valuable. This is why I don't understand why the value of saving a person is the same as the value of the saving the whole humanity. May be it is because it has the same effect on our cognitive structure. Or maybe any single human is as valuable as the truth itself. The same way that there are morals governing the changes one implies in other humans, there are also morals in implying change to the truth. Change in itself is not a value. Change should be subject to rules and standards set by someone. If you like you can assume that these standards should be set by humans or you can assume that these standards should be set by God! The fact is, standards defined by human would not be universal. You need a higher authority to set the standards and values. Since one should say to which direction the truth should be directed to. Or to which direction the humanity should go. This is a matter of more important than being decided by collective consciousness. Because the collective consciousness may also have hidden cognitive abilities yet to be discovered. Therefore we cannot put our trust in what collective consciousness understand about humanity and the direction it leads humanity towards them. This position we have assumed as a hidden assumption in it. And that is the changes in truth are not be determined.

Are changes in truth predetermined?

Can the truth go in the wrong direction? Can the humanity as a whole choose a false pass? A false pass in cognition or in actions? The study of history teaches us that yes it is a possibility. Therefore, the changes in truth are not predetermined. This is why it make sense to talk about morals of change and why truth has to be alive and has to have free-will and cognitive structure. Because of the similarities with human cognition, it is not strange assumption if we assume that changes in the cognitive structure of truth are predetermined. But that doesn't mean the

changes in the essence of truth are predetermined. And indeed this is not the case for human being and in fact this is why it makes sense to believe that human has free-will. So, the essence of truth can be changed and in fact, this is why it matters to change truth. But why is there any value in changing the essence of Truth? Why is there any value in leading the truth to certain direction? Why should we encounter with the free-will of truth even if we can? Shall we not leave the truth alone? The fact is anyone who tried to discover truth and then to interact with truth does Influence the truth anyway. If it is importance for such a person to which path the truth is lead. Then he/she tries to influence the truth to some direction or away from wrong directions anyway. It is like the everyday life that people we like them and care for them, we also try to influence them in the right direction.

On the concept of free-will in movement of essence

Do we have free-will to influence essence of others? Of course not against their will. If someone is willing to be influenced, such influence is a fact of life. But then if such an influence happens is by the will of influencer or influencee? Of course the will of influencee is a must. And the will of influencer is not present very often. But it is also a fact that the will of influencer could convince the influencee to accept such an influence.

On the concept of free-will in movement of truth

If one wishes to influence the truth in some direction, this means that one should set up for a close and personal relation with the truth. Then one needs to convince the truth to be influenced and this could be result of years of friendship and having and liberated soul. Many are influencees of the truth rather than influencers. This is in fact the reason why it is valuable to influence the essence of Truth. This way one can influence the direction of humanity. Who are the heroes of such an influence? Could one say that Plato and Aristotle and many philosophers following them have had such elevated souls? Could it be that Abraham, Moses, Jesus and Muhammad have had such influences? This brings us

to the question that if “Truth” is God itself or a “name” of God? Could names of God be influenced by humans? Let theological philosophy tackle these questions.