Modeling of Interactions between Physics and Mathematics

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There is a spectrum between what mathematicians and physicists do. Moving from mathematics to physics, we find many instances in which, physics influences the way mathematicians do mathematics. In fact, there are instances that pure mathematicians practically do physics. There are mathematicians whose field of research is classical mechanics, relativity, quantum mechanics, string theory or supper-symmetry. Moving from physics to mathematics, there are many instances in which, physicists do mathematics. Physicists do differential equations, infinite dimensional group representations, operator theory, algebraic topology, algebras and operads, deformation of metric, and category theory, while doing physics. It is hard to say if mathematics imitates physics or vice versa. In fact, there is a two way relationship between them and our goal is to model the interaction between the two.

Similar developments

There are many instances in which, mathematics is developed similar to physical theories. Energy integrals are computed in circumstances where there is no concept of potential involved. Mathematical expectation is very similar to the concept of center of gravitation. Classical mechanics over fiber bundles makes no physical sense if we are not dealing with tangent bundle. Deformation quantization makes sense in situations where there is no deformation of physical theories involved.

There are also many circumstances in which physics is developed similar to mathematical theories. Although, it took some time for quantum mechanics to find the language of operator theory, quantum theory imitates operator theory. Although it waited until the end of 20th century for the idea to seriously develop in mathematics, one could say that relativity imitates deformation of metric, which is a purely mathematical idea. Gauge theory imitates fiber bundles, quantum field theory imitates cobordism theory and conformal field theory imitates algebras over operads.

Physical and mathematical invasion

When parts of mathematics are considered as parts of physics, we say that physics has invaded mathematics. Parts of the theory of and particular differential equations, infinite dimensional representations of particular groups, deformation quantization of particular spaces, quantum groups, geometric quantization, Lorenz spaces and space-time in general, and super symmetry are all mathematical theories which are considered as parts of physics.

*When parts of physics are considered as parts of mathematics**,** we say that mathematics has invaded physics. Classical Mechanics, quantum theory, solution of Einstein equations,* *solution of wave equations, super mathematics are all parts of physics which are considered as branches of pure mathematics.*

There are many paradigms in physics which have roots in mathematics. The paradigms of differentiation, integration, operators, algebras over operads, spaces, singularity all come from mathematics but have physical interpretation. On the other hand, there are many paradigms in mathematics which have roots in physical concepts. Paradigms of length, area, volume, measure, center of gravity, energy, stability and harmonicity come from physical theories, but have developed generalizations and abstractions in mathematics.

Making assumptions

Important assumptions in physics use mathematics as the language of their formulation. The role of mathematics in their formulation is to make assumptions and their implications computable. Assumptions on space and on space-time, assumptions on observers and on observables, assumptions on measurement, are all formulated in the language of mathematics.

Many important assumptions in mathematics have roots in physical interpretations. The role of physics in their formulation is to connect mathematics to nature and everyday life intuition. Definition of manifolds and geometric spaces, assumptions on geometric spaces, assumptions on Hilbert spaces, structures on algebras, assumptions on group representations are all formulated using the physical intuition.

Theorization and problem solving

There are standards of a good theory in physics. Compatibility with related paradigms, compatibility with experiments, compatibility with atlas of concepts, computability, disprovablity, relations with measurements, simplicity and the most important of all being approved by experiments are some of standards of a good theory for physicists.

There are also standards of a good theory in mathematics. Computability, illumination, relations with other theories, insightfulness, generalizability, existence of a toy theory, and simplicity experiments are some of standards of a good theory for mathematicians. Physicists and mathematicians have different theorization standards.

Physicists and mathematicians have different problem solving habits. Some of problem solving habits in physics are the following: patience, divergent thinking, criticizing conjectures, looking for equivalent formulations, fluency in working with ideas and concepts, and looking for simpler models.

Some of problem solving habits in mathematics are the following: tasting the problem, gaining personal view towards the problem, considering relations with similar theories, considering generalizations, checking special cases, performing a few steps computationally, and thinking simple. This does not mean that physicist and mathematicians do not share any problem solving skills, but we want to emphasize that they have different problem solving personalities.

Teaching habits and learning habits

Physicists and mathematicians have different habits of teaching. Teaching habits of physicists includes the following features: drawing formulas, using geometric imagination, recognition of simple from difficult, decomposition and reduction to simpler problems, handling jumps of the mind, estimating how much progress has been made, finding the trivial propositions quickly, formulating good conjectures, being creative and directed in constructions, understanding an idea independent of the context. For physicists imagination and intuition comes before arguments and computations.

Mathematicians have other habits of teaching. Deciding where to start, listing different strategies to attack the problem, mathematical modeling in different frameworks, deciding about using symbols or avoiding symbols, deciding about what not to think about, organizing the process of coming to a solution, deciding how to put down the proof, clarifying the logical structure, writing down side results, putting down the full proof after finishing the arguments, notifying important steps in form of lemmas, considering the mind of reader

Physicists and mathematicians have different habits of learning. Learning habits of physicists includes the following features: considering concept relations, comparing similar assumptions for theorization, comparing the related paradigms, criticizing conjectures, looking for equivalent formulations, obtaining fluency in working with ideas and concepts, looking for simpler models. Physicists care about atlas of concepts, paradigms of physical theories, computability, disprovability, measurements and physical intuition.

Mathematicians have other habits of learning. Questioning what are logical implications, considering relations with similar theories, considering generalizations, checking special cases, performing a few steps computationally, thinking simple, trying to solve similar problems are important features of learning for mathematicians. Mathematicians care about understanding simple cases completely, understanding the ways one finds generalizations, understanding the logical structure, finding the exact assumptions needed, managing how to solve similar problems, flexibility of computation techniques.

Contributions of one to the other

The scientific life of physicists has many contributions to the methods of mathematical thought: introduction of new mathematical concepts, motivating computations, motivating theorizations, motivating assumptions, and motivating mathematical conjectures. There are fields of research moving from physics to mathematics. Physics motivates introduction of important special cases of theories.

Mathematicians also have many contributions to the physical thought: providing mathematical formulations for physical theories, mathematical rigor, categorizing exceptions, mathematical computations, expansion of the realm of physical thought, study of the generalizations, criticism, guidance.

The method of mathematical thought has also contributions to the scientific life of physicists: rigor, postulation, logical structure, problem solving, critical thinking, abstraction, categorization.

The science of physics also has contributions to the mind of mathematicians: relation with applications, philosophical thought, intuition, computation, divergent thinking, searching for the truth, relation with nature.

Physics and mathematics form a pair

Mathematics plays the role of father and physics plays the role of mother. There are pairs inside them: Quantum theory and Hilbert spaces, relativity and space-time, classical mechanics and manifolds, dynamical systems and differential equations. Mathematics is the father. The role of father is providing ideas and intuitions, management of relations with other theories, providing the global structure, determining how to generalize. Mathematics furnishes the soul. Physics is the mother. The role of mother is providing appropriate formulation and language, management of internal relations between sub-theories, providing the local structures, determining how to solve problems. Physics furnishes the body.

Mathematical physics is the fruit of this marriage. The similarities with mathematics are the following: mathematical physics is based on geometric ideas and intuitions. Mathematics provides the global structure and determines relations with other theories. The similarities with physics are the following: Mathematical physics is based on physical formulation and language. Physics manages internal relations between sub-theories according to physical intuition and determines methods of solving problems.

A word on personality of mathematical-physicists

Mathematical-physicists have double standards: Mathematical standards criticize geometric ideas and intuitions. Management of relations with other theories is according to mathematical standards of theorization. Mathematical intuition provides the global structure and determines how to generalize. Physical standards provide appropriate formulation and language, and manage internal relations between sub-theories. Physical standards provide the local structure and determine how to solve problems.