Psychology of Theorization in Mathematics and Physics

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There are two kinds of mathematicians or physicists: Problem Solvers are analyzers and move from local to global and they are interested in important special cases. Theoreticians are wholists and move from global to local and they are interested in various generalizations. Theorization and problem solving are parallel skills for mathematicians and physicists to further develop mathematics and physics. Skills of problem solving develop in early ages, but theorization skills develop much further in the scientific life of mathematicians and physicists. It rarely happens that someone in the age of Galois is that able in theorization in mathematics or physics. Comparing the research of Abel and Galois clearly indicates the difference between a problem solver and a theorizer.

Some of theorization skills are more basic than others, because they are pre-skills to others or because they are related to early stages of developing theories. Some other skills of theorization are more advanced because they have many pre-skills or because they develop in later stages of developing theories.

The skills of making assumptions and developing theories

1) Making assumptions based on experiments.

2) Testing assumptions and theories.

3) Generalization of approved assumptions and theories to wider scopes.

4) Recognition of relation between two assumptions or two theories.

5) Repair and surgery of assumptions and theories based on experience.

6) Comparing the strength and weakness of different assumptions and theories.

7) Searching for truth in making assumptions and developing theories.

1) Making assumptions based on experience

We have to start from some assumptions. Otherwise, we face cycles or infinite sequences of backward implications (Aristotle). Experience for physicists means that our assumptions should be disprovable by experiments (Lakatosh) and experience in mathematicians means considering important and simple special cases (Newton). The realm of assumptions should be the same as the realm of experience. There are several realms of experience (Plato). One should develop a language for each and every realm of experience so that one could state assumptions (Vigotsky).

2) Testing assumptions and theories

Testing assumptions in physics means performance of more careful experiments. Testing theories in physics means comparing implications of a theory with implications of well-established theories and their assumptions. One should design appropriate experiments to test theories or assumptions. Testing assumptions in mathematics means that one should check important special cases. Testing theories in mathematics means that one should check if implications could be interpreted by natural statements. The concept of natural statement is based on well-established theories.

3) Generalization of assumptions and theories to wider scopes

Generalization of assumptions and theories in mathematics is defined by implication. Generalization of assumptions in physics is defined by analogy. Generalization of theories in physics is defined by making a new theory which is more computationally and conceptually able than the previous one in explaining nature. By generalization one can unite the realms of two theories in physics and mathematics. There are natural barriers to generalization of assumptions and theories. Sometimes one can't unite two given theories.

4) Recognition of relation between two assumptions or two theories

In mathematics and physics recognition of relations between assumptions usually leads to unification of theories. In mathematics recognition of relations between theories forms a paradigm. In physics recognition of relation between theories is a motivation for searching for a new formulation containing both theories. In physics one is interested in the structure of relations between concepts. Two theories, for which the structures of their concepts are the same, are considered equal in the eyes of physicists. In mathematics one is interested to find relations between two theories for the further development of mathematics.

5) Repair and surgery of assumptions or theories based on experiments

In mathematics one does surgery on assumptions or theories in order to repair implications. In physics one does surgery on assumptions in order to repair analogies. Surgery of theories in physics is performed by developing a new theory in similar circumstances so that it contributes to the atlas of concept relations. In mathematics surgery and repair could lead to unification of assumptions or theories. In physics surgery and repair could lead to replacement of a formulation by a more powerful formulation of a theory.

6) Comparing the strength and weakness of assumptions and theories

In mathematics strength and weakness of assumptions are assessed by fluency and naturality of implications. In physics strength and weakness of assumptions are assessed by computational flexibility. In physics strength and weakness of theories are assessed by explaining the nature and clarifying the concept relations. Weakness of theories in mathematics is motivation for looking for a more natural framework. Weakness of theories in physics is motivation for looking for a more powerful and more flexible framework.

7) Searching for truth in making assumptions and developing theories

Truth is physics is understood by concept relations. Truth in mathematics is understood by analogies. In mathematics one compares two or three theories and find dictionaries between them in order to look for background truth. In physics one compares tens or hundreds of examples in order to look for the true concept relations. In physics concepts are defined by relations. In mathematics concepts are just a model of the truth.

Psychology of problem solving against theorization

Problem solvers race against time. They take advantage of a clean mind and try to manage their thought, language, and feelings and mathematical behavior. They try to develop intuition.

Theoriticians on the other hand, race against truth. They take advantage of a clear wisdom and try to manage the use of wisdom, different formulations, personality of theorization and mathematical qualities. They also try to develop intuition.

Motivations for problem solving and theorization

Even kids enjoy problem solving. People like to revive their childhood memories. One can manage problem solving by local considerations. Problem solving could be performed by understanding special cases which is not very abstract.

Theorization on the other hand, needs global considerations and thus develops wisdom. Theorization develops intuition better than problem solving. Theorization manages personality. Finally, theorizers are better problem solvers. Since, theorization could be used as a tool for problem solving.

Why and how problem solvers become theorizers

Personality of problem solvers motivates theorization. Intuition developed by problem solving is one of the main skills for theorization. Forming and management of personality is easier for theorizers. One has to create a language to express intuition which is the job of wisdom and theorizers could educate and manage wisdom more easily.

Some problems need development of new languages for the solution. Sometimes generalizing problems need theorization. Sometimes recognizing similarity between problems or finding dictionary between them needs generalizing mathematical structures.

The complementary role of problem solvers and theorizers

Theorizers make the framework, problem solvers check the details. Theorizers decide which problems are important and problem solvers solve them. Problem solvers show directions to theorizers. Theorizers teach problem solvers how to think about a problem. Problem solvers produce arguments and theorizers decide which arguments are illuminating and why.

Problem solving and theorization are pairs. There are pairs in physics: wave-particle duality, relativity versus quantum theory, real versus imaginary numbers, electricity versus *magnetism, dynamics versus kinematics, Bozonic-Fermionic symmetry, mirror symmetry. There are also pairs in mathematics: geometric versus algebraic thinking, continuous versus discrete thinking, local versus global thinking, categorical versus internal thinking, homology versus cohomology, boundary versus derivation, commutativity versus non-commutativity, rigidity versus deformation. There are also pairs of formulations: Lagrangian versus Hamiltonian mechanics, small-scale versus large-scale formulations of mechanics, differential forms versus vector formulation of electromagnetism, differentials versus finite differences, infinitesimals versus limits, Hopf-algebra versus geometric formulation of affine algebraic groups, geometric versus algebraic formulation of algebraic curves. Some pairs are dual pairs and some pairs are formed by analogy. Some pairs are alternatives and some have different perspectives towards the same phenomena.*

Paradigms and the role of language

There are paradigms in physics and mathematics: paradigm of wave equation, paradigm of energy, paradigm of mechanics, paradigm of center of gravity, paradigm of intersection theory, paradigm of algebraic geometry, paradigm of derivative, paradigm of integral are examples of paradigms.

Language is the ultimate tool for forming paradigms. One can't conceptually relate two theories, without joining their lingual formulation. This means that people trust language but not intuition. Because they can pretend to speak but they can't even pretend to communicate mental images directly.

Marriage of theories

Marriage of pairs gives rise to new theories. Wave-particle marriage give rise to quantum mechanics. Relativity-quantum marriage gives rise to field theory. Real-imaginary marriage gives rise to complex numbers Electricity-magnetism marriage give rise to Maxwell equations. Geometric versus arithmetic thinking give rise to algebraic thinking. Continuous versus discrete thinking give rise to fundamental theorem of calculus. Local versus global thinking give rise to superposition. Boundary versus differential gives rise to Poincare duality.

The role of father theory is to provide ideas and intuitions, to suggest formulations, to manage relations with other theories, to provide the global structure, to determine how to generalize, and to furnish the soul.

The role of mother theory it to provide and choose appropriate formulation and language, to manage internal relations between sub-theories to provide the local structures to determine how to solve problems, and to furnishe the body.