

"The Truth Finder Mathematician"

or

"Mathematician: The Truth Finder"

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We start from Platoism and reality of imagination and end up in Sadra's philosophy of many layers of Truth all of which could be subject to change. Then we try to figure out implications on philosophy of mathematics and build up a new philosophy of unity of Mathematics, Which says that mathematics a whole is incarnation of truth. This belief has implications on the nature of mathematical truth.

Introduction

In a previous paper "on different points of View towards practical philosophy of mathematics" we introduced a new philosophy called "Incarnation of truth and unity of mathematics" and explored implications on learning Mathematics and teaching mathematics and performing research and evaluation of research and understanding history of mathematics. in this paper, we shall try to build foundation of this new philosophy of mathematics and base it on Sadra's philosophy of essential movement or movement of essence in which truth has several layers of existence in which there is incarnation from more abstract layers to lower layers and elevation from less abstract layers to higher layers. Above that, there is change in every layer of truth. But the more abstract the layer is, the more difficult the change is and the slower. The author acknowledges support by an Oswald Veblen grant from Institute for Advanced Study.

On Platoism

In Plato's philosophy of truth, truth incarnates and it is in the form of imagination. Not Imagination of microcosmos, but imagination of macrocosmos incarnates in imagination of microcosmos, and that is how mathematics is incarnated truth. Plato leaves no room for imagination to be elevated from microcosmos to macrocosmos, and believes the mathematical truth lives and have always lived in the realm of imagination of cosmos. In this point of view, mathematics is the thoughts of God and imagination of cosmos is the realm of the mind of God, and mathematics is the language in which God thinks. we slightly modify Plato's

philosophy by admitting the imagination to be elevated from microcosmos to macrocosmos. The makes room for the mind of human to be elevated to the mind of cosmos and vice versa, the mind of macrocosmos to incarnate in the mind of human.

In this manner, mind of macrocosmos will act as a means of communication between human minds. There is no need to assume mathematics was always existant in the realm of imagination of cosmos and it is eternal truth which manifests in the mind of human is the form of mathematics we recognize. This leaves us with a mathematics with two layers of being: microcosmos level or the level of mind, and macrocosmos level or the level of universe.

On Imagination

After DesCartes, human's realms of existence Is limited to mind and body. New Platoism assumes the same levels of being for macrocosmos. Microcosmos and macrocosmos being Related both in the level of mind and in the level of body. this formulation of Plato`s thought developed to the point that philosophers spoke about the world of imagination on a more abstract level of being than the material world. The world of imagination containing the mind of humans and mind of universe. Then the question comes, what is imagination? If it could be created in the mind of humans, how could be that imagination is the same as truth? How could be that imagination could incarnate and manifest? Is it could incarnate and manifest it should be elevated also. But there is no room for elevation in the realm of imagination. This makes us think of upper continuum having several layers of existence, And there should be layers more abstract than the layer of imagination, this makes us to think of the concept replacing mind and imagination, which used to call the concept of truth, which has several layers of existence and among them there is manifestation and elevation and these layers form a hierarchy which is in fact a hierarchy of worlds through which the mathematical truth elevates and then incarnates.

On the concept of truth

Such a concept of Truth calls for reformulation of the mind and body problem. There must be several levels of cognition for the microcosmos and the corresponding levels of cognition for the macrocosmos. Assuming that Cognitive contents of microcosmos and macrocosmos which are similar live in the same world, we shall have a hierarchy of cognitive levels which is also a hierarchy of beings or in other words a hierarchy of beings or in other words a hierarchy of levels of existence through which there is movements of cognitive content or what we call truth In both directions of manifestation and elevation. Is it that the more abstract level of cognition are, the less personal and more collective they are? That could be true. Because the manifestation and elevation could take place between the cognitive system of different people. My deep understanding of some mathematical truth may elevate to deeper levels in the cognition of another mathematician, and deeper levels of cognition of another mathematician, could only manifest in my cognition up to a lower level only. Therefore, the deeper the level of truth is, the more collective is the nature of cognition. This is the wisdom

of elevation of truth and existence of several layers of cognition rather than a simple layer which was called imagination. This is by no means saying that the realm of imagination is unreal.

On Reality of imagination

I understand the reality of our personal imagination by presence and the reality of the imagination of macrocosmos by The long distant relations between personal imaginations. But we do not understand reality of imagination of cosmos by presence. So there comes the question if the realm of imagination of macrocosmos is real? How do we know that cosmos must think and make decisions? I ask, how do we know that other people think and make decisions. We know this by contradictions between our wills. Will of universe also contradicts ours. So why not assuming that Universe has consciousness. Now I ask if we have knowledge by presence on higher levels of our personal cognition. My answer is that not everybody, but some people do recognize manifestation and elevations in their cognitive structure. Those who do not are also able to build up such a hierarchy in their cognitive structure. In these higher levels of consciousness one has will and feels that the macrocosmos is sometimes against our will. This will be a reason we could assume macrocosmos has the same cognitive structure as the microcosmos. But what if someone argues that everyone understands by presence only finitely many layers of cognition. Why should we believe they are infinitely many layers of cognition, or maybe not enumerable? May be, layer consciousness forms a Spectrum!

On reality of Truth

Not having knowledge by presence on all levels of Truth, why shall we assume reality of Truth? Finiteness of layers of truth has an Implication and that is: there exists a layer which is called “essence” consisting of the highest level of Truth. The nature of essence will be that it doesn't change! This is against the knowledge of truth by presents. Our practice shows that truth should change. There is no essence to human layers of cognition and no essence to Human layers of being. Because this is against the reality of free will, which we have knowledge by presence. Therefore to argue that truth is real, we stand upon the fact that our free will is real. Also we have knowledge by presence that truth is manifested to our cognitive structure from outside. So there are higher levels of cognition outside our being. This is why we believe they are infinite layers of Truth. What if one claims that layers of cognition is limited in human and macrocosmos both. But there are higher levels of cognition in macrocosmos! Then, we argue that this implies the existence of essence for human and for macrocosmos and contradict the free will of human and macrocosmos, over which we have knowledge by presence. This is why we have to assume realms of truth although we have not Personally tested all these levels of knowledge.

On incarnation and elevation of truth

Truth moves up and down in our cognitive structure and the macrocosmic cognitive structure. So it could be generated as cognition in microcosmos and then elevated to the level of macrocosmos. We shall not assume existence of ultimate truth independent of human cognition outside the creative touch of a human being. This means that platonic truth could be made by human mind. Therefore could be changed by human mind, and therefore it is not absolute. The truth created in one's mind could elevate to the microcosmos and manifest in another person's mind. This is why mathematics is accessible to everybody. This is why mathematical truth is subject to change. One could ask: what is the wisdom of assuming that macrocosm has the same cognitive structure as the microcosmos. The answer is, this way macrocosmos behaves like a realm of communication between cognitive structures of human beings. There is another question here: if the manifestation and elevation are subject to the will of recognizer? This question can be asked both in microcosmos and in macrocosmos. This is in fact the same standard question which has been elaborated by many: Is the act of cognition automatic or by Will. The answer is cognition is like breathing. There is an automatic version. But you can change it by your will.

On platonic manifestation and elevation of imagination

Now the question is why assuming manifestation and elevation of imagination is not enough for our purposes? Why do we assume layers of Truth? Why do we assume several layers of truth? This assumption is made to theorize essential change or change in essence of the cognitive structures of microcosm and macrocosm and why not assuming free will for macrocosmos the same way that we feel free will for microcosm by our knowledge by presence? This way we have unified Kant and Plato. As we discussed before, we have knowledge by presence that imagination is real. Therefore, we can assume that imagination is one of the several layers of existence of truth. This point of view is very Platonic. Because, in order to assume a realm of imagination for macrocosmos, we also have to assume imagination is real. This is a clue to better understanding of the platonic point of view. But Plato had to assume imagination of the macrocosmos is more abstract than imagination of the microcosmos. Because imagination should be elevated from micro world to the macro world. We don't need to assume this, because truth has several layers of existence in our perspective. For us the Cognitive structure of the macrocosmos is exactly the same in microcosmos.

The path to the ultimate truth

The elevation to higher and higher levels of truth put us in a situation that we cannot help but to assume that there is an idealized ultimate truth we are approaching to, and that there is a path for the truth it should pass to get to the ultimate position. Although ultimate truth is not accessible, but it is a result of idealization of the cognitive structure. Assuming that this result exist doesn't hurt. Although, we must be careful not to assume it is pre occupied by knowledge unknown to us. This is kind of assuming the existence of an eventuality state of being or becoming which is very similar to the idea of life after death in religion. The point about the path to the ultimate truth is that there is no predestined path. But there is a path to be

formed according to the will of the cognitive being leading to eventuality of its cognition. This realm could be the same for cognition of microcosmos and macrocosmos. We do not need to assume a separate idealization for every cognitive entity. Cognitive layer of being which are related to each other by the manifestation and elevation relationship, give us an infra structure for a unified eventuality for the path of truth. One can assume manifestation and interpretation of truth in all levels of being are manifestation of the ultimate truth.

The truth has multiple interpretations

The truth is manifestable and it has several manifestations. These several manifestations are called interpretations, because manifestations are not unique. But why do we assume the path of the truth is towards unity and manifestation on the other hand towards multiplicity. The movement towards unity is present in the platonic image of truth. We can ask the question locally on the structure of cognition. Why it is that elevation is movement towards unity and manifestation is movement toward multiplicity? Why more abstract means more unified? In fact this is a fact on which we have knowledge by presence in our own personal cognitive structure. The more abstract truth becomes the more unified it seems. We assume the same structure in macrocosmos because it shares the eventuality realm with microcosm. This means that the eventuality of truth should be the ultimate unity and material interpretation of truth should be the ultimate multiplicity. This is confirmed by several parallel cognitions seemingly similar in essence. There are several explicit examples in the realm of mathematics. For example several dictionaries between unrelated fields of research that show eventual similarities between separate theories. Also, generalization of theorems usually tend to be more abstract.

The ultimate truth can change, but very slowly

The fact that truth can change implies that ultimate limiting truth could also change. Otherwise, this idealizing structure closes our wings of cognition. But there is a long path towards the ultimate truth which is a limiting object and this makes the change in ultimate truth very slow. Such a slow movement in ultimate level of being gives an ultimate concept of time in the realm of ultimate truth. This makes the answer to the question of “what is mathematics” in subject to change. This brings us question of how the answer has changed in history and how mathematics has developed through time. We are definitely no longer live in the time of Euclid and we cannot afford to work with his definition of mathematics under the influence of Aristotle. This forces us to question our approach to mathematics. Does the mathematics of our time confirm our approach towards the structure of mathematical knowledge? If not, how is it to influence the path of mathematics and what is will be implications in future history of mathematics. The answer is that, this approach brings us to control the future answer to what is mathematics according to our understanding of human cognition and human abilities and human needs.

The mathematical truth

I am not sure if the mathematical truth as we define it today is closed under elevation and manifestation. In other words, we do not know if the mathematical truth elevates to a truth which is mathematical in our working definition of mathematics. Even if we assume that there is a realm of mathematical truth closed under elevation and manifestation, this does not mean that other truth which is not mathematical does not incarnate in the realm we choose to call mathematics. Therefore it is not conceivable to have a realm of cognitive specified to mathematical knowledge. Keeping in mind that mathematics itself change its definition in time we should confess that there could not exist a concept of “mathematical” truth. This is because any truth could manifest in the realm of mathematics. Or in better words, mathematics could be mirror to all layers of truth and cognition of the microcosmos and macrocosmos in the scene that mathematical cognition could accept any truth, or in other words every truth admits mathematical modeling. In fact, this is what we are doing in this paper. We are trying to develop a mathematical model for human cognition and also the cognitive structure of macrocosmos if I dare to say.

On the truth manifested in mathematics

The truth finder mathematician looks for the original version of the truth which is manifested in the realm of mathematics. This is kind of reverse interpretation which we call initialization with the meaning of moving to the origins of something. Definitely we don't want to end up with a philosophy of mathematics saying everything in our cognitive structure is mathematics. Therefore, we must be ready to accept initialization of a mathematical truth be non-mathematical, and be prepared to live with non-mathematical truth manifested inside mathematics. We know that our definition of mathematics only accepts parts of the truth in all layers of existence but eventuality of truth in all layers of existence implies unity of ultimate truth and the question is does there remain any room to talk about unity of mathematical knowledge, without assuming that all our knowledge is a kind of mathematical knowledge? In order to answer this question we have to elaborate on unity of truth. This is a difficult task, since the realm of unity is formed by idealization and we do not have knowledge by presence in this realm. Does it make sense to make a mathematical model for the realm of unity of truth or the realm of truth as a whole?

On the unity of truth

Assuming unity of truth the structure of truth accompanied by realm of unity looks like a cone consisting a single truth and all manifestations of this single truth. This has implications on our cognitive understanding of the truth. All local parts constituting the structure of truth can be understood in a single language. One can call this language mathematics or any other name but they are all compatible with each other cognitively. Another implication is that although two independent truths can manifest is a single common truth but there is always a single joint truth manifesting to both manifesting truths. This has serious mathematical implications which we will discuss in the next section. But a model for that structure is considering a simple truth in the same level of abstraction lower than the ultimate truth and

consider all its manifestations. This will be a cone again but not an infinite cone like the totality of the structure of truth. Or another model could be a truncated cone. Consisting a number of correlated truths. One such model for the whole truth is the structure of all mathematical truth. Many philosophers used mathematics as a toy model to study the structure of whole truth, and this could be motivation for mathematician to do mathematics.

On unity of mathematics

Mathematics today is based on two correlated main concepts of number and shape, which are the origins of theory of number systems and the theory of space. Perhaps unity of concepts of number and shape could result in a single cone consisting of the mathematical truth. The mathematics today is a truncated cone manifested by these two concepts of number and shape. At the time of Pythagoras these two concepts were more or less unified. Unifying number and shape means that there is a simple mathematical language which could explain all the mathematical truth in a simple unified form. Assuming that there exists a simple mathematical concept manifesting to all known mathematics has several practical implications for example, a mathematical truth cannot have two independent generalizations. These two generalizations must be manifestations of a single truth which accepts all other three as manifestations of itself. For example, this means that Hirzebruch-Riemann-Roch cannot have two independent generalizations like Grothendieck-Riemann-Roch and Atiyah-Singer index theorem. They must be implied by a common theorem, which is a generalization to both.