On The Concept of Pure and Applied Mathematics

Arash Rastegar

Institute for Advanced Study, Princeton, USA.
Sharif University of Technology, Tehran, Iran.

Abstract: The concepts of pure and applied mathematics seem to be fairly defined and separated ways of doing mathematics. In this paper, we show that there is a hierarchy of concepts of applied mathematics which are of different levels of abstractions. The most abstract one is a modern concept of pure mathematics which is introduced in this paper. And the last least abstract one is the well-known concept of applied mathematics. We will show that mathematics could be regarded as applied in many senses.

Introduction

Euler is the father of the concept of pure mathematics. After Euler this concept developed and changed in 19th century and in 20th century. The goal of this paper is to introduce a modern concept of pure mathematics for 21th century. The main idea is that in many research papers in pure mathematics the techniques and concepts used to do mathematics are predetermined. It is not the case that mathematicians are ready for the problem itself to suggest solution techniques fit for the problem which were not present before. Rather than that pure mathematicians usually try to attack problems that are expected to be solved by the same well known techniques as before. This is in a sense kind of applied mathematics. But this time applications of mathematics to mathematics itself. The fact is, this approach blocks innovation. Pure mathematics should be free from its own history when tackling new problems. This way of doing mathematics could contribute to the mathematical thinking more efficiently.

On the concept of application

It has always been inspiring to me reading “the usefulness of useless knowledge” by Flexner. This means that knowledge must be useful. But being useful depends on the purpose also. A general tool like a screwdriver or knife could be useful in many ways. So is mathematics. Mathematics as a science which is one of bases of all sections of knowledge must prove to be useful. The different ways to use mathematics determine the level of abstractness of the mathematics which is used. this level of abstractness could be as common as a solving everyday life problem, as mathematics was useful as such for ever, for example in art and trade and as pure as not being limited to any predetermined mathematical techniques solely for the purpose of developing cognitive structure or searching for the truth! These applications have happened through stages in history of mathematics. They make the concept of pure mathematics more and more abstract by finding more abstract applications of mathematics. Archimedes is father of application of mathematics to engineering. Although mathematics has been used before him for architecture for thousands of years before him, for example in building the Egyptian pyramids. Applications of mathematics to astronomy as a part of basic sciences was always present there but reminds us the name of Ptolemy. Although mathematical modeling of the movement of stars goes back to thousands of years before him. The concept of education as a purpose for doing mathematics and also truth finding goes back to Plato and is our final destination.

On applications of Mathematics to everyday life

Applications of mathematics in counting and measurements is the most basic use of mathematics in everyday life which was used for trade, architecture, art, tool making and even accounting of wealth. Algebraic equations appeared after religious rules for dividing the inheritance. Many algorithm of arithmetic calculations simplified computations used in trade or astronomy. These earlier stages were developed in China, India, Persia and Egypt and Greece. But there was a more pure approach to mathematics present in Greek geometry and number theory. That is why we have them today as Euclidian geometry and Diophantine equations. Although advances in Euclidean geometry contributed a lot to art and architecture, number theory remained a more or Less pure part of mathematics. The first millstone put forward for the concept of pure mathematics was the long live book of Euclid "The Elements". Pythagorian theorem was the basis for trigonometry and applied technique and also Pythagorean triples which goes back to thousands of years before Diophantine. The concept of proof was raised as the basis of this ancient concept of pure mathematics. Solving equations using geometric drawing techniques was also another incarnation of the concept of pure mathematics. But with the rise of the concept of application of mathematics to engineering solving equations become part of the new concept of applied mathematics. Quadratic equations become important after Galileo and Kepler. Although the geometry of conic sections were developed 2000 years before that in Greece.

On applications of mathematics to engineering

Application of mathematics to architecture goes back to Egypt and beyond. But it is customary to call Archimedes as the first engineer who discovered the rule of levers. This is in fact a more abstract application which goes through application of mathematics to physics and then to engineering. Therefore Archimedes is the father of mathematical physics. The first machines (moving tools) were built and used using mathematical modeling. But application of mathematics to physics were also used in many of these machines. Newton was preceded by analytic geometry of Descartes unifying Euclidean geometry and algebra, which could be also considered as the contributions of mathematics to the concept of space. Descartian coordinates helped engineering directly also by advancement of mathematical modeling. At this time also, the concept of pure mathematics was limited to revolutions in Euclidean geometry and advanced in solving new Diophantine equation. Mathematical modeling of the shape is not the only contribution of mathematics directly to engineering which is not through physics. Mathematical theory of movement was also part of engineering which eventually lead to kinematics as a branch of mechanics. Application of mathematics to everyday life did not die after appearance of applications to engineering. Same for direct applications to engineering which are even present-today. With the rise of computers, mathematical modeling of the engineering problems has gone through new stages.

On applications of mathematics to fundamental sciences

The applications of mathematics to physics started from Archimedes and became mature by contributions of Newton. Einstein came up with the idea of space and time, and Feynman came up with field theory. Quantum mechanics was mathematically formulated by Von Neuman. Mathematical biology also contributed to basic sciences by applying dynamical systems. Diverse applications of mathematics to fundamental sciences made mathematics recognized as the father of all basic sciences. Since all basic sciences have their own philosophical background and methodology, intervene of mathematics never mixed different branches of basic sciences. Soon, in 20th century many branches of mathematics itself became philosophically self-contained and one could speak of contributions of whole mathematics to development of these branches of mathematics. Applications of different branches of mathematics to a single branch never resulted in mixture of branches of mathematics, since each branch has its own philosophical methods and questions, which fairly defines and separated branches of mathematics. this brings us to another realm which we still don't recognize as pure mathematics since the methodology and skills and concepts to be dealt with are predetermined. Mathematician attacks a problem by a predetermined personality and cognitive structure. This approach is doing mathematics does not contribute to the question of what new things are mathematically understandable.

On applications of mathematics branches of mathematics

There are four main branches in mathematics: algebra, analysis, geometry and combinatorics. This division of mathematics to four main branch, goes back to 19th century. Analysis started by development of calculus by Newton and Leibniz and before that geometry and number theory were regarded as the main branches. Geometry and analysis are continuous versus algebra and combinatorics which are discrete. Geometry and combinatorics are pictorial, versus algebra and analysis which are verbal. These philosophical bases for different branches makes their definition solid and not deformable. Sub-field are formed by contributions of one field to the other like algebraic geometry and algebraic combinatorics as new sub-branches contributed to geometry and combinatorics. Non-commutative geometry should be regarded as contributions of geometry and algebra, and there are also contributions of combinatorics to algebra which are part of algebraic graph theory. This is how new branches of mathematics are formed. Analysis contributes sub-fields to geometry like complex geometry and geometry contributes sub-fields to analysis like other kinds of non-commutative geometry done by Connes. This way of doing mathematics is traditionally called pure, but in our setting it is regarded as applied mathematics, for the reasons explained in previous section.

On pure mathematics after Euler

After study of different layers of abstractness of applied mathematics, we shall study the history of the concept of pure mathematics. Of course, Greek mathematicians had pure-concerns in doing geometry and number theory and even before that. List of Pythagorean triples goes back to thousands of years. But the modern concept of pure mathematics started with Euler in eighteenth century. Euler worked on problems rooted in applied mathematics but in order to solve them he had pure concerns. For example, he studied solutions of the wave equation which is an applied problem, but he was also interested in non-continuous solutions which is a pure concern. Therefore, the eighteenth century pure mathematics was formed by pure solutions to applied problems. Finding Pythagorean triples is also pure in the same manner. (3, 4, 5)-triple was used in architecture to produce right angles. Being interested in all solutions of Pythagorean question is a pure concern which led to Diophantine equations which was an ancient field in pure mathematics. Ruler and compass geometry also had applications in architecture, and same for Euclidean and spherical trigonometry. The latter had applications in astronomy but Greek geometry was pure development of ruler and compass geometry. Muslim mathematicians also developed a dictionary between Euclidean trigonometry and spherical trigonometry which had roots in pure mathematics.

On Pure mathematics in 19th century

In 19th century, applications of mathematics to other parts of mathematics lead to another concept of pure mathematics. Analogies between different fields of mathematics was a motivation to develop the new fields of mathematics which were formed by applications of mathematics to mathematics. Although eventually we called this kind of doing mathematics applied, but in 19th century and even later this kind of doing mathematics was considered pure mathematics. An example of these analogies is development of algebraic number theory and field theory under the influence of function field arithmetic which was developed by Riemann after introduction of Riemann-surfaces. another source for the concept of pure mathematics in 19th century was building firm mathematical foundations for calculus developed by Newton and Leibniz. For 200 yours calculus was being used although it didn't have firm foundations. Therefore, one can say theorization was another concern for development of pure mathematics in 19th century. Before that problem solving was the sole reason to develop pure mathematics. Efforts of Cauchy and Weierstrass lead to new developments of calculus and efforts of Gauss, Dedekind, Kronecker, and Galois lead to new developments in number theory. efforts of Gauss, Riemann, Klein and Poincare lead to new developments in geometry. Developments in hyperbolic geometry was also based on analogies.

On pure mathematics in 20th century

In 20th century many branches of mathematics became independent in their development. They no longer lay on researchers which made them become into existence. These branches developed their own methods, skills, and philosophical foundations which became the source of their further development. Answering certain questions and solving certain problems became crucial, based on the new philosophical foundations. For example, study of the Galois group had an important role in development of algebraic number theory, another theoretical concern was developed by theoretical conjectures which guided new developments in many fields of mathematics. Poincare conjecture, Mordell conjecture, Weil conjecture, and Langland program were the forefront of developments in Manifold theory, Number theory, Algebraic geometry. These three fields were particularly rich because of such theoretical conjectures which lay the foundations for a new concept of pure mathematics. There is a saying attributed to Andre Weil which claims good mathematicians prove theorems, great mathematicians look for analogies and greatest mathematicians are concerned with analogies between analogies. This proverb explains very well the spirit of pure mathematics in 20th century. Of course study of analogies goes back to Muslim mathematicians concerned with analogies between Euclidean and spherical trigonometry, but there were important of instances in 19th century mathematics concerned with analogies. This became dominant in 20th century.

On the modern concept of pure mathematics

Here we shall suggest a modern concepts of pure mathematics for the new era of 21th century. In previous papers we have developed and introduced concept of doing mathematics for the purpose of development of cognitive structure and also for the sake of discovering of truth. We shall suggest that these concerns could lay foundations for a modern concept of pure mathematics different from the ones we already discussed. We believe that these motivations are more advanced and more abstract than those initiated from the problem solving mathematics and theorizing mathematics. The main philosophy is the point that, development of pure mathematics should contribute to cognitive structure of human race and also should discover the nature of truth by interpretation of mathematical truth. In this perspective, mathematics is a huge laboratory for making experiments trying to discover the nature of truth. This of course will have many pioneer implications in contemporary philosophy of truth and also philosophy of mind. In a way, this new concept of pure mathematics is regarding modern mathematics as applications of mathematics in development of cognitive structure and in study of the nature of truth. This adds two new layers to the hierarchy of applied mathematics towards pure mathematics.