Cognition types for mathematicians

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Some people have wrong believes about cognition types: They believe verbals become scientists; pictorials become artists; kinetics become sportsmen; wholists become philosophers; and analyzers become Scientist. To us, cognition abilities of verbals are associated to speech and hearing; and those of pictorials are associated to vision and image processing.

Verbals talk to themselves while thinking. Thoughts for them consist of a sequence of words, and words are kind of symbols representing meanings. So, verbals are good in using symbols. Pictorials think in terms of concepts and relations between them. They do not use words for thinking. They rather use an abstract form of pictures for visualization.

Wholists study systems and move from whole to parts; and analyzers start from coherence of parts and move from parts towards whole. Wholists pay attention to global aspects of the subject of cognition and try to understand the subject as a system or as a whole. Analyzers are engaged in local considerations and try to understand the subject of cognition as a collection of correlated parts which should appropriately fit together.

Statistically most of the verbals are analyzers, which is a convenient combination of cognition types. Verbals work with words and symbols, which naturally form parts which fit together to form a sentence, which plays the role of a whole. Therefore verbals naturally move from parts to whole, which is the manner of analyzers. Analyzers, on the other hand, are good in putting parts together to form a system or a whole, which is exactly what verbals do to form a sentence from words.

Statistically most of the pictorials are wholists, which is a convenient combination of cognition types. Pictorials work with images, which are naturally wholes consisting of parts. Therefore, pictorials naturally move from whole to

parts, which is the manner of wholists. Wholists, on the other hand, are good in understanding a system or a whole, which is exactly what an image is, which is what pictorials are good in dealing with.

Parallel Streams of Thought in Mathematics

Geometric thinking and algebraic thinking are parallel streams of thought in mathematics. Many ideas in mathematics can be formulated both geometrically and algebraically. Like the solution of polynomial equations of low degree. Sometimes progress in understanding the geometric picture of an idea forces us to look for a better algebraic formulation, and sometimes a generalization in the algebraic setting extends the concept of space and the realm of doing geometry.

Discrete thinking and continuous thinking are also parallel streams of thought in mathematics. Many ideas in mathematics can be formulated both discretely and continuously. Like the idea of derivative versus finite difference. Sometimes progress in understanding the discrete picture of an idea, forces us to look for a better continuous formulation, like the discrete version of Taylor series appearing before the continuous version. Sometimes our continuous understanding precedes our discrete conceptions. One can categorize mathematics as: discrete and algebraic, which we call algebra; discrete and geometric, which we call combinatorics; continuous and algebraic, which we call analysis; continuous and geometric which we call geometry.

Main Branches of Mathematics and Cognition Types

Most algebraists are verbal and analyzer. They have to be verbal to be good in working with algebraic symbols. An algebraic formula is like a sentence consisting of symbols, which are like words set aside each other. They have to be analyzer, because of the nature of the step by step proof in algebra. An algebraic proof consists of parts, which are put together to form a stream of consciousness having beginning and end, inside the realm of an order, like the realm of “time”.

Although most algebraists are verbal and analyzer, it could happen that a pictorial mind is engaged in algebra. A pictorial and analyzer fits algebra better than pictorial and wholist, because of the linear nature of arguments in algebra which is the art of an analyzer. It happens often that algebraic ideas can be

reformulated in geometric language, which has advantages for pictorials, like in algebraic geometry. Pictorials try to understand algebra by translating it to geometry.

About pictorial and wholist algebraists, we should keep in mind that they can never be as successful as natural algebraists who are verbal and analyzer in what an algebraist usually tries to do. They have to develop their own art. Verbal and wholists are natural logicians. They are also good in philosophy. These rare wholist algebraists are good in generating ideas how to attack a problem and good in further development of a theory, which are arts of wholists.

Most analysts are verbal and analyzer. They have to be verbal to be good in working with algebraic symbols. Analysis is nothing but algebra plus topology and order. An analytic formula is very much like an algebraic formula except it may have an infinite number of summands. They have to be analyzer because of the nature of the step by step proof in analysis, which is very similar to algebra. Proves in analysis also consist of parts, which are put in a linear order.

Although most analysts are verbal and analyzer, it could happen that a pictorial mind is engaged in analysis. A pictorial and analyzer fits analysis better than pictorial and wholist, because of the linear nature of arguments in analysis. It happens often that analytic ideas can be reformulated in geometric language, which has advantages for pictorials, like in geometric function theory. Pictorials try to understand analysis by translating it to geometry.

About pictorial and wholist analysts, we should keep in mind that, they can never be as successful as natural analysts, who are verbal and analyzer in what an analyst usually tries to do. They have to develop their own art. In fact it happens often that geometric ideas play a crucial role in analysis. Verbal and wholists are natural probability theorists. They are also good in philosophy. These rare wholist algebraists are good in generating ideas how to attack a problem and good in further development of a theory, which are arts of wholists.

Most geometers are pictorial and wholist. They are pictorial, because they are good in geometric intuition. They deal with images and geometric imagination, which has everything to do with vision. They have to be wholist, because of the nature of the global thinking in geometric arguments. An argument in geometry is

like a global picture, which is not linear, because it is in “space” not in “time”. Because geometers understand “space” globally, but “time” locally. This is why geometers traditionally study a fixed object not a family of objects.

Although most geometers are pictorial and wholist, it could happen that a verbal mind is engaged in geometry. A verbal and wholist fits geometry better than verbal and analyst, because of the global nature of arguments in parts of geometry. It happens often that geometric ideas can be reformulated in algebraic language, which has advantages for verbals, like in algebraic geometry. Verbals try to understand geometry by translating it to algebra.

About verbal and analyzer geometers, we should keep in mind that, they can never be as successful as natural geometers, who are pictorial and wholist. They should develop their own art. These rare geometers are good in algebraic manipulations, which is the art of verbal and good in step by step arguments, which is the art of analyzers. Pictorial and analyzer geometers are good in local geometric arguments. This is the art of doing infinitesimal geometry, which was started by Newton.

Most combinatorists are pictorial and wholist. They are pictorial because they use geometric models for combinatorial objects like graphs and permutations. Geometric models are based in vision. They have to be wholist, because of the global nature of counting in combinatorial arguments. An argument in combinatorics is based on introducing a global picture of the subject of counting, where consideration about parts come afterwards. Therefore, they move from whole to part, which is the manner of wholists.

Although most combinatorists are pictorial and wholist, it could happen that a verbal mind is engaged in combinatorics. A verbal and wholist fits combinatorics better than verbal and analyst, because of the global nature of counting in parts of combinatorics. It happens often that combinatorial ideas can be reformulated in algebraic language which has advantages for verbals, like in algebraic combinatorics. Verbals try to understand combinatorics by translating it to algebra.

About verbal and analyzer combinatorists, we should keep in mind that, they can never be as successful as natural combinatorists, who are pictorial and wholist. They have to develop their own art. These rare combinatorists are good in

algebraic manipulation, which is the art of verbals and good in dealing with cases, which is the art of analyzer. These combinatorists are often interested in algebraic combinatorics. Pictorial and analyzer combinatorists are combinatorial geometers which deal with geometry locally.

Problem Solving Versus Theorization

Wholists and analyzers have different approaches in problem solving. Analyzers are calculators, step by step thinkers, logical, linear thinkers. Wholists are strategists, global thinkers, intuitionists, divergent thinkers. Analyzers try to understand the problem by looking at important examples and special cases. Wholists try to understand the problem by considering different possible generalizations.

Wholists and analysts also have different approaches in theorization. Analyzers theorize by fitting parts in the neighborhood of each other and are weak in global formulations. Wholists theorize by global considerations, and are weak in fitting parts to each other. Analyzers theorize by considering important examples and special cases. Wholists theorize by considering generalizations.

Proving Versus Conjecturing

For verbal and analyzers proof is step by step, formal, logical, linear, either in positive or negative direction i.e. from beginning to end or vice versa, flowing in the realm of time, putting parts together to form a whole sometimes divided into cases, which are formulated verbally, formally breakable to propositions and lemmas, sometimes computational, sometimes using formal ideas in mathematical logic, sometimes non-coherent, constructive, partly non-translatable to a flow of ideas and concepts, which fails to give a global picture.

For pictorial and wholists proof is translatable to flow of ideas and concepts, nonlinear, global, dividing whole to parts which hardly fit together exactly, intuitive, sometimes non-constructive, sometimes divided into cases which are formulated geometrically, of a coherent nature, formulated in a single geometric language, giving a perspective towards importance of the result, introducing a perspective towards the geometry of concepts circling around the proof and influencing the atlas of concepts.

For verbal and analyzers conjecturing is a formal art, which has several pre-skills: Reinterpreting an algebraic expression; Reproducing a new formal language by reformulating a series of related ideas and concepts; Reproducing a new formal language to connect two flows of computations; Using formal language to introduce new concepts; Formal generalizations; Development of atlas of concepts.

For pictorial and wholists conjecturing is an intuitive art, which has several pre-skills: Reproducing a new geometric language reformulating a series of related ideas and concepts; Reproducing a new geometric language to connect two global geometric pictures; Using geometric objects to introduce new concepts; Intuitive generalizations; Geometrization of formal phenomena; Using analogies and dictionaries.

The Art of Writing Mathematics

Wholists are not good writers: Wholists start from writing titles of sections. Each paragraph is a set of global generalities. Most sentences can be omitted without attracting any attention. Sections hardly match and fit together. Proofs lack computational details. Proofs are not computationally complicated. Propositions and lemmas do not completely fit. Intuition works in price of rigor.

Analyzers are good writers: The logical structure of paper is firm. The paper has a linear structure. Sections, propositions and lemmas fit completely. Rigor and outmost generality forces analyzers to miss the nicest formulations. Introduction of their papers fail to give a global perspective and summarizes the linear structure of details. The paper does not have a global structure.

Verbal Mathematicians versus Pictorial Mathematicians

There are people for whom the verbal and pictorial skills none overcome the other. This is strength. They are both rigorous and intuitive. They are both formal and geometric. They are good in computations and good in geometrization. They translate fluently between the geometric and algebraic languages. They translate fluently between discrete and continuous formulations.

Verbals who become pictorial have moved towards perfection. They move to intuition after passing through rigor. They move towards geometrization after passing through algebrization. They find a global picture after being concerned

with local information. They translate to the language of concepts and ideas after performing computations.

Pictorials who become verbal have also moved towards perfection. They move from intuition to rigor. They move towards algebrization after passing through geometrization. They deal with local information after finding a global picture. They deal with the language of concepts and ideas before performing computations. Pictorials who become verbal fly higher than verbals who become pictorial.

Wholist Mathematicians Versus Analyzer Mathematicians

There are people for whom the wholistic and analytic skills none overcome the other. In conjecturing they benefits from both sides, which is strength. In organizing proofs they can't decide to start from whole or parts. In theorization they can't decide between global formulations or step by step approach. In writing papers they can't decide between the two approaches of wholists and analysts. These are weaknesses.

Wholists who become analysts have also moved towards perfection. They start as wholists and review as analyzers. They start with intuition and review by rigor. They deal with local information after finding a global picture. They deal with the language of concepts and ideas before performing computations. Wholists who become analyzers are usually pictorials who become verbal.

Analyzers who become wholists have also moved towards perfection. They start as analyzers and review as wholists. They start with rigor and review by intuition. They find a global picture after dealing with local information. They deal with the language of concepts and ideas after performing computations. Analyzers who become wholists are usually verbals who become pictorial.

Verbal-wholists are natural philosophers. They may be interested also in physics because of their interest in philosophy. Not all philosophers are verbal. For example: Eastern philosophy. Not all philosophers are wholist. For example: Analytic philosophy. Verbal-wholists are rare creatures, since most verbals are analyzers. Verbal-wholists naturally move towards becoming pictorial and analyzer.

Pictorial-analyzers are natural combinatorists. Not all combinatorists are pictorial. For example: Algebraic combinatorists. Not all combinatorists are analyzers. For example: Combinatorial counting. Pictorial-analyzers are rare creatures, since most pictorials are wholists. Pictorial-analyzers naturally move towards becoming verbal and wholist.

Doing Mathematics and Communicating Mathematics

Mathematicians with different cognition types think differently about doing mathematics. For pictorials doing mathematics is a philosophical, theorizing, conceptual activity. For verbals doing mathematics is a computational, problem solving, formal activity. For wholists doing mathematics is a global, encyclopedic, diverse activity. For analyzers doing mathematics is a local, particular, converging activity.

Mathematicians with different cognition types think differently about mathematical communication. For pictorials mathematical communication is an intuitive, pictorial, nonlinear activity. For verbals mathematical communication is a rigorous, verbal, linear activity. Wholists communicate with almost all mathematicians. Analyzers communicate with experts in fields related to their research.

Developing Mathematics and Perspectives towards Mathematics

Mathematicians with different cognition types behave differently in developing mathematics. Pictorials develop mathematics by relating different research fields. Verbals develop mathematics by reformulating formal languages. Wholists develop mathematics extending paradigms. Analyzers develop mathematics by deeply understanding local implications and local structure of their research field.

Mathematicians with different cognition types have different philosophical perspectives in mathematics. For pictorials mathematics has several layers of abstractness, which are correlated. For verbals mathematics has a single layer and all mathematical activities are at the same level of abstractness. For wholists mathematics is about analogies and dictionaries. For analyzers mathematics is about proofs and computations.

Mathematics Education and Cognition Types

Mathematicians with different cognition types have different behavior in mathematics education. Pictorials try several different approaches to mathematics education. Verbals experience and develop a single approach in education of mathematics. Wholists start from theorems and move towards examples. Analyzers start from examples and extract theorems from important examples.